Electoral competition with endogenous leadership

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Abstract

We study electoral competition as a timing game. Two candidates who differ in their valences and competence run for office. Competence is the probability that the winner implements her policy platform after the election. In continuous time, candidates do not move simultaneously because under simultaneous moves, there only exists a mixed strategy Nash equilibrium. This creates an incentive to wait to observe the outcome of the rival's randomization—a type of second-mover advantage. We obtain a unique subgame perfect Nash equilibrium in which the stronger candidate leads. Her platform is distorted towards the status quo policy, thus offering a new explanation for status quo biases. We also analyze leadership in a variant of the game with an incumbent and a challenger.

Keywords: timing, continuous-time game, second-mover advantage, electoral competition, status quo bias

JEL classification: C72, C73, D72

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1 Introduction

We study electoral competition where candidates can choose when to announce their policies. Much economic analysis assumes an order of moves, but leadership arises endogenously in our setting. When a static game having only a mixed strategy Nash equilibrium is "adapted" to a continuous-time dynamic setting, waiting for a split second to observe the outcome of the rival's randomization is generally more profitable than moving simultaneously with her. We thus find that simultaneous-move mixed strategy equilibria in a static setting convert to sequential pure strategy equilibria in the continuous-time setting. This general insight is widely applicable.

Our game has two candidates competing for office, who differ in their appeal to the voters (their "valence"). It is well-known that in static settings, such games often have only a mixed strategy Nash equilibrium.[1] We add a novel feature to this game by assuming that candidates can differ also in their competence. Conditional on winning the election, a more competent candidate is more likely to implement her announced platform rather than simply maintaining the status quo. Competence offers a new explanation for why the status quo can affect electoral competition.

Before converting the game to continuous time, we first study a static version with an exogenous order of moves. This analysis is an input for the later generalization to a continuous-time game. Under sequential moves with an exogenous order, each candidate has a second-mover advantage: following delivers a higher payoff to a candidate than leading. Moreover, the second-mover advantage is greater for the weaker candidate, who (apart from special cases) loses with certainty as the leader, but wins with positive probability as the follower. The stronger candidate, by contrast, wins with positive probability both as the leader and as the follower. If the stronger candidate leads, she generally chooses a policy platform near the center of the policy space, but distorted towards the status quo. If the weaker candidate leads, the stronger candidate can win the election with certainty by matching the leader's policy platform. This explains why the weaker candidate has a particularly strong second-mover advantage.

Results differ if both candidates have the same valence but one candidate is more competent. Here, if the weaker candidate leads and adopts the status quo policy as her platform, the other candidate loses her competence advantage: by matching the leader's platform, both candidates maintain the status quo policy after the election, so competence does not matter. In this case, the weaker candidate still has a second-

[1] When matching her rival's policy platform, the candidate with higher valence is unambiguously preferred by voters and thus elected with certainty. The weaker candidate must thus differentiate themself. The strategic situation resembles anti-coordination games such as Matching Pennies.

mover advantage, but she can win with positive probability even when leading.

We further show that the game generally does not have a pure strategy Nash equilibrium under simultaneous moves.[2] Candidates' mixing over policy platforms then creates a strong incentive to deviate from simultaneous moves. This explains why in the continuous time version of the game (analyzed in Section 4), where each candidate is free to decide about her policy platform and about the timing of this choice, only sequential subgame perfect Nash equilibria (SPNE) in pure strategies exist.

To transform the game to continuous time, we add a simple time-dependency to candidates' payoff functions, reflecting their preference for early announcements. The only incentive to wait in the dynamic setting is to enjoy the second-mover advantage, following the leader's move immediately after observing it. Determining the identity of the leader is one of our main goals in the dynamic setting.

To solve the game in continuous time, we use the machinery developed in our companion paper (Karp et al., 2025). There, we provide a set of general assumptions useful for analyzing timing games, and establish the relation between continuous-time games and their discrete-time counterparts.[3] Our theorems show that results in the continuous-time setting are not artifacts of the special properties of continuous time. Instead, the results are shared by "nearby" games in discrete time. Here we extend that theory by providing equilibrium results for games with a second-mover advantage.

We use these results to solve the electoral competition game in continuous time. If one candidate is stronger in both characteristics (valence and competence), then this candidate leads in the unique SPNE.[4] The other candidate follows this move immediately and obtains her second-mover advantage but nonetheless has a lower probability of winning the election than the leader. Just as in the static game, when the stronger candidate leads, the leader's policy in the dynamic game is distorted towards the status quo. The stronger candidate chooses a position in the policy space that would have enabled the weaker candidate to appeal to the (expected) median voter. This strategy forces the weaker candidate to resort to a more extreme platform as the follower, to win the election under more extreme realizations of the median voter's bliss point. Endogenizing the order of candidates' moves, along with their policy platforms, thus

[2]There is only one special parameter configuration where a pure strategy equilibrium exists under simultaneous moves. When adapting the electoral competition game to continuous time, this special configuration is ruled out by assumption.

[3]Ambrus and Lu (2015) introduce a continuous-time framework for coalitional bargaining and show that Markov perfect equilibria are the only SPNE of their model that can be approximated by SPNE of nearby discrete-time bargaining models.

[4]This holds for our specification of time dependence used in Section 4. As we argue in Section 3.2, there are many ways in which a static game can be adapted to continuous time. Obviously, results will depend on this. We use a parsimonious setup that remains "close" to the static game.

offers a new explanation for a “status quo bias”.[5]

We also analyze a variant of our dynamic electoral competition game with an incumbent and a challenger. Whereas the incumbent is always a candidate, the challenger can only win the election if he announces a policy platform. Because that is costly, the challenger only joins the fray if their fixed costs are sufficiently small. By contrast, the incumbent can win the election also if she maintains her default policy. Provided that the challenger’s fixed costs are sufficiently small, the incumbent generally leads at the beginning of the game and the challenger follows immediately by announcing a platform. Otherwise, nobody announces a (new) platform and the challenger stays out. Under some conditions, leadership may also be reversed so that the challenger leads late in the game. This variant illustrates how candidates’ strategies depend in a non-trivial way on time. In some subgames, the challenger’s strategy is to announce a platform that just deters the incumbent from announcing a new platform.

The key takeaway is that imposing simultaneous moves on players can be seriously restrictive, unless there are specific reasons why players *must* move simultaneously.[6] In our electoral competition game, candidates never choose to move simultaneously if they are free to decide when to announce their policy platforms. Under simultaneous moves, only a mixed strategy equilibrium exists; but each candidate would rather deviate from her own mixed stragegy, waiting a split second to observe the outcome of her rival’s randomization. Endogenizing the order of players’ moves in continuous time turns out to be fairly simple, based on our general results for continuous-time games with a second-mover advantage and results from the static electoral competition game. Our main results depend only on some general properties of players’ payoffs, and are insensitive to functional form assumptions.

Related literature

Our paper uses methods of continuous time modeling for the game-theoretic analysis of electoral competition. It thus combines different strands of literature.

Our electoral setting assumes that candidates do not know the “ideal” policy of the median voter when they choose their policy platforms. Candidates are purely office-motivated, each seeking to maximize the probability of her election. When the

[5]Fernandez and Rodrik (1991) show that a status quo bias arises when the individual gainers from reform cannot be identified beforehand, explaining resistance against policies that on aggregate are efficiency-enhancing. See also Samuelson and Zeckhauser (1988) for experimental results on status quo biases.

[6]With penalty kicks, the goalie typically cannot “wait an instant” to condition her response on the kicker’s action, so here a mixed strategy equilibrium is sensible.

election takes place, the uncertainty is resolved and voters behave rationally. Duggan (2005) calls this the stochastic preference model.

"Valence" is another well-known feature from the literature. When candidates differ in their valences, the stronger candidate can win the election with certainty despite not knowing the median voter's ideal policy, simply by matching the weaker candidate's platform. The weaker candidate must thus position himself away from the stronger candidate to win with a positive probability. This explains why the timing of candidates' policy announcements is crucial: the weaker candidate, who loses with certainty when leading, has a strong second-mover advantage. When simultaneous moves are imposed on the candidates (regardless of their preferences) typically only mixed strategy equilibria exist (Aragones and Palfrey, 2002; Groseclose, 2001; Hummel, 2010). Ashworth and Bueno de Mesquita (2009) include uncertainty about both the median voter's ideal point and the candidates' valence.

The irrelevance of the status quo for candidates' strategic positioning in the policy space limits most models of electoral competition. Persson and Tabellini (2000, p.151) write: *"A more bothersome feature ... is the unimportance of the status quo. In these models of electoral competition, and in particular in the median-voter model, history plays no role."* Introducing competence as a characteristic makes the status quo policy relevant. Voters evaluate candidates not only by their announced policies, but also consider the probability that a winning candidate will deliver on their promises.

Few papers have studied the effects of competence in electoral competition. Miller (2011) comes closest to our approach; his "effectiveness" corresponds to our "competence", and he also includes valence. However, Miller (2011) considers the extreme case of purely policy-motivated candidates with strong biases. Abstracting from uncertainty about the median voter's bliss point, for most parameter constellations, the stronger candidate wins the election. The weaker candidate's sole interest is to moderate the stronger candidate's policy choice. The weaker candidate thus positions herself at the median voter's bliss point whereas the stronger candidate chooses a position as far away from the median voter's bliss point as possible while still winning the election.

Desai and Tyson (2025) combine valence and competence into one characteristic called "capability". Capability affects the probability that a winning candidate implements her policy platform, similar to "competence" in our model; but voters also attach intrinsic value to capability, much as they do to valence. As in Miller (2011), the candidates in Desai and Tyson (2025) are purely policy-motivated. They find that in equilibrium both candidates choose an expected policy more moderate than their ideal point, with the weaker candidate moving closer to the center than the stronger candidate. Gouret and Rossignol (2019) model competence as a multiplier on the voter's (dis)utility for a candidate. As with our approach, a voter prefers the winning candidate to have low competence when her policy platform is far from the voter's ideal policy. However, Gouret and Rossignol's (2019) model does not contain a status quo.

To the best of our knowledge, using a continuous-time setup to analyze the timing of candidates' policy announcements in electoral competition, is novel. We apply the framework developed in our companion paper, Karp et al. (2025), which builds on Simon and Stinchcombe (1989). Our simplifying restriction to two players, each of whom can move at most once, is suitable for applications such as price or electoral competition. However, our generalization of Simon and Stinchcombe's (1989) setup that allows for infinite action sets, such as intervals, expands the set of potential applications, as our electoral competition game highlights.[7] We provide conditions under which agents do not move simultaneously, leaving only sequential move equilibria.[8]

Continuous time offers an important advantage over discrete time for analyzing timing games: the former allow the possibility of moving "sequentially yet at the same time" (Simon and Stinchcombe, 1989). The second-mover can follow the leader's move immediately, with no delay. Players' action choices as leader and follower typically differ from their action choices under simultaneous moves, because the follower can condition her action choice on the leader's action, and thus exploit her second-mover advantage. In a discrete-time model, by contrast, it is generally not possible to obtain a sequential pure strategy equilibrium with *impatient* players. The follower would strictly prefer to deviate, implementing her best response to the leader's action simultaneously rather than sequentially. Thus, only simultaneous actions choices (as in a simultaneous-move game) can be rationalized in the discrete-time setting.

However, simultaneous move equilibria are often implausible. As explained earlier, in games that do not have a pure strategy Nash equilibrium under simultaneous moves, each player prefers to wait and observe the outcome of her rival's randomization, rather than move simultaneously with her. The continuous-time model, where players are free to decide when to move and what action to choose, overcomes these technical issues. Our framework allows for subgame perfect Nash equilibria in which players move simultaneously or sequentially. The non-existence of a pure strategy simultaneous move Nash equilibrium then explains why we obtain only sequential equilibria in the

[7] Simon and Stinchcombe (1989) assume that action sets are finite. Our companion paper, Karp et al. (2025), analyzes different variants of price competition to illustrate the general applicability of our continuous time machinery for analyzing games with infinite action sets.

[8] The non-existence of a pure strategy Nash equilibrium when players are compelled to move simultaneously (as in our electoral competition game) is only one possibility to rule out simultaneous moves in continuous time; we offer alternative conditions that lead to the same result.

dynamic electoral competition game.

The literature on continuous-time games is growing. Other authors have used such settings to study timing games, but usually without allowing players to also choose actions from a non-trivial set, e.g., Hoppe and Lehmann-Grube (2005), Hendricks, Weiss, and Wilson (1988). Other authors use continuous-time settings with inertia, e.g., Bergin and MacLeod (1993), Calford and Oprea (2017), Park and Xiong (2024). We do not constrain players' ability to choose the timing of their moves, so we abstract from inertia. Kamada and Sugaya (2020) endogenize policy platforms and their timing in electoral competition games where candidates' opportunities to modify their positions arrive stochastically.

Earlier approaches to endogenize the order of moves in two-player games include Hamilton and Slutsky (1990) who use a two-period setting. Because the discrete-time setup cannot accommodate impatient players, these authors abstract from discounting and other sources of impatience. Other authors use risk dominance (Harsanyi and Selten, 1988) to endogenize the order of moves in two-period applications. Examples include van Damme and Hurkens (1999, 2004) who study quantity and price competition games. Our approach using continuous time is computationally simpler, and does not rely on specific functional form assumptions. This simplicity and generality highlights the benefits of using continuous-time modeling for games that have traditionally been analyzed by imposing an order of moves.

The remainder of this paper is organized as follows. Section 2 introduces our electoral competition game and analyzes it for different timing regimes (with an exogenous order of moves). Section 3 summarizes our continuous-time framework and presents results for games with a second-mover advantage. In Section 4, we adapt our electoral competition game to continuous time, and use our results from Section 3 to solve the game. A variant of our dynamic electoral competition game with an incumbent and a challenger is analyzed in Section 5. Section 6 concludes. Proofs and more general results on continuous time games with a second-mover advantage are in the Appendix.[9]

2 Electoral competition game

The median voter's ideal policy is a random variable $l \sim U[-1, 1]$. Candidate $i \in \{A, B\}$ announces policy $x_i \in \mathbb{R}$ before learning the realization of l. If i wins she implements her announced policy with probability $p_i > 0$, and retains the status quo

[9]The Online Appendix considers additional cases, discusses mixed strategies, and contains a preliminary welfare analysis.

policy $-1 < s < 1$ with probability $1 - p_i$; p_i measures the candidate's competence. If i wins, voters obtain a utility premium v_i, unrelated to the policy; v_i is i's valence.

The median voter, who knows her ideal point, obtains the expected utility

$$u_l(i, x_i) = -p_i(l - x_i)^2 - (1 - p_i)(l - s)^2 + v_i$$

from choosing candidate i. The candidate who gets this agent's vote wins the election. The voter chooses candidate i if $u_l(i, x_i) > u_l(j, x_j)$, $j \neq i$. In case of a tie, the voter chooses candidate A with exogenous probability $\alpha \geq 1/2$.[10]

Candidates are purely office-motivated, so candidate i's payoff equals her probability of winning the election, denoted π_i. This model is tractable in the dynamic environment where candidates decide when to announce their policy platforms. First, however, we analyze the game under three exogenous timing regimes: either A or B leads or they choose their platforms simultaneously. In the latter case, the game often does not have a pure strategy equilibrium.

We begin with the median voter's decision problem. If this voter chooses candidate i, the expected policy is

$$a_i \equiv p_i x_i + (1 - p_i)s \Leftrightarrow x_i = \frac{a_i - (1 - p_i)s}{p_i}. \tag{1}$$

To simplify the algebra, we use a_i instead of x_i as the strategic variable[11]. We refer to a_i as both candidate i's *action* and as her expected policy (conditional on winning), whereas x_i is i's *policy platform*. With this transformation, the utility of the median voter who chooses candidate i is:

$$U_l(i, a_i) = -2(l - s)(s - a_i) - \frac{(s - a_i)^2}{p_i} - (l - s)^2 + v_i. \tag{2}$$

We adopt the following notation: $\Delta a \equiv a_A - a_B$, $\Delta v \equiv v_A - v_B$, $\Delta p \equiv p_A - p_B$, and $\Delta U_l \equiv U_l(A, a_A) - U_l(B, a_B)$. Additionally, let

$$q_i \equiv -\frac{(s - a_i)^2}{p_i}$$

be the "competence-weighted effect" of candidate i's expected policy distance from the status quo s. We also define $\Delta q \equiv q_A - q_B$; Δq is the difference between candidates'

[10] The assumption $\alpha \geq 1/2$ is reasonable given that A is the stronger candidate, and it reduces the number of case distinctions in the proof of Proposition 2 (below). Results remain qualitatively unchanged if the assumption is relaxed, as shown in Online Appendix D.1.

[11] Note that by $x_i \in \mathbb{R}$, a candidate can choose any $a_i \in \mathbb{R}$ and specifically any $a_i \in [-1, 1]$.

competence-weighted effects. For $a = a_A = a_B$ we have:

$$\Delta q|_{\Delta a=0} = \frac{\Delta p}{p_A p_B}(s-a)^2. \tag{3}$$

Our definitions imply

$$\Delta U_l = v_A - v_B - \frac{(s-a_A)^2}{p_A} + \frac{(s-a_B)^2}{p_B} + 2(l-s)(a_A - a_B). \tag{4}$$

This utility difference can be simplified to

$$\Delta U_l = \Delta v + \Delta q + 2(l-s)\Delta a. \tag{5}$$

The preference of the voter with ideal point l between the two candidates depends on the candidates' valences, competence-weighted effects, and location effects. Higher valence is always an advantage to a candidate. The effect of greater competence depends on the relation between the candidate's position and the status quo. In the special case where the candidates announce the same expected policy ($a_A = a_B$), equation (3) implies that greater competence makes a candidate more attractive.[12]

Provided that $\Delta a \neq 0$, from equation (5), we obtain the critical point $\bar{l}$ at which the voter is indifferent between the two candidates:

$$\bar{l}(a_A, a_B) = s - \frac{\Delta v + \Delta q}{2\Delta a} \tag{6}$$

$$= s - \frac{p_A p_B(v_A - v_B) - p_B(s-a_A)^2 + p_A(s-a_B)^2}{2p_A p_B(a_A - a_B)}. \tag{7}$$

Suppose that the indifferent median voter is "interior": $-1 < \bar{l}(a_A, a_B) < 1$.[13] If $\Delta a > 0$, then we call A the rightist candidate and she wins with probability $\pi_A(a_A, a_B) = \frac{1}{2}(1 - \bar{l}(a_A, a_B))$. If instead, A is the leftist candidate ($\Delta a < 0$), then A's winning probability is $\pi_A(a_A, a_B) = \frac{1}{2}(1 + \bar{l}(a_A, a_B))$.[14]

If $\Delta a = 0$, equation (5) simplifies to

$$\Delta U_l|_{\Delta a=0} = \Delta v + \Delta q|_{\Delta a=0}. \tag{8}$$

[12]With $a_A = a_B \neq s$ the expected policy is the same under both candidates, but the policy platform of the more competent candidate is necessarily closer to the status quo. Therefore, the variance of the policy is smaller when the more competent candidate wins. Thus, the preference for the more competent candidate reflects the voter's implicit risk aversion.

[13] If the solution to equation (6) does not satisfy $-1 \leq \bar{l} \leq 1$, then it is understood that $\bar{l}$ takes a boundary value. The proof of Proposition 1 uses this convention.

[14]We use the abbreviated form π_A when there is no risk of confusion.

Hence, if one candidate dominates in both valence and competence (i.e., $\Delta v > 0$ and $\Delta p > 0$) and in addition $\Delta a = 0$, the voter prefers the dominant candidate for all l. If each candidate dominates in a different characteristic (and $\Delta a = 0$), then the voter prefers candidate A if $\Delta v > -\Delta q|_{\Delta a=0}$.

We emphasize the case where candidate A is dominant in both competence and valence. There we obtain a clear ranking of candidates' incentives to lead or follow, facilitating our later dynamic analysis. We also consider the special cases where candidates differ only in their valence ($\Delta p = 0$) or only in their competence ($\Delta v = 0$). Online Appendix D.2 examines the mixed case where each candidate dominates in one of these characteristics.[15]

2.1 Case: $\Delta v > 0$ and $\Delta p > 0$

With $\Delta v > 0$ and $\Delta p > 0$, A is more competent and has higher valence, and therefore is the stronger candidate. To exclude the extreme case where A wins with certainty both as the follower and as the leader, we assume

$$\Delta v < p_B \left(1 - \frac{\Delta p}{p_A} s^2\right). \tag{9}$$

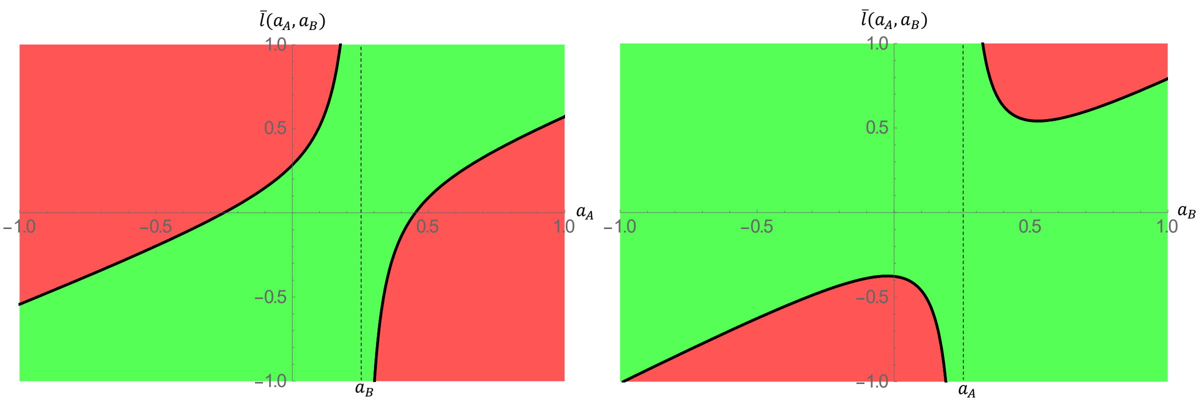


Figure 1: Black curves show the critical $\bar{l}(a_A, a_B)$, for $p_A = 0.8$, $p_B = 0.6$, $s = 0.5$, $\Delta v = 0.1$. The left panel holds $a_B = 0.25$ fixed and varies a_A. The right panel holds $a_A = 0.25$ fixed and varies a_B. For a given policy pair, 1/2 times the height of a region equals the winning probability for A (green) and B (red).

Figure 1 helps to visualize the difference between the candidates' incentives, holding fixed $\Delta v, p_A, p_B, s$. In both panels, the black curves show the graphs of the critical

[15] This case can also be analyzed in continuous time, but requires complex case distinctions due to the conflicting effects of competence and valence.

voter's position, $\bar{l}(a_A, a_B)$. The left panel shows these graphs for fixed $a_B = 0.25$ as a_A varies, and the right panel shows the graphs for fixed $a_A = 0.25$ as a_B varies. Given an action pair, one-half the height of a region (green for A and red for B) equals the probability that the agent wins. The figure shows that candidate A has an incentive to move close to her rival, whereas B does better by moving away. This difference is key to understanding why B's payoff (her election probability) is zero if she leads, and positive if she follows.

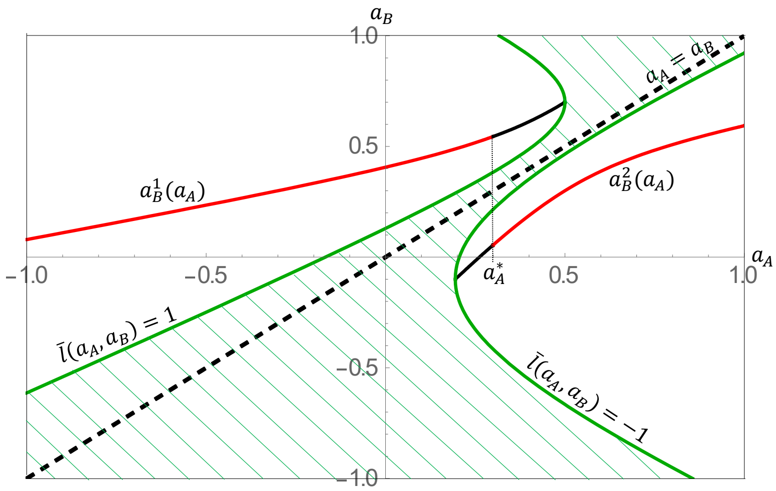


Figure 2: Best response correspondences, for $p_A = 0.8$, $p_B = 0.4$, $s = 0.5$, $\Delta v = 0.1$. Candidate A wins with probability 1 for policies in the green striped area. The red curves show B's best response correspondence, which is discontinuous and multi-valued at $a_A = a_A^*$. On these curves, B has positive probability of winning.

The following proposition describes the best response correspondences and the equilibria under different timing assumptions. Figure 2 illustrates the results.

Proposition 1. *Assume that $\Delta v > 0$, $\Delta p > 0$, and Inequality (9) holds. (i) Candidate A's best response correspondence is a closed interval that includes $a_A = a_B$ (the striped green area in Figure 2). B's best response correspondence is single valued except at the discontinuity point $a_A^* \equiv (1 - p_B)s$, where there is a downward jump (the red curves in Figure 2). (ii) As follower, A wins with certainty, regardless of B's choice. As leader, A chooses $a_A = a_A^*$ and B wins with positive probability, which is less than A's winning probability. (iii) The candidates' best response correspondences are disjoint, so there exists no pure strategy Nash equilibrium under simultaneous moves.*

Candidate A wins with certainty when following B's announcement, simply by choosing a position close to B's. Inequality (9) ensures that at A's optimal action as leader, $a_A = a_A^*$, B wins with positive probability by following optimally. Because

$a_A = a_A^*$ minimizes B's winning probability, B has a still higher probability of winning for any other choice of a_A. Therefore, B's best response correspondence lies outside the set where A wins with certainty; that set is A's best response correspondence. This disjointness of best response correspondences implies that a pure strategy Nash equilibrium does not exist under simultaneous moves.

Part (ii) notes that A's winning probability is greater than B's, even when A leads. Thus, the intrinsic electoral strength associated with $\Delta v > 0$, $\Delta p > 0$ more than offsets the loss in the second-mover advantage.

If the stronger candidate A leads, her optimal action $a_A^* = (1 - p_B)s$ corresponds to the policy platform $x_B = 0$, the policy that maximizes the *other* candidate's appeal to the expected median voter (if B were the sole candidate).[16] The equilibrium policy platform of candidate A, x_A^*, corresponding to her equilibrium action a_A^*, moves away from 0, towards s; that is, it is "distorted towards the status quo policy".[17] As follower, B wants to differentiate herself from A. As leader, A takes advantage of this incentive by choosing the platform corresponding to $x_B = 0$, thereby inducing B to move away from what would otherwise be B's favored platform.

2.2 Two limiting cases

Here we consider two limiting cases: (i) a simple case where $\Delta v > 0$ with $\Delta p = 0$ and (ii) the more complex situation where $\Delta v = 0$ with $\Delta p > 0$. These cases highlight the differing roles of competence and valence. The first case is closer to existing literature, but we consider it under different timing regimes.

Remark 1. When $\Delta v > 0$ with $\Delta p = 0$, the equilibrium is qualitatively the same as in Proposition 1 (which assumes $\Delta v > 0$ with $\Delta p > 0$). In the special case where $p_A = p_B = 1$, the equilibrium does not depend on the status quo policy; in this case, A chooses the political center $a_A^* = 0$ when she leads.

The anti-coordination structure of the game remains even when $\Delta p = 0$. As noted above, previous models have been criticized because their equilibria do not depend on the status quo, and in that respect are independent of history. The Remark shows that in a more general model the status quo s does affect the equilibrium unless both candidates are perfectly competent ($p_A = p_B = 1$).

[16] Conditional on B being able to implement her announcement, $x_B = 0$ is the ideal point for the expected median voter. If B is not able to implement her announcement, that announcement does not affect the outcome.

[17] Using equation (1), A's equilibrium policy is $x_a^* = \frac{\Delta p}{p_A} s$. The distortion towards the status quo thus only vanishes in the special case where $p_A = p_B$, i.e., candidates are equally competent.

The second limiting case eliminates A's valence advantage (setting $\Delta v = 0$) but retains her competence advantage ($\Delta p > 0$). In this case, a potential complexity arises. Namely, a candidate may prefer to choose an action as close as possible to her rival's, but without matching it. We then say that the candidate "shades" her rival's action. As Tirole (1988, p.234) points out (in the context of a Bertrand game with constant but asymmetric marginal costs), an equilibrium may then fail to exist "in a strict sense", because "shading" (or "undercutting") the rival's action by ϵ is mathematically not well-defined. In our game, this problem arises if either (i) A leads, $s = 0$, and $\alpha > 1/2$; or (ii) B leads, $s \neq 0$, and $\alpha < \frac{1+|s|}{2}$. In each of these cases, the follower shades the leader's action choice (see the proof of Proposition 2 for details).

In spite of this complication, we are able to establish the following result (recall that α is the exogenous probability that A wins in the event of a tie):

Proposition 2. *(i) In the electoral competition game with $\Delta v = 0$ and $\Delta p > 0$, there exists a pure strategy Nash equilibrium under simultaneous moves if and only if $s = 0$ and $\alpha = 1/2$. (ii) If A leads they choose $a_A = a_A^*$, and B wins with positive probability. (iii) If B leads they choose $a_B = s$, and B wins with positive probability unless $\alpha = 1$. (iv) Each player obtains a higher winning probability as follower than as leader (second-mover advantage).*

The important difference, relative to the case where $\Delta v > 0$, is that B wins with positive probability also as the leader, unless $\alpha = 1$ (Proposition 2.iii). However, B still does better as the follower, and A's behavior as leader is unchanged. Apart from a parameter set of measure zero ($s = 0 \wedge \alpha = 1/2$, Proposition 2.i), there is still no pure strategy Nash equilibrium under simultaneous moves.

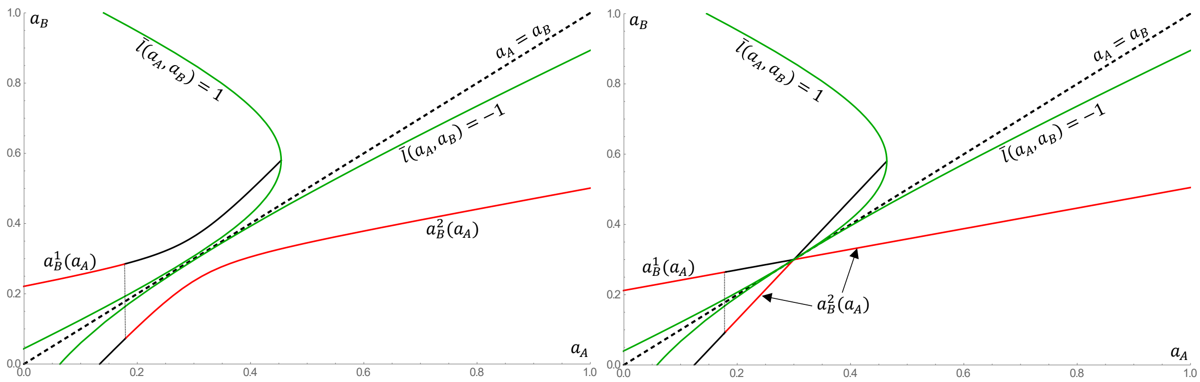


Figure 3: Combinations of a_A and a_B where $\bar{l}(a_A, a_B) = 1$ resp. $\bar{l}(a_A, a_B) = -1$ (green curves), and B's best response correspondence (red curves), for $p_A = 0.8$, $p_B = 0.4$, $s = 0.3$. Left panel: for $\Delta v = 0.01$. Right panel: for $\Delta v = 0$.

Figure 3 illustrates this situation, using $s = 0.3$. The left panel corresponds to

$\Delta v = 0.01$ and the right panel corresponds to $\Delta v = 0$. The green curves in both panels show the loci where $\bar{l} = \pm 1$. These are tangent to the 45° line (and thus to each other) at $a_A = a_B = s$ for $\Delta v = 0$. The red curves in both panels show the two arms of B's best response correspondence, with the discontinuity at $a_A^* = (1 - p_B)s$. The left panel, corresponding to the case in Proposition 1, shows these arms bounded away from the 45° line, as is also the case in Figure 2. The right panel shows the lower arm tangent to the 45° line and to the two $\bar{l}$ loci at $a_A = a_B = s$.[18] This difference causes the qualitative changes described above.

For all $a_A = a_B \neq s$, A wins with certainty. At such a point, the candidates implement the same expected policy, but the variance of the outcome is smaller under the more competent candidate A (see Footnote 12). The median voter then prefers the more competent candidate. This advantage disappears at $a_A = a_B = s$, so here A wins with the exogenous probability α.

It appears from Figure 3 that $a_A = a_B = s$ is a pure strategy Nash equilibrium. However, if for example A takes $a_B = s$ as given, A has the option of matching B's action and winning with probability α, or "shading" B's action by adopting a position close to but not equal to s. B has the same opportunity if they take $a_A = s$ as given. The proof of Proposition 2 shows that one of the two agents always has an incentive to defect from $a_A = a_B = s$ unless $s = 0$ and $\alpha = 0.5$—the only parameter combination at which there is a pure strategy Nash equilibrium under simultaneous moves.

A's behavior as leader (Proposition 2.ii) is the same as when $\Delta v \neq 0$, but B's strategic situation as leader is different (Proposition 2.iii). If B were to adopt any position other than s, A wins with certainty by matching that position. However, by choosing $a_B = s$, knowing that A will then choose to either match or shade B's position, B's payoff is always positive, unless $\alpha = 1$. A comparison of B's payoffs as follower and leader shows that B is still always better off as follower.

3 Continuous-time framework

We introduce a continuous-time framework for the analysis of two-player games. We then embed the electoral competition game from Section 2 into this framework to endogenize both the timing of candidates' policy announcements and their platforms. Continuous time (unlike discrete time) is not "well ordered", in that there is no "first instant" after time t. Simon and Stinchcombe (1989) show that, as a result of this,

[18] The tangency occurs on the lower arm because in this figure $s > 0$. For this reason, the figure shows only the positive quadrant.

agents who move at the same time t might do so simultaneously *or* sequentially. In the former case, neither agent can condition her action on her rival's action. In the latter case, this conditioning is possible. It is thus important to record the order of players' moves, and not just their timing, for a complete characterization of the outcome of a continuous-time game; Karp et al. (2025) provide further details.[19]

In the electoral competition game, as in many others, the lack of a pure strategy equilibrium is due to the static assumption that *requires* agents move at the same time. Sometimes, analytic convenience or convention is the only reason for that requirement. More plausibly, agents can choose when to move, and can do so simultaneously or sequentially. The continuous-time setting allows for these possibilities. If a static game having only a mixed strategy Nash equilibrium, is "adapted" to continuous time (thereby allowing agents to choose when to act) there is always a profitable deviation from simultaneous moves: waiting for a split second to observe the outcome of the rival's randomization and conditioning one's response on that outcome is better than moving simultaneously with her. In such games, there exists no simultaneous move SPNE, leaving only sequential move SPNE equilibria.

We first provide an intuitive summary of the continuous-time framework; Appendix B.1 gives the formal details. We next establish a link between continuous-time games and their static counterparts. We then provide an intuitive description of our main results for continuous-time games with a second-mover advantage. Appendix B.2 contains a formal statement of these results.

3.1 Setup

We provide an intuitive summary of the continuous-time framework, along with notation used to adapt our electoral competition game to continuous time. Karp et al. (2025) specialize Simon and Stinchcombe's (1989) machinery for continuous-time games by considering only two players, each of whom can move at most once on the time interval $[0, 1)$; it generalizes their analysis by allowing players' action sets to be compact subsets of $\mathbb{R}^n$ (e.g., an interval), rather than finite sets.[20] The specialization is adequate for our electoral competition model, and the generalization is essential, because candidates can choose any platform in an interval. Other games, e.g., involving firm entry and pricing decisions, also require this generalization.

[19] That paper provides a more thorough analysis of the relation between discrete- and continuous-time games, and shows that this continuous-time framework has many other applications.

[20] The generalization requires modified regularity assumptions and different proofs for establishing the relation between continuous- and discrete-time games. See Karp et al. (2025) for further details.

Players can refrain from moving, maintaining a default action. In our electoral setting, the default action might be deciding not to run for office (by not announcing any policy), or retaining a party-specific default policy. The default action, denoted ω_i, can also be interpreted as waiting. This default action remains implemented until the player chooses a new action, a_i, from the compact set A_i^-. If player i never moves, ω_i remains implemented at all times. Player i's full action set is $A_i = A_i^- \bigcup \omega_i$. We assume perfect and complete information, so if player j moves before player i, the latter can react to j's prior move (her timing and action choice). As a follower, i can move immediately after j, which in continuous time means moving at the same calendar time t as the leader, but nonetheless following j (Simon and Stinchcombe, 1989). In this case, the follower can condition her action on the leader's action, obtaining a second-mover advantage without incurring any cost of delay. The follower can also move after some delay, or refrain from moving altogether.

The time when player i moves is denoted t_i. If a player never moves in $[0, 1)$, we set $t_i \equiv 1$ and $a_i \equiv \omega_i$. (Note that time $t = 1$ is not part of the game.) The strategy of a player can be split up into a "leader's part", describing when the player first plans to move and what action she takes, conditional on the other player not moving earlier, and a "follower's part" that describes the timing and action when following an observed prior move of the rival. This description of strategies emphasizes that agents might move at the same point in time either simultaneously or sequentially. In the first case, neither agent can condition her action on the other agent's action. In the second case, the follower can condition her action on the leader's action and on the timing. The equilibrium actions may differ in these two cases. Payoffs $\Pi_i(t_i, a_i, t_j, a_j)$ are continuous in players' timing and action choices, and embed any discounting or other sources of impatience.

The new results for analyzing general games with a second-mover advantage (Section 3.3) require additional notation. Section 4 uses these results to study electoral competition. If player i reaches time $t \in [t_j, 1)$, after an observed prior move by player j at time t_j with action choice a_j, and i behaves optimally at t, i obtains the payoff

$$\Pi_i^s(t|t_j, a_j) \equiv \max_{t_i^s \in [t,1],\, a_i^s \in A_i} \Pi_i(t_i^s, a_i^s, t_j, a_j). \tag{10}$$

The solution to this optimization problem consists of the functions $t_i^{s*}(t|t_j, a_j)$: the optimal time to move, and $a_i^{s*}(t|t_j, a_j)$: the optimal action to take as the second mover. If the solution is not unique, we assume that both players know which pair of maximizers the follower chooses.

Conditional on leading at time $t \in [0,1)$, whether or not it is optimal to do so, we define the leader's (player i's) maximum payoff as

$$L_i(t) \equiv \max_{a_i \in A_i^-} \Pi_i(t, a_i, t_j^{s*}(t|t, a_i), a_j^{s*}(t|t, a_i)). \tag{11}$$

We denote player i's optimal action, conditional on leading at time t, as $a_i^L(t)$. We again assume that the maximizer a_i in equation (11) is unique, or if not, that both players anticipate which maximizer i will choose if this player leads at time t.

If player j leads at time $t \in [0,1)$, implementing her optimal leader's action $a_j^L(t)$, then the follower, player i, obtains the payoff

$$F_i(t) \equiv \Pi_i^s(t|t, a_j^L(t)). \tag{12}$$

This definition embeds player i's optimal behavior as a follower, through equation (10). Note that $F_i(t)$ is not the payoff i gets when moving *at* t, but rather the payoff from optimally following j's time t move; it might be optimal for i to move immediately after j (i.e., also at calender time t), at a later point in time, or not at all.

3.2 A bridge between static and continuous-time games

Here we discuss the relation between the type of static game from Section 2 and the dynamic game sketched in Section 3.1. We would like to know how the predictions of the static game change if the order of moves is endogenous. To that end, we need to adapt the static game to a dynamic setting, requiring that we take a stand on the relation between the two games.

An obvious way to specialize our continuous-time setup in Section 3.1 to produce a static game is (for all t) to give players the payoff $\Pi_i(t_i, a_i, t_j, a_j)$ and action sets A_i^-, as in the dynamic game, but require that they move simultaneously at t. We refer to this as the *restricted game* at t, because it excludes the option of waiting, ω_i.[21] Our assumptions about $\Pi_i(t, a_i, t, a_j)$ (listed in Appendix B.1) ensure that there exists a Nash equilibrium (possibly only in mixed strategies) to the restricted game (Zhou et al., 2011).

There are many ways to convert a static game to a dynamic game. However, to determine the consequence of endogenizing the order of moves, the minimal requirements for this conversion are (i) the action sets A_i^- are the same in the static and dynamic settings; and (ii) for all $t \in [0,1)$, the best response correspondences are the same in the original static game with payoff $\pi_i(a_i, a_j)$ and in the restricted game with

[21]This restricted game is of interest only if neither player has previously moved.

payoff $\Pi_i(t, a_i, t, a_j)$. When these two conditions are met, we say that the static and restricted games are "structurally equivalent". With this equivalence, the sets of pure strategy Nash equilibria are the same in both games; thus, the non-existence of a pure strategy Nash equilibrium in one game implies non-existence of a pure strategy Nash equilibrium in the other game. (As noted above, a Nash equilibrium always exists.)

The restricted game provides a simple way to obtain a sufficient condition for the non-existence of a simultaneous move equilibrium in the dynamic game. The dynamic game allows players to wait at t, whereas the restricted game does not. If there is a simultaneous move equilibrium in the dynamic game (more precisely: a SPNE that entails simultaneous moves in some subgame(s) of the dynamic game such that no player has previously moved), it is also a Nash equilibrium in the restricted game for some t; in this case, the requirement to move simultaneously is not binding. Therefore, if for all t no equilibrium in the restricted game is an equilibrium of the dynamic game, then there exists no simultaneous move equilibrium in the dynamic game.

This observation is useful because we know that (for almost all parameter values), the only simultaneous move equilibrium in our static electoral competition game is in mixed strategies. Given structural equivalence, there exist only mixed strategy equilibria in every restricted game. However, the equilibrium in the dynamic game cannot involve mixing over actions. If player i were to use her restricted game mixed strategy in the dynamic game, the rival j prefers to wait and condition her response on the outcome of i's mixing—amounting to a second-mover advantage for j. In continuous time, j incurs no cost from this "delay". These observations imply[22]

Proposition 3. *Suppose that for all $t \in [0, 1)$ the restricted game and the static game are structurally equivalent and that the static game has no pure strategy Nash equilibrium. Then there exists no simultaneous move SPNE in the dynamic game.*

This proposition means that (for almost all parameter values), there are no simultaneous move SPNE in any dynamic game whose restricted game representation is structurally equivalent to our static game. This result simplifies analysis of the dynamic game, by enabling us to focus only on sequential move equilibria. The result therefore simplifies the goal of determining the endogenous order of moves in the electoral competition game and in other games with similar features. Often, we obtain a unique SPNE with sequential moves in the dynamic game.

[22]The preceding discussion provides a sketch of the proof of this claim, so we do not provide a formal proof. Although the claim is "almost immediate", we state it as a proposition to emphasize its importance. Online Appendix D.5 contains a discussion of mixed strategies in the context of Theorem 1. There, we expand the scope of Proposition 3 to cover players' randomization over the timing of their move (Proposition 3').

The profession is so habituated to mixed strategy equilibria that we may forget that these sometimes arise from the assumption that agents *must* move simultaneously, even if they prefer not to do so. That assumption might be adopted for simplicity, without a compelling economic rationale. However, by using a continuous-time formulation of the game, we discover that in some cases the dynamic game is both more tractable and its predictions more plausible than its static counterpart. Section 4 illustrates this using the electoral competition model in Section 2.

Because our focus is the electoral competition game, we emphasize the situation where non-existence of a simultaneous move equilibrium in the dynamic game arises from the non-existence of a pure strategy equilibrium in the related static game (Proposition 3). However, there may be entirely different reasons for non-existence of a simultaneous move equilibrium in the dynamic setting. Waiting might produce a direct benefit, independently of the actions taken. For example, in an innovation game, the second-mover may be able to imitate the leader's technology. This delay provides a direct incentive to wait, independently of the leader's action choice. This type of direct benefit would also exclude simultaneous move SPNE in the dynamic game, even if there were a pure strategy Nash equilibrium in the static game. For our general dynamic game discussed in the next subsection, we therefore adopt:

Assumption 1. For every $t \in [0, 1)$ in the continuous-time game, at subgames where neither player has previously moved, at least one player strictly prefers to wait at t rather than to use her restricted-game Nash equilibrium strategy, given that the other player adopts her restricted-game Nash equilibrium strategy at t.

Given that the restricted game based on our dynamic game is structurally equivalent to our static game (which in general has only mixed strategy equilibria), Proposition 3 implies that Assumption 1 is automatically satisfied for such games. However, in other games, the assumption can be satisfied for other reasons. The characterization of SPNE provided in the next subsection builds on Assumption 1, but holds independently of the underlying reason why a player would deviate from simultaneous moves at any t.

3.3 SPNE in dynamic games with a second-mover advantage

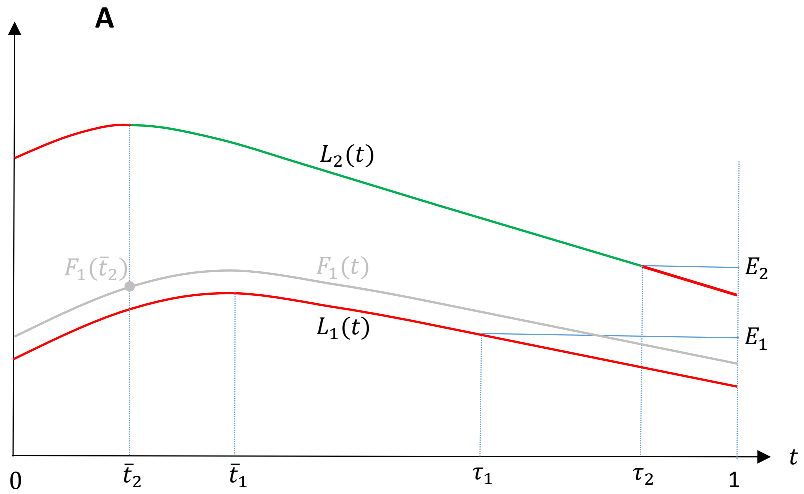
We hereafter consider games with a second-mover advantage, as in the electoral competition game. In continuous time, player i has a "strict second-mover advantage" at t if, assuming that the other player moves at t, i strictly prefers to wait until t, rather than lead shortly before t.[23] As in Section 3.1, we provide an intuitive summary of our results in the main text, and present the general and formal analysis in Appendix B.2. For example, our continuous-time framework allows for endogenous discontinuities in players' action choices that can result in discontinuous payoffs. Theorem 1 in Appendix B.2 accommodates such complexities. Here, our goal is to convey intuition without overloading the main text with technical details.

Figure 4 illustrates two cases that make it easy to understand both our notation and the role of "patience" in determining which player leads in equilibrium. E_i denotes player i's endgame payoff obtained if *nobody* moves in $[0, 1)$. In both panels, player 1 (but perhaps not player 2) has a strict second-mover advantage at all $t \in [0, 1)$. For both players, the payoff from leading at t, $L_i(t)$, is single-peaked with i's ideal time located at $\bar{t}_i \geq 0$. The possible non-monotonicity of $L_i(t)$ accommodates non-strategic reasons for a player to delay moving. For example, the timing of an advertisement or electoral campaign might be important; in a market entry game, there may be reasons why entering prematurely might not be optimal. In addition, for each player i there is a first decision node, a critical time denoted τ_i, beyond which i does not want to lead provided that the other player also does not lead for the rest of the game. In both panels, $\tau_2 > \tau_1$, reflecting an asymmetry between players. The main difference between the panels is that $\bar{t}_1 > \bar{t}_2$ in panel A, but $\bar{t}_2 > \bar{t}_1$ in panel B.

Our approach works backwards in time to identify conditions for a unique SPNE. (See Appendix B.2 for details.) When there is a fixed cost of choosing an action (e.g., an entry or campaigning cost), or other reasons not to choose an action late in the game (e.g., an outside option obtained from never moving) then neither player will move near the end of the game. If, due to an asymmetry, the latest time when a player would lead differs for the two players, we can identify a range of subgames with a unique leader. If, in addition, the leader's payoff is decreasing with time, and the other player has a second-mover advantage, we can reason backwards in time towards earlier subgames. Under certain conditions, we thus find a unique equilibrium in the overall game. Figure 4 shows the curve $L_i(t)$ as green if (according to Theorem 1 in Appendix B.2) i leads at time t, and red if i waits.

Players' (im)patience, which plays a central role, has three dimensions. (i) A second-mover advantage gives an agent a reason to delay moving (if the other player

[23] Note, that we will require a strict second-mover advantage only for a range of subgames for one player, whereas Assumption 1 must hold at every $t \in [0, 1)$. Assumption 1 is used to rule out simultaneous moves in any subgame. The strict second-mover advantage arises from players' endogenous action choices when leading vs. following and permits us to reason backwards in time, ruling out switches in the leadership role across ranges of subgames. In isolation, it does not rule out simultaneous moves. However, both assumptions capture the idea of a "second-mover advantage".



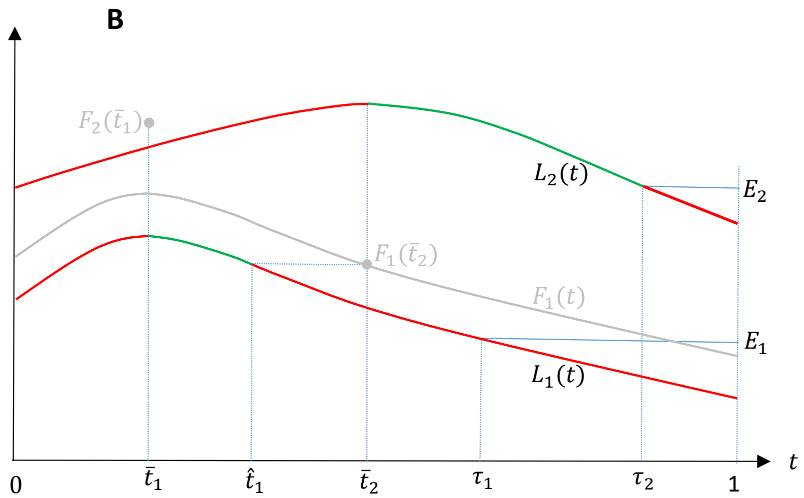


Figure 4: Payoff functions $L_i(t)$ and $F_i(t)$ in stylized examples with a second-mover advantage for player 1, and $L_2(t)$ monotonically decreasing for $t > \bar{t}_2$ (player 2's ideal point for leading); **panel A**: example where $\bar{t}_1 > \bar{t}_2$; **panel B**: example where $\bar{t}_2 > \bar{t}_1$; Green color: player i leads at t in unique SPNE, Red: player i waits (see Theorem 1).

is expected to lead). (ii) An earlier critical time τ_i (after which leading is no longer profitable) means that from an earlier time onward, player i prefers to wait until the end of the game. (iii) A later ideal time to lead, $\bar{t}_i$, creates a third reason for patience.

Theorem 1 establishes the existence of a unique pure strategy SPNE in the overall game under a minimal set of conditions.[24] This general result accommodates additional complexities, such as discontinuities in players' action choices and payoffs. Here

[24]Mixed strategy equilibria are discussed in Online Appendix D.5, where we argue why under the conditions of Theorem 1, there cannot be any SPNE that involves mixing over the timing and/or over players' action choices.

we emphasize the intuition, captured by the two panels in Figure 4. The following corollary summarizes the cases of interest for our later analysis of dynamic electoral competition.

Corollary 1. *Suppose that Assumption 1 holds, and: (i) $\tau_2 > \tau_1$; (ii) $L_2(t)$ is strictly decreasing over $[\bar{t}_2, \tau_2)$; (iii) player 1 has a strict second-mover advantage at every t in this range; and (iv) either $\bar{t}_2 = 0$ or if $\bar{t}_2 > 0$, player 1 has no incentive to preempt player 2 by leading over $[0, \bar{t}_2]$. Then player 2 leads in the unique pure strategy SPNE.*

Figure 4A illustrates a case where these conditions are satisfied. In this panel, player 1's optimal time for leading, $\bar{t}_1$, is later than player 2's, which (together with the other conditions) rules out an incentive for player 1 to preempt her rival early in the game. Theorem 1 in Appendix B.2 does not require all of the features (e.g., continuity everywhere, or that $F_1(t) > L_1(t)$ holds for *all* $t \in [0, 1)$) shown in Figure 4.

The proof of Theorem 1 (in Appendix B.3) begins with decision nodes $t \in (\tau_2, 1)$, shown in Figure 4. In this interval, neither player is willing to lead, provided that the other player also does not lead. The only reason that a player might be willing to lead would be to preempt a move by the other player. But the other player would also only lead, if this player fears that otherwise, the first player would lead. This configuration requires an infinite sequence of intervals over which players switch their roles as leader and follower, because there can be no final interval over which a player leads. That infinite sequence of switches violates one of our general assumptions for continuous-time games (Assumption 2 in Appendix B.1), and is thus ruled out.[25] Hence, at any decision node $t \in (\tau_2, 1)$, both players wait in any SPNE.

Building on this result, we can work backwards in time towards earlier decision nodes. At decision nodes shortly before τ_2, player 1 still prefers to wait rather than lead, provided that player 2 does not move. But player 2 now strictly prefers leading over waiting, since $L_2(t)$ is strictly decreasing in this range. Hence, there cannot be any SPNE where both players wait at such decision nodes, or where player 1 leads. This conclusion identifies unique SPNE strategies such that player 2 leads and player 1 waits at those nodes. The color coding in Figure 4 represents this fact.

This result enables us to work backwards over $[\bar{t}_2, \tau_2)$. Lemma 2 in Appendix B.3 shows that if player j leads at a point in time t_h and player i has a strict second-mover advantage at all $t \in [t_l, t_h]$, and $L_j(t)$ is strictly decreasing over such interval,

[25] Without imposing Assumption 2, Theorem 1 can be maintained if another assumption is imposed or strengthened that rules out an "infinite sequence of switches". For example, if at least one player has a strict second-mover advantage also in the range $t \in (\tau_2, 1)$ (where Condition 1.i does not require it) with a strictly positive δ (see Definition 1 in Appendix B.1) that applies for all t in this range, the result can be restored.

then players cannot switch their roles as leader and follower in this interval. This fact allows us to pin down a unique SPNE for all subgames starting at $t \in [\bar{t}_2, 1)$. Theorem 1 states that player 2 leads at every $t \in [\bar{t}_2, \tau_2)$ whereas player 1 waits, and both players wait at every $t \in [\tau_2, 1)$. In earlier subgames, both players wait if player 1 has no incentive to preempt player 2, as required by assumption (iv) in Corollary 1.

Figure 4B illustrates a case where assumption (iv) in the Corollary is violated. Here, player 1 prefers to preempt player 2 by leading at $\bar{t}_1$ instead of waiting until player 2 leads at $\bar{t}_2$. In equilibrium, player 1 leads at $\bar{t}_1$. This figure shows the equilibrium effect of the relative importance of the different types of impatience. Player 1 would like to follow, but is unwilling to wait until player 2 becomes willing to lead.[26]

Appendix B.2.1 further expands the scope of Theorem 1, allowing for a broader range of potential applications, by relaxing some of the monotonicity assumptions underlying Theorem 1. In particular, it allows for an upwards jump or an increasing segment in $L_i(t)$ (beyond $\bar{t}_i$), while preserving the overall equilibrium characterization of Theorem 1. This is achieved by "ironing" the curve $L_i(t)$, thus replacing it by a modified curve $\bar{L}_i(t)$ where the increasing part or jump is replaced by an interval over which this player waits (rather than leads). This procedure merely requires that within this range, the other player does not want to preempt player i's next planned move (at the end of this range). We use this procedure in Section 4.1; Figure 9 in Appendix B.2.1 provides another perspective.

Equipped with these general results for continuous-time games with a second-mover advantage, we embed our electoral competition game in a framework where players are free to decide when to choose their actions. We emphasize that the results summarized above (and extended and formalized in Appendix B) hold beyond our specific application. The results can thus be used to analyze other continuous-time games with similar features, in particular players' impatience and a second-mover advantage.

4 Electoral competition in continuous time

We embed the electoral competition model from Section 2 into a continuous-time setting, endogenizing the order of agents' moves. The dynamic game forces us to make modeling choices that do not arise in the static setting. In the dynamic setting

[26] If a player deviates from her equilibrium strategy, thereby reaching an off-equilibrium subgame, the ensuing equilibrium differs from the equilibrium to the original game starting at $t = 0$. For example, in Figure 4B, if player 1 deviates by waiting beyond $\bar{t}_1$, sending the game to the subgame at $\bar{t}_2$ where neither player has moved, then the resulting equilibrium is for player 2 to lead immediately and player 1 to follow immediately. See Appendix B.3 for the full set of equilibrium conditions.

we need to include the option of waiting, ω_i, which opens the possibility that an agent never announces a platform, thus requiring additional parameters.

The payoffs $\Pi_i(t_i, a_i, t_j, a_j)$ and $\pi_i(a_i, a_j)$ must be "structurally equivalent" (Section 3.2). In addition, Section 3.3 shows that the functions $L_i(t)$ and $F_i(t)$, induced by the functions $\Pi_i(t_i, a_i, t_j, a_j)$, are key in determining the order of announcements. Thus, we need to specify the time dependence of $\Pi_i(t_i, a_i, t_j, a_j)$. There are many ways to do this, depending on our research goals and the electoral setting of interest.

Our goal is to determine the endogenous order of moves in a dynamic game that is as "close as possible" to our static game, i.e., to introduce as few extraneous complications as possible. We therefore choose a parsimonious representation of time dependence. Here, Theorem 1 enables us to easily determine that the strong candidate leads in the unique SPNE, and the weak candidate obtains their second-mover advantage. The simplicity of this process is inherently valuable, and it shows that the result is insensitive to details of the static game, i.e., to $\pi_i(a_i, a_j)$. Of course, results depend on the specification of time dependence of payoffs.[27] In Section 5, we analyze a variant of the game where, in some cases, the weaker candidate leads.

4.1 Dynamic specification and analytic results

When both candidates announce a policy platform, we assume that i's payoffs is

$$\Pi_i(t_i, a_i, t_j, a_j) = (1 - t_i)\pi_i(a_i, a_j) - c_i. \tag{13}$$

The linear time-dependency of the payoff has the merit of simplicity but it is not crucial for our results. In the static game, $\pi_i(a_i, a_j)$ is i's probability of winning when candidates choose actions (a_i, a_j). In the dynamic game, candidate i incurs a fixed cost c_i from running for office, e.g., for campaigning. Delaying entry harms the candidate, without benefiting the rival. For example, it may raise the cost of running a campaign, reduce opportunities for fund-raising, or expose the candidate to the risk of being replaced. The fact that player i's payoff in equation (13) does not depend on the time j announces her policy, is not restrictive when candidates are impatient. The impatient candidate waits only to obtain the second-mover advantage. Once the leader moves, the follower responds immediately, as her payoff falls over time, or refrains from making a move altogether.

Our results do not rely on the assumption of a fixed (entry or campaigning) cost. An alternative assumes that candidates have outside options, so a candidate i who

[27]See the discussion of the two panels in Figure 4 in Section 3.3.

never announces a policy platform obtains a fixed payoff, E_i. Hence, announcing a policy platform at $t_i \geq 0$ is profitable for candidate i (assuming that the rival also announces a platform) only if

$$(1 - t_i)\pi_i(a_i, a_j) \geq E_i.$$

Subtracting E_i on both sides and reinterpreting the overall payoff Π_i as the *additional* payoff (above the value of the outside option) then reproduces the formulation in equation (13), where c_i is replaced by E_i (no longer reflecting a fixed entry cost).

Another alternative is that the entry cost increases with time. Campaigning costs may be higher if a candidate chooses her platform shortly before the election, because the late announcement requires more expensive media to get her message out.[28] For example, if the campaigning cost equals $c_i/(1 - t_i)$, we retrieve the formulation in equation (13) by multiplying both sides by $1 - t_i$, and reinterpreting the payoff. This different interpretation does not change the results.

The payoff in equation (13) assumes that both candidates announce a platform. We need additional notation for the situation where one candidate leads at a time when the other candidate decides that it is best to never announce a policy as follower. Using equation (11), we denote the leader's payoff in this situation as

$$\max_{a_i \in A_i^-} \Pi_i(t, a_i, t_j^{s*}(t|t, a_i), a_j^{s*}(t|t, a_i)) = (1 - t)\overline{\pi}_i - c_i. \tag{14}$$

The left side is evaluated at $t_j^{s*}(t|t, a_i) = 1$ and $a_j^{s*}(t|t, a_i) = \omega_j$. These equalities state that j's optimal response to i's lead is to never announce a platform. The parameter $\overline{\pi}_i \leq 1$ measures i's success when j abandons the competition. If $\overline{\pi}_i = 1$, then j's withdrawal benefits i. But it is also possible that $\overline{\pi}_i < 1$, so that candidate i can be negatively affected by j's withdrawal.[29] By assumption, we rule out the possibility of a strategic manipulation of the leader's action choice a_i^L with the goal to influence the follower's decision whether to announce a platform (or to refrain from doing so for the rest of the game). Intuitively, if i leads, anticipating that j follows immediately with her policy announcement, then by choosing a_i^*, i already minimizes j's winning probability, leaving no room for "entry deterrence". However, if j's withdrawal harms candidate i, i might lead with a more "favorable" (to j) platform to keep j in the race.

[28]Think of campaigning via television instead of Facebook. Our variant analyzed in Section 5 includes such fixed costs that increase with time.

[29]For example, Kamala Harris was nominated by the Democrats following Joe Biden's withdrawal from the 2024 US presidential election. The Republican candidate, Donald Trump, who finally won the election, sharply criticized Biden's withdrawal at the time.

Such considerations are excluded here, but play a role in Section 5 where we analyze a variant of the dynamic electoral competition game with an incumbent and a challenger.

If neither player moves in $[0, 1)$, then both candidates obtain a payoff of zero, i.e., $\Pi_i(1, \omega_i, 1, \omega_j) = 0$ (in line with purely office-motivated candidates). If only player j moves in $[0, 1)$, but player i never moves, player i obtains a payoff of zero: $\Pi_i(1, \omega_i, t_j, a_j) = 0$ for all $a_j \in A_j^-$ and $t_j < 1$.

We assume that candidate A is the stronger candidate in the static game: either (i) $\Delta v > 0$ and $\Delta p \geq 0$, or (ii) $\Delta v = 0$ and $\Delta p > 0$. For case (ii), we assume the pre-election status quo is non-zero ($s \neq 0$) to ensure that there does not exist a pure strategy Nash equilibrium in the static game. By Proposition 3, there then exists no simultaneous move SPNE in the dynamic game. In the subgames where candidate B announces a policy as a follower (i.e., does not use the default ω_j for the rest of the game), candidate A always chooses a_A^* as the leader by our result in Section 2.2. To exclude trivial results, we assume that the stronger candidate's valence advantage is limited, i.e., condition (9) holds.

If $-1 \leq \bar{l}(a_A, a_B) \leq 1$, where $\bar{l}(a_A, a_B)$ is given by equation (7), A's payoff in the static game from Section 2 is

$$\pi_A(a_A, a_B) = \frac{1}{2} \cdot \begin{cases} 1 - \bar{l}(a_A, a_B), & \text{if } a_A > a_B \\ 1 + \bar{l}(a_A, a_B), & \text{if } a_A < a_B. \end{cases}$$

B's payoff is $\pi_B(a_A, a_B) = 1 - \pi_A(a_A, a_B)$. For Δa sufficiently close to zero and $\Delta v > 0$, the condition $-1 \leq \bar{l}(a_A, a_B) \leq 1$ is not satisfied, and A's payoff is $\pi_A = 1$ (which corresponds to winning with probability one in the static game). The only parameter combination that produces a tie is $\Delta v = 0$, $\Delta p > 0$ and $a_A = a_B = s$. For this case, candidate A obtains the payoff α and candidate B's payoff is $1 - \alpha$.

We denote i's winning probability as follower or leader in the static game as, respectively, π_i^F and π_i^L. For a given set of parameter values $(s, p_A, p_B, \Delta v, \alpha)$, these can be derived from the results of Section 2.[30] Hereafter, we treat π_i^F and π_i^L as parameters, highlighting that our results depend on general features of $L_i(t)$ and $F_i(t)$, but not on details of the underlying game. Many different games can produce leader and follower payoff functions with similar general features. Based on our electoral

[30]For $\Delta v = 0$, when the weaker candidate leads, the stronger candidate may not have a best response as this candidate may prefer to shade the weaker candidate's action choice (see Section 2.2 and the proof of Proposition 2.iii). However, the limit value for the winning probability of each candidate is still well-defined. Moreover, in Proposition 4 we will show that in the dynamic game the stronger candidate leads in equilibrium.

competition game, we restrict the payoff parameters by assuming:

$$\text{(i) } \pi_j^F = 1 - \pi_i^L, \ \text{ (ii) } \ \pi_i^F > \pi_i^L, \ \text{ (iii) } \pi_A^L > \pi_B^L, \ \text{ and } \ \text{(iv) } \overline{\pi}_A \geq \overline{\pi}_B; \tag{15}$$

(i) is a property of probabilities; (ii) states that both candidates have a second-mover advantage; (iii) follows from Propositions 1 and 2 and our assumption that A is the stronger candidate. The new condition (iv) is consistent with A being stronger. It states that A's payoff when B abandons the competition is higher than B's payoff when A abandons the competition; we need this condition to accommodate the possibility that a candidate never announces a platform.

Using Equations (12) and (14), i's payoff as follower in the dynamic game is

$$F_i(t) = \max\{(1-t)\pi_i^F - c_i, 0\}. \tag{16}$$

Candidate i follows the leader immediately (by announcing a platform) for

$$t < t_i^{crit} \equiv \max\{1 - \frac{c_i}{\pi_i^F}, 0\} \tag{17}$$

and never enters for $t \geq t_i^{crit}$.[31] Thus, i's payoff, conditional on leading at t, is

$$L_i(t) = \begin{cases} (1-t)\pi_i^L - c_i & \text{if } t < t_j^{crit} \\ (1-t)\overline{\pi}_i - c_i, & \text{if } t \geq t_j^{crit}. \end{cases} \tag{18}$$

To rule out trivial cases, we assume that the entry costs are small enough that early in the game, A is willing to lead and B is willing to announce a policy as the follower: $c_A < \pi_A^L$ and $c_B < \pi_B^F$, so $L_A(0) > 0$ and $F_B(0) > 0$. Additionally, we assume $c_A \leq c_B$, also in line with the notion that A is the "stronger" candidate.

Figure 5 illustrates $L_i(t)$ and $F_i(t)$ that satisfy these assumptions. For both candidates, $F_i(t)$ consists of a linearly decreasing segment with $F_i(0) > 0$, followed by a constant segment with the value zero, beginning at $t_i^{crit} < 1$. In addition, $t_A^{crit} > t_B^{crit}$. $L_i(t)$ is piecewise linearly decreasing, with a potential discontinuity at $t = t_j^{crit}$; if $\overline{\pi}_i = \pi_i^L$ (so i is indifferent whether j announces a platform as the follower or drops out of the race), the function is continuous.

We introduce two additional restrictions needed to apply Theorem 1. The first one implies that the weaker candidate B does not lead at $t \geq t_A^{crit}$, where A would no longer enter as the follower. If B were to lead at t_A^{crit}, her payoff would be $(1 - t_A^{crit})\overline{\pi}_B - c_B$.

[31] Assumption 3 in Appendix B.1 (right continuity) breaks the tie at $t = t_i^{crit}$.

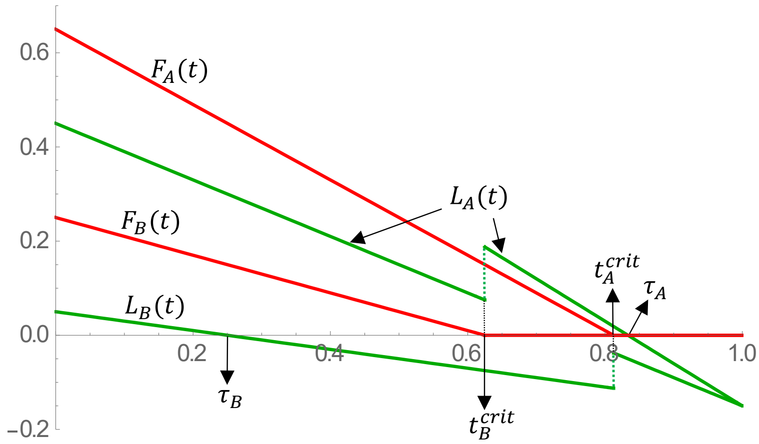


Figure 5: $L_i(t)$ (green) and $F_i(t)$ (red) in the dynamic electoral competition game, for $\pi_A^L = 0.6$, $\pi_B^L = 0.2$, $\overline{\pi}_A = 0.9$, $\overline{\pi}_B = 0.6$, $c_1 = c_2 = 0.15$

We require that this payoff is negative, i.e., using Definition (17), that

$$c_B \pi_A^F > c_A \overline{\pi}_B \quad (\text{“}B \text{ does not lead late in the game”}). \tag{19}$$

With this restriction, B never leads at any $t \geq \max\{0, 1 - c_B/\pi_B^L\} \equiv \tau_B$.[32] This condition is fairly weak. We already assumed that $c_A \leq c_B$, and conditions (15) imply that $\pi_A^F > \pi_B^F$, and $\pi_B^F > \pi_B^L$. The new condition is thus always satisfied if $\overline{\pi}_B \leq \pi_B^L$, but can be satisfied also for values of $\overline{\pi}_B$ larger than π_B^L.

Our second restriction arises if A obtains a higher payoff by leading shortly after (rather than shortly before) the time beyond which B would choose to maintain her default action ω_B as the follower, rather than responding with a policy announcement; this occurs if $\overline{\pi}_A > \pi_A^L$. This case produces an upward jump in $L_A(t)$ at $t = t_B^{crit}$, as Figure 5 illustrates. Here, we apply the "ironing" procedure summarized in Section 3.3 and described in detail in Appendix B.2.1. This procedure eliminates the non-monotonicity in $L_A(t)$, producing the flat interval over $[t_A^\sharp, t_B^{crit}]$ shown in the graph of $\bar{L}_A(t)$ in Figure 6.[33]

To apply Theorem 1, our second (additional) restriction is that B does not want

[32]There are three possible cases for τ_A. If both segments of $L_A(t)$ intersect the horizontal axis, then τ_A is the larger of the two intersection points. If there is only one intersection, this point is τ_A. If neither segment intersects the horizontal axis, then $\overline{\pi}_A < \pi_A^L$ (so $L_A(t)$ has a downwards jump at t_B^{crit}); here, τ_A equals the point of discontinuity, i.e., $\tau_A = t_B^{crit}$.

[33] Setting $L_A(t_A^\sharp) = L_A(t_B^{crit})$ yields $t_A^\sharp = 1 - (1 - t_B^{crit})\overline{\pi}_A/\pi_A^L = 1 - c_B\overline{\pi}_A/(\pi_B^F \pi_A^L)$.

to preempt A by leading in the interval when A waits. This condition is[34]

$$\pi_A^L \pi_B^F > \overline{\pi}_A \pi_B^L \text{ (“}B\text{ does not preempt while }A\text{ waits”).} \tag{20}$$

This condition is also fairly weak: because $\pi_B^F > \pi_B^L$, it trivially holds for any $\overline{\pi}_A \leq \pi_A^L$, but can be satisfied also for values of $\overline{\pi}_A$ larger than π_A^L.

We now apply Theorem 1 to solve the dynamic electoral competition game. The following proposition considers subgames where neither player has previously moved. In all other subgames, where one candidate has already moved, the other candidate solves a simple optimization problem and thus follows the leader's move optimally.

Proposition 4. *In the dynamic electoral competition game, under assumptions* (13), (15), (19), *and* (20), *there is a unique pure strategy SPNE. In this equilibrium, candidate A leads at* $t = 0$ *and B follows immediately. The equilibrium strategies are:*

- *(i) if* $\overline{\pi}_A \leq \pi_A^L$ *or* $L_A(t_B^{crit}) \leq 0$ *(or both), A leads at* $t < \tau_A$*, and waits thereafter, while B always waits;*
- *(ii) if* $\overline{\pi}_A > \pi_A^L$ *and* $L_A(t_B^{crit}) > 0$*, strategies are the same except that A waits for* $t \in [t_A^\sharp, t_B^{crit})$*.*

If Condition (19) *fails, then both players have a first-mover advantage in off-path subgames at* $t \geq t_A^{crit}$*, but the equilibrium strategies at earlier subgames are unchanged, so the* outcome *of the game remains unique: A leads at* $t = 0$ *and B follows immediately.*

Figure 6 graphs the candidates' payoffs if they lead, $\bar{L}_A(t)$ and $L_B(t)$, for the case $\overline{\pi}_A > \pi_A^L$. Here, the color-coding identifies the candidates' strategies, red indicating “wait” and green indicating “lead” (as in Figure 4 in Section 3.3). Candidate B's equilibrium strategy is to wait at every subgame where neither player has moved. Candidate A leads at every such subgame before τ_A, except over the interval $[t_A^\sharp, t_B^{crit})$, where A waits. Our key finding is that the stronger candidate A leads, i.e., announces her policy platform before the other candidate. Most importantly, candidates do not announce their platforms simultaneously if they are free to decide when to move.

Figure 6 illustrates a case where both candidates obtain a positive payoff from leading near the beginning of the game. Nevertheless, the stronger candidate leads in the unique equilibrium. The explanation is that the candidates recognize that there

[34]This condition is $L_B(t_A^\sharp) < 0$, which is equivalent to inequality (20). To establish this equivalence, use $L_B(t_A^\sharp) < 0 \iff c_B > (1 - t_A^\sharp)\pi_B^L$. Using the formula for $t_A^\sharp$ in Footnote 33 we obtain $(1 - t_A^\sharp)\pi_B^L = c_B \overline{\pi}_A \pi_B^L / (\pi_B^F \pi_A^L) < c_B \iff \overline{\pi}_A \pi_B^L / (\pi_B^F \pi_A^L) < 1$. Condition 3 in Appendix B.2.1 refers to this as a “no preemption” condition.

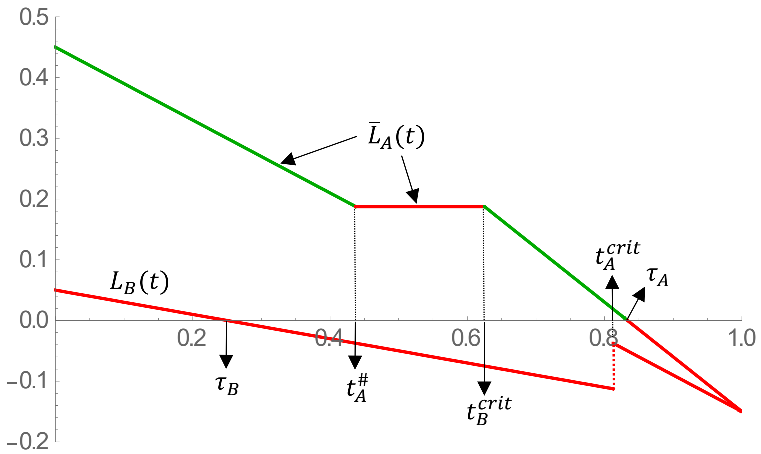


Figure 6: SPNE strategies in the dynamic electoral competition game: $\bar{L}_A(t)$ and $L_B(t)$, green: player leads in subgame beginning at t, red: player waits; for $\pi_A^L = 0.6$, $\pi_B^L = 0.2$, $\overline{\pi}_A = 0.9$, $\overline{\pi}_B = 0.6$, $c_1 = c_2 = 0.15$

are later (off-path) subgames at which the weaker candidate would not lead (because doing so returns a negative payoff), but the stronger candidate is still willing to lead. Reasoning backwards in time, the stronger candidate thus starts leading from an earlier time onward and ends up leading immediately at $t = 0$. Both candidates would prefer to follow rather than lead, but only the weaker candidate enjoys her second-mover advantage. Here, innate strength creates a strategic disadvantage. Similar results have been found by other authors, but using different methods, e.g., van Damme and Hurkens (1999 and 2004).[35]

Applying Theorem 1 requires only checking that the conditions of the theorem hold. For the game here, this is straightforward, making it easier to characterize the sequential equilibrium than to compute the mixed strategy equilibrium – which occurs only when agents are *assumed* to move simultaneously. That assumption is sometimes imposed by convention, but not motivated. In our electoral competition game, agents never choose to move simultaneously if they are free to decide when to announce their platforms. The strong candidate announces her policy first and the weak candidate responds immediately, conditioning her response on the leader's announcement.

The stronger candidate A leads in equilibrium and uses the same platform as in the static game. Section 2 shows this platform is distorted towards the status quo policy s, unless both candidates are equally competent. Our dynamic electoral competition game, where leadership emerges endogenously, thus offers a novel explanation for a

[35] Using a price competition application, our companion paper, Karp et al. (2025), shows that entry deterrence can reverse the equilibrium order of moves such that the weaker player leads. Also in our variant analyzed in Section 5, depending on the parameters, the weaker candidate may lead.

status quo bias in electoral competition.

Online Appendix D.3 provides a partial welfare analysis, comparing the efficiency of the sequential equilibrium to a setting where a planner chooses candidates' policy platforms to maximize the expected welfare of the median voter. Numerical results suggest that the planner chooses policies that are farther apart than the equilibrium policies, thereby accounting for more extreme realizations of the median voter's bliss point.[36]

4.2 Numerical example

A simple numerical example illustrates how the parameters of the dynamic game can be derived from the details of the underlying electoral competition game.

Suppose $E > 0$ is the value of an outside option for each candidate if they decide not to run for office. Equivalently, we set $c_A = c_B = E$.[37] The payoff in the dynamic game is thus interpreted as the payoff from entering net of the value of the outside option. If both players enter, we get $\Pi_i(t_i, a_i, t_j, a_j) = (1 - t_i)\pi_i(a_i, a_j) - E$ (see equation 13). The multiplier $(1 - t_i)$ can be interpreted as the probability of still being in the race at time t_i if the candidate does not announce a platform before then. For instance, this candidate may be favored by party i, but faces a risk of being replaced by another candidate over time if they do not enter early. If i enters the race, but her rival candidate j does not, i's payoff is $\Pi_i(t_i, a_i^L, 1, \omega_j) = (1 - t_i)\overline{\pi}_i - E$ (see equation (14)).

As an example, let $\overline{\pi}_i = \frac{2}{3}\pi_i^L$. This example implies that if candidate j drops out, she is replaced by another candidate of the same party who is, in expectation, slightly stronger than candidate j. Here, candidate i does not benefit if j does not run for office. Finally, we have $\Pi_i(1, \omega_i, 1, \omega_j) = \Pi_i(1, \omega_i, t_j, a_j) = 0$: staying out always yields the value of the outside option to candidate i, irrespective of the behavior of her rival.

Suppose further that candidates are equally popular among voters: $\Delta v = 0$, but A is more competent: $p_A > p_B$. Recall that only in this case, candidate B can win the election with positive probability also as the leader (see Section 2). We also assume that $s > 0$ and $\alpha = 1/2$. With this specification of the electoral competition game, we get $a_A^L = a_A^* = (1 - p_B)s$ if candidate A leads. Using the results from the proof of Proposition 2 (see Appendix A), we thus find that

$$\pi_A^L = \frac{1}{2}\left(1 + s\sqrt{\Delta p/p_A}\right), \text{ and } \pi_B^F = 1 - \pi_A^L.$$

[36]The more centralized policies in the dynamic electoral competition game are reminiscent of the "Median voter theorem" in a static electoral competition game (without competence or valence issues).

[37]When considering to lead at t, player i compares the resulting payoff with the endgame payoff if nobody moves in $[0, 1)$. An equivalent comparison arises when moving comes with a fixed cost.

If B leads, he chooses $a_B^L = s$ (as $\Delta v = 0$), which yields a winning probability of

$$\pi_B^L = \frac{1-s}{2}, \text{ and } \pi_A^F = 1 - \pi_B^L.$$

The two inequalities in Proposition 4, (19) and (20), are easily checked: with $c_A = c_B = E$, inequality (19) simplifies to $\pi_A^F > \overline{\pi}_B$, which is satisfied as $\overline{\pi}_B < \pi_B^L$ and $\pi_A^F > \pi_B^F > \pi_B^L$. Also condition (20) is satisfied, as $\pi_A^L > \overline{\pi}_A$. Assigning some numerical values to the parameters E, p_A, p_B, s yields a fully specified example.[38] The SPNE in the dynamic game is as described by Proposition 4. The example illustrates the simplicity of relating the parameters in the dynamic game to those in the static one.

5 Variant with an incumbent and a challenger

In Section 4, we focused on cases where a player who moves incurs a fixed cost or where players obtain fixed "endgame payoffs" E_i if nobody moves in $[0, 1)$. Here, we illustrate with the help of a variant of this game that the payoff of a player from "not making a move" can also arise from the electoral competition game itself. Specifically, we analyze the interaction between an incumbent and a potential challenger. Whereas the challenger must announce a policy platform in order to join the fray, the incumbent is automatically a candidate.[39]

Let A be the incumbent and B the challenger. A's default policy is $\omega_A \geq 0$.[40] If A makes a move, she announces a platform $a_A \neq \omega_A$. Doing so, she incurs a fixed cost that is increasing with time (e.g., making a credible announcement shortly before the election requires higher efforts than early on). For simplicity, we assume that this cost is linear in t and given by $c_A t$ for candidate A. Similarly, if B announces a platform (and thus runs for office), he incurs a cost of $c_B t$ ($c_i > 0$ are parameters). If B never announces a platform, his payoff equals zero (he loses the election with certainty). In the latter case, the incumbent (A) wins with probability 1, irrespective of whether she announces a new platform or maintains her default policy.

For this variant, we use the simplest version of our electoral competition setup (see Section 2): we assume that candidates differ only in their valences ($\Delta v > 0$), but not

[38] For example, let $E = 0.1$, $s = 0.4$, $\Delta p/p_A = 1/4$. This yields $\pi_A^L = 0.6$, $\pi_B^F = 0.4$, $\pi_B^L = 0.3$, $\pi_A^F = 0.7$, and with $\overline{\pi}_i = \frac{2}{3}\pi_i^L$ we get $\overline{\pi}_A = 0.4$ and $\overline{\pi}_B = 0.2$. The critical time beyond which B is unwilling to make a move as the follower is $t_B^{crit} = \frac{3}{4}$. We also find $\tau_A = \frac{3}{4}$ and $\tau_B = \frac{2}{3} < \tau_A$.

[39] For example, the incumbent's default policy may coincide with the status quo. But this is irrelevant here because under our set of assumptions (see below), in particular perfectly competent candidates, the status quo is irrelevant.

[40] The case where $\omega_A < 0$ is the mirror image and does not add something novel, so we ignore it.

in their competence. The incumbent (candidate A) is stronger in that she has a higher valence.[41] For simplicity, we set $p_A = p_B = 1$. We maintain assumption (9) from Section 2, which (for $p_A = p_B = 1$) simplifies to $\Delta v < 1$. In Section 2, we argued that if the candidate with lower valence (here: B) moves first, he loses the election for sure if the stronger candidate matches the leader's action choice (or chooses a platform sufficiently near to it), see Proposition 1 and Remark 1. However, this conclusion is only valid if the follower indeed chooses an action.

In this variant, however, A does not need to announce a new platform to win the election with a positive probability. Depending on B's announcement, this candidate may decide to maintain their default policy. This, in turn, gives B a possibility to lead, and yet win the election with a positive probability (in contrast with our findings from Section 2). It is, thus, not obvious which candidate will lead or follow in equilibrium. However, it is clear that the challenger (candidate B) *must* make a move in order to win with a positive probability, while the incumbent can also refrain from making a move and still win the election.

From Section 2, we can use the following (simplified for $p_A = p_B = 1$):

$$\bar{l} = \frac{a_A^2 - a_B^2 - \Delta v}{2(a_A - a_B)}.$$

For $\Delta a = a_A - a_B > 0$, we obtain the winning probabilities (as long as $\bar{l} \in [-1, 1]$):

$$\pi_A = \frac{1 - \bar{l}}{2} = \frac{1}{2}\left(1 + \frac{\Delta v - a_A^2 + a_B^2}{2(a_A - a_B)}\right), \text{ and } \pi_B = 1 - \pi_A. \tag{21}$$

Without loss of generality, we will focus on the case where $\Delta a > 0$.

Conditional on both players making a move, if A leads, the follower (B) chooses their best response:

$$a_B^R(a_A) = a_A - \sqrt{\Delta v}.$$

(The other solution, $a_A + \sqrt{\Delta v}$, violates $\Delta a > 0$.) As the leader, A then chooses the platform a_A that just makes B indifferent between these two responses. This maximizes A's (and thus minimizes B's) winning probability (equation (21)):

$$a_A^* = 0 \Rightarrow \pi_A^L = \frac{1}{2}\left(1 + \sqrt{\Delta v}\right), \pi_B^F = 1 - \pi_A^L.$$

[41] However, we allow the challenger's fixed cost of announcing a platform to be smaller. Ashworth and Bueno de Mesquita (2008) present a model with an incumbency advantage that arises because incumbents tend to have greater ability than challengers. This aligns with our assumption $\Delta v > 0$.

If B leads (and A makes a move as follower), then A wins the election with probability 1 by simply matching B's platform (see Section 2).

However, the situation is different when taking into account candidates' fixed costs. Let us denote the payoffs in the dynamic game by Π_i.[42] We obtain:

$$\Pi_A = \begin{cases} \pi_A(a_A, a_B) - c_A t, & \text{if } A \text{ annouces at } t \\ \pi_A(\omega_A, a_B), & \text{if } A \text{ never announces a (new) platform.} \end{cases}$$

Note, that in the first case, $\pi_A(a_A, \omega_B) = 1$ if B never announces a platform.

For B's payoff, we find:

$$\Pi_B = \begin{cases} \pi_B(a_A, a_B) - c_B t, & \text{if } B \text{ enters at } t \\ 0, & \text{if } B \text{ never announces a platform.} \end{cases}$$

The second line follows from $\pi_B(a_A, \omega_B) = 0$ for all a_A (including $a_A = \omega_A$).

Also in the dynamic setting, conditional on leading, A always chooses $a_A^* = 0$: If B makes a move as the follower, this maximizes A's winning probability (see above); if B does not move, then A wins with probability 1 for any policy announcement, so $a_A = a_A^* = 0$ is still an optimal leader's policy for A.

If B leads, this candidate can only win with a positive probability if he sets a_B such that A does *not* announce a platform as the follower. To maximize his own winning probability, B may choose a_B such that A is just indifferent between making a move as the follower and not.[43] Then B's optimal leader's platform is determined by the following indifference condition for A (see equation (21)):

$$\pi_A(a_A^R(a_B), a_B) - c_A t = \pi_A(\omega_A, a_B).$$

We already know that A's best reply is $a_A^R(a_B) = a_B$ (A matches B's platform) and thus $\pi_A(a_A^R(a_B), a_B) = 1$. Hence, this condition simplifies to

$$\pi_A(\omega_A, a_B) = 1 - c_A t.$$

Since we focus on the case where $\Delta a > 0$, let $a_B^L < \omega_A$.[44] Using (21), the above

[42] To ease notation, we suppress the dependency on t_i, a_i on the left side, and we do not treat the cases where one candidate never moves separately.

[43] As a tie-breaking rule, it is convenient to assume that A does not move if he is indifferent between making a move (as follower) and not.

[44] Given $\omega_A \geq 0$, leading with some $a_B^L > \omega_A$ yields no advantage to B and can be neglected.

indifference condition for A thus reads:

$$\pi_A(\omega_A, a_B) = \frac{1}{2}\left(1 + \frac{\Delta v - \omega_A^2 + a_B^2}{2(\omega_A - a_B)}\right) = 1 - c_A t.$$

Solving for $a_B = a_B^L$, B's optimal leader's policy, we find

$$a^L_{B_{1,2}} = -1 + 2c_A t \pm \sqrt{(1 + \omega_A - 2c_A t)^2 - \Delta v}. \tag{22}$$

Hence, there are two solutions. Intuitively, there are two ways in which B can adopt a "sufficiently weak" policy position vis-a-vis A, such that A is just indifferent between moving and not: B can either choose a_B sufficiently near to ω_A (so A wins with a high probability even without making a move), or choose a_B sufficiently far away from ω_A, so that this platform appeals to the median voter only under extreme realizations of the voter's bliss point. If they exist, both solutions lead to identical payoffs, so we can pick either one of them. The challenger's (B's) resulting payoff is then

$$\Pi_B^L = 1 - \pi_A(\omega_A, a_B^L) - c_B t.$$

Using $a^L_{B,1}$ or $a^L_{B,2}$ (see above) to replace a_B^L, it is easy to verify that the leader's payoff for candidate B then simplifies to:

$$\Pi_B^L = (c_A - c_B)t. \tag{23}$$

And because the indifference condition is satisfied, A's resulting payoff is simply

$$\Pi_A^F = 1 - c_A t.$$

However, the two solutions for a_B^L in equation (22) may not exist, because the expression under the square root can become negative. This holds beyond the point where the two solutions in equation (22) coincide, denoted $\bar{t}_B$, which is easily calculated:[45]

$$\bar{t}_B = \frac{1 + \omega_A - \sqrt{\Delta v}}{2c_A} > 0. \tag{24}$$

For illustration, see Figures 7 and 8.

For small t (left part of Figure 7), B's optimal leader's action is $a^L_{B,1}$ (orange curve) or $a^L_{B,2}$ (blue). The former curve is downward sloping, because with increasing t, A's

[45] As will become clear further below, this corresponds to the maximum of the leader's payoff $\bar{t}_2$ in Figure 4 (Section 3.3), hence this choice of notation.

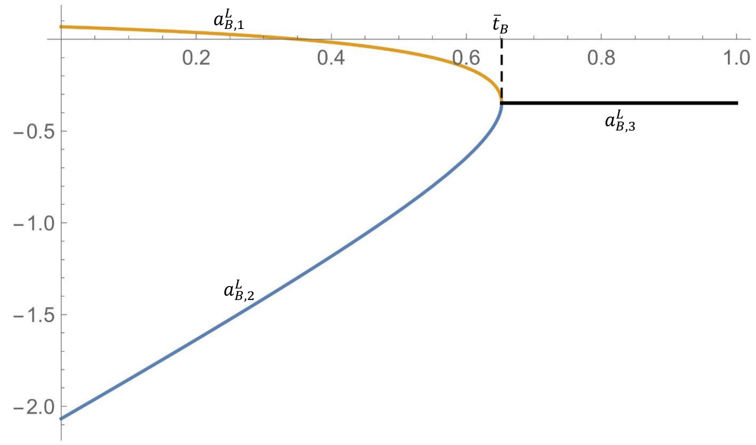


Figure 7: B's optimal leader's action (a^L_B) depending on t; $a^L_{B_{1,2}}$ (orange/blue): candidate A is indifferent between announcing a new platform and not; $a^L_{B,3}$ (black): B chooses his preferred leader's policy while A maintains her default policy ω_A; for $\omega_A = 0.2$, $c_A = 0.5$, $c_B = 0.3$, $\Delta v = 0.3$

fixed cost of announcing a platform increases. Thus, B can choose a policy more distant from A's default policy to win the election with a higher probability.[46] Conversely, the curve $a^L_{B,2}$ is upward sloping because, with increasing fixed costs of candidate A, B does not need to differentiate as much to prevent A from making a move.

The ideal policy for B, if A would *never* move as the follower (irrespective of B's action choice), is obtained by maximizing:

$$\pi^L_B = 1 - \pi_A(\omega_A, a^L_B),$$

without the indifference constraint for A (that we used earlier). This unconstrained optimization yields (using equation (21)):

$$a^L_{B,3} = \omega_A - \sqrt{\Delta v}, \text{ and } \pi^L_B = \frac{1}{2}\left(1 + \omega_A - \sqrt{\Delta v}\right). \tag{25}$$

Intuitively, beyond the point $\bar{t}_B$ where $a^L_{B,1}$ and $a^L_{B,2}$ coincide (see Figure 7), as the leader, B can choose their most preferred leader's action $a^L_{B,3}$ and still, A does not make a move as the follower.

The resulting payoffs $L_i(t)$ and $F_i(t)$ in the dynamic game are illustrated in Figure 8. Intuitively, up until $\bar{t}_B$, the leader's payoff of candidate B is increasing with

[46]Recall that as the weaker candidate, B must (to some extent) differentiate away from A's platform to win with a higher probability, because this makes their platform more appealing under more extreme (here: more leftist) realizations of the median voter's preferred policy.

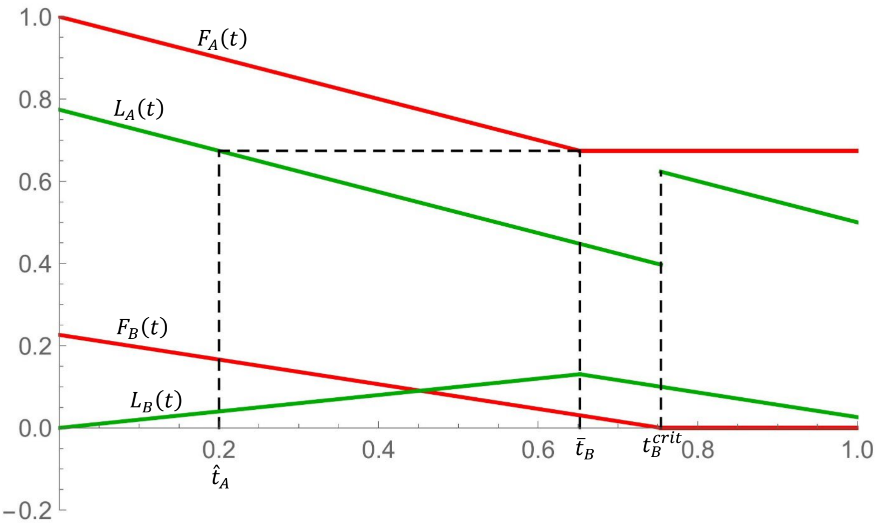


Figure 8: Payoffs in the variant with an incumbent (A) and a challenger (B), green: optimal leader's payoff ($L_i(t)$), red: optimal follower's payoff ($F_i(t)$), for $\omega_A = 0.2$, $c_A = 0.5$, $c_B = 0.3$, $\Delta v = 0.3$

time, because A is less tempted to announce a new platform. So B can lead with a platform that allows him (B) to win with a somewhat higher probability. Beyond $\bar{t}_B$, B leads with his preferred leader's action $a^L_{B,3}$; B's winning probability is thus constant, but the payoff $L_B(t)$ is decreasing with time because of the increasing fixed cost $c_B t$.

Existence of the increasing segment of $L_B(t)$ requires that $c_B < c_A$ (see equation (23)). Without this condition, the stronger candidate A can use their incumbency advantage to make sure that B will not enter the race and refrain from announcing a platform (see below).

To complete the derivation of the leader's and follower's payoff functions ($L_i(t)$ and $F_i(t)$), we compute the latest point, denoted t_B^{crit}, at which candidate B is still willing to announce a policy platform as the follower (see Figure 8). It is located at the point where $F_B(t)$ reaches zero (because B's outside option is to obtain a payoff of zero by not running for office). Straight-forward calculations yield:

$$t_B^{crit} = \frac{1 - \sqrt{\Delta v}}{2c_B} > 0. \tag{26}$$

We finally compute the latest time, τ_B, at which candidate B is willing to lead. Using B's winning probability from equation (25) and subtracting the fixed cost, $c_B t$, the indifference point where B's leader's payoff becomes zero is

$$\tau_B \equiv \min\left\{\frac{1+\omega_A-\sqrt{\Delta v}}{2c_B}, 1\right\} > 0.$$

Conditional on B not moving, there is no $t \in [0,1)$ at which candidate A is willing to lead, as A's payoff from both players never moving equals 1 while her leader's payoff is below 1 for all $t \in [0,1)$. Hence, $\tau_A = 0$.

Summing up the above findings, we obtain the following payoffs in the dynamic game: For the incumbent, we get:

$$L_A(t) = \begin{cases} \frac{1}{2}\left(1+\sqrt{\Delta v}\right) - c_A t, & \text{for } t < t_B^{crit} \\ 1 - c_A t, & \text{for } t \geq t_B^{crit}. \end{cases}$$

$$F_A(t) = \begin{cases} 1 - c_A t, & \text{for } t < \bar{t}_B \\ \frac{1}{2}\left(1-\omega_A+\sqrt{\Delta v}\right), & \text{for } t \geq \bar{t}_B. \end{cases}$$

And for the challenger, we get:

$$L_B(t) = \begin{cases} (c_A - c_B)t, & \text{for } t < \bar{t}_B \\ \frac{1}{2}\left(1+\omega_A-\sqrt{\Delta v}\right) - c_B t, & \text{for } t \geq \bar{t}_B. \end{cases}$$

$$F_B(t) = \begin{cases} \frac{1}{2}\left(1-\sqrt{\Delta v}\right) - c_B t, & \text{for } t < t_B^{crit} \\ 0, & \text{for } t \geq t_B^{crit}. \end{cases}$$

The ordering of t_B^{crit} and $\bar{t}_B$ is ambiguous, with $t_B^{crit} > \bar{t}_B$ if and only if (using equations (24) and (26))

$$\frac{c_A}{c_B} > \frac{1+\omega_A-\sqrt{\Delta v}}{1-\sqrt{\Delta v}}. \tag{27}$$

Also the existence of these critical points within the time frame $[0,1)$ depends on the parameters. These ambiguities necessitate case distinctions in the proof of Proposition 5. Additionally, in the case where $\min\{\bar{t}_B, t_B^{crit}\} \geq 1$, i.e., if

$$\sqrt{\Delta v} \leq \min\{1+\omega_A - 2c_A, 1-2c_B\},$$

results may depend on the relation between the highest leader's payoff for candidate A (at $t=0$): $\frac{1}{2}\left(1+\sqrt{\Delta v}\right)$ and her follower's payoff if candidate B leads just before

$t = 1$: $1 - c_A$. The former is greater than the latter if

$$\sqrt{\Delta v} > 1 - 2c_A. \tag{28}$$

We will argue below that if this condition does not hold, then leadership is reversed in the outcome of the game so that the weaker candidate announces their platform first.

We summarize our results for this game in the following

Proposition 5. *In the variant with an incumbent and a challenger, if assumption* (28) *holds and $c_A > c_B$, the incumbent (A) leads at $t = 0$ and the challenger (B) follows immediately by announcing a policy platform. If $c_A < c_B$, nobody announces a (new) platform, so the challenger stays out and the incumbent wins the election.*

Remark 2. If assumption (28) does not hold and $c_A > c_B$, the incumbent never announces a (new) policy platform, and the challenger enters by announcing a platform just before the end of the game.

The proof applies Theorem 1, with some qualifications. The candidate with the higher valence: A (the incumbent) corresponds to player 1 in Figure 4B (Section 3.3). Note, that $\tau_B > \tau_A$. The result confirms that for many parameter values, the stronger candidate leads (as in Proposition 4, Section 4). But there are also some important differences.

As in Figure 4B, when $c_B < c_A$, for candidate B there exists an "ideal time for leading", $\bar{t}_B > 0$. This results from B's optimal leader's strategy that entails an action choice that just deters the follower, A, from announcing a new platform. First, with increasing t it becomes easier to deter A from moving (as A's fixed costs are increasing). But then the possibilities of leading with a more profitable platform are exhausted for B, so that his leader's payoff declines again (due to his own increasing fixed cost). However, in the SPNE outcome of the overall game, these interesting dynamics are overshadowed by the stronger candidate's incentive to lead early in the game.

The proof of Proposition 5 also shows that for some parameter constellations, in later subgames (that can only be reached if candidates deviated from their optimal behavior in earlier subgames), an equilibrium may fail to exist at least in a strict sense. This problem is related with a general feature of continuous time games such as ours: if a player only finds it optimal to choose an action at an isolated point in time, and the same type of behavior is not optimal shortly before or shortly after this time, strategies are more difficult to formalize. To see this, consider a payoff structure where, say, candidate B wants to move as late in the game as possible, but before the end of the game, i.e., within the interval $[0, 1)$. Obviously, in continuous time, a "latest point" just before $t = 1$ does not exist, so that under such payoff structures, a SPNE *cannot* exist in a strict sense. This problem arises under some parameter constellations. But it does not affect the incentives of players to lead or wait earlier in the game, so that the result of Proposition 5 can nevertheless be validated. Note, that this issue is not specific to the game analyzed here, but it is a general difficulty of any continuous time game in the same overall setting (see Simon and Stinchcombe, 1989).

For other parameter constellations, in particular when condition (27) is violated, a similar problem arises, but at a point inside the interval $[0, 1)$: t_B^{crit}. Also here, candidate B prefers to lead only *at* time t_B^{crit}, but not shortly before this time (because $L_B(t)$ is increasing). And B cannot lead after this time, because here, there is a unique SPNE in which A leads. B's strategy then violates right-continuity (Assumption 2, see Appendix B.1). In this case, the technical issue can be addressed by simply relaxing the requirement of right-continuity for players' strategies at t_B^{crit}. This permits B to lead only at this point, while A starts leading in an open interval following t_B^{crit}.[47]

Aside from these technical issues that are relevant only in off-path subgames (later in the game), the proof of Proposition 5 builds on the existence of an interval in the beginning of the game where candidate A is unambiguously better off leading, because the highest payoff she may receive from leading or following later in the game is lower than her leader's payoff at $t = 0$. This holds true if assumption (28) is satisfied. If it is violated, then both candidates have a strict second-mover advantage over the entire range $[0, 1)$,[48] with $L_A(t)$ decreasing and $L_B(t)$ increasing everywhere, and $L_A(0) < \lim_{t\to 1} F_A(t)$. In this case, the only sensible outcome of the game is that candidate A never moves (she achieves a strictly higher payoff if B leads at any $t \in [0, 1)$, or if B never moves), while B announces a platform just before the election takes place (immediately before $t = 1$). We formulate this as Remark 2, rather than as part of Proposition 5, because in a strict sense, there exists no SPNE that supports this outcome. See Online Appendix D.4 for a further discussion of this issue.

[47] The assumption of right-continuity is often useful as a tie-breaker, and it helps to simplify the formalization of strategies and their mapping into outcomes, see Karp et al. (2025) for further details on this. For a large variety of games, the assumption does not limit the analysis of SPNE, because players generally find the same kind of behavior optimal not only at a specific point in time, but also shortly before or after it. But there are exceptions to this rule, as the proof of Proposition 5 illustrates. Relaxing the right-continuity requirement, the general theorems in Karp et al. (2025) that relate discrete and continuous time games cannot immediately be applied without some adjustments, but the simplicity of the game of interest here suggests that this is not a serious limitation. See Online Appendix D.4 for a supplementary discussion of the right-continuity assumption.

[48] It is easy to verify that the intersection point of $L_B(t)$ and $F_B(t)$: $t = (1 - \sqrt{\Delta v})/(2c_A)$ coincides with the time where $F_A(t)$ equals $L_A(0)$; hence, both players have a second-mover advantage over the entire range if assumption (28) is violated.

6 Conclusion

Analyzing a game in continuous time that, in different variants, has often been analyzed as a one-shot game before, offers an entirely new perspective. In our setting, electoral competition becomes a timing game, where issues like leadership, second-mover advantages, and strategic delay play a key role. Such considerations are typically absent in "conventional" one-shot games. We find that our approach makes the analysis more realistic, because (as we demonstrate), when the static version of the game only has a mixed strategy Nash equilibrium, candidates never choose to move simultaneously when they are free to decide about the timing of their moves (as well as about their platform choices). By including the feature of competence, we also offer a new explanation for status quo biases in electoral competition.

Apart from these new perspectives on electoral competition, we offer a novel methodology that allows one to transfer also other games that have previously been analyzed by imposing simultaneous moves on the players to continuous time. The key building blocks of our general framework are presented and discussed in greater detail in our companion paper, Karp et al. (2025).[49] Here, our focus is on applying this methodology to analyze games with a second-mover advantage. For such games, under a small set of intuitive conditions, we identify a unique SPNE entailing sequential moves by the players. The characteristics that determine which agent follows – a source of "patience" – are context-specific. An agent who is more patient in all dimensions follows in equilibrium. If each agent is more patient in a different dimension, the identity of the follower depends on the relative strength in the different dimensions.

We believe that, apart from electoral competition, there are various other potential applications where our tools can be fruitfully applied, for example in the area of industrial organization. A possible research agenda thus involves reconsidering static games, to determine when the assumption of simultaneous moves is descriptive. Where that assumption is made for reasons of (apparent) tractability rather than plausibility, the tools we provide may offer an alternative way to think about the game. Additional features that may be added to our continuous-time modeling framework include asymmetric information or exogenous changes to the payoff structure. Allowing players to choose the timing of their moves freely in games that have traditionally been analyzed as static ones, promises new insights also when using laboratory experiments.[50]

[49] We summarized our general continuous time setup in Section 3.1 and presented it in some more detail in Appendix B.1.

[50] Calford and Oprea (2017) pioneered experiments based on the continuous-time framework of Simon and Stinchcombe (1989). Our general framework (Section 3) drastically expands the set of games that can be analyzed by including infinite action sets, such as intervals.

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Appendix

Appendix A contains the proofs for the results of Section 2. Appendix B formalizes the continuous-time framework introduced in Section 3, based on our companion paper (Karp et al., 2025), and presents and proves general results for games with a second-mover advantage. Appendix C contains the proofs for the results of Section 4.

A Proofs for Section 2

Proof of Proposition 1. Part i. We first confirm the description of A's best response correspondence, and then turn to B's best response correspondence.

Using equations (3) and (5), when $\Delta a = 0$ the median voter strictly prefers candidate A regardless of the realization of this voter's ideal point; in this case, candidate A wins with certainty. Given the continuity of the model in actions, for all realizations of the median voter's ideal point, this voter strictly prefers candidate A whenever $|\Delta a|$ is sufficiently small. Therefore, there is an interval with positive measure, that includes the 45^o line in Figure 2, such that for all policies in this interval candidate A wins with certainty.

We now show that this interval is candidate A's *entire* best response correspondence: no point outside this interval is a best response to a_B. To establish this claim, we show that for any pair of positions at which candidate A's probability of winning is less than 1, the candidate does strictly better by moving her policy toward the 45^o

line. We use the critical value $\bar{l}(a_A, a_B)$ in equation (7) and compute the derivative

$$\frac{\partial \bar{l}(a_A, a_B)}{\partial a_A} = \frac{p_B(a_A - a_B)^2 + \Delta p(s - a_B)^2 + p_A p_B \Delta v}{2 p_A p_B (a_A - a_B)^2} > 0. \tag{29}$$

This relation implies that if $\pi_A < 1$, candidate A has an incentive to move closer to candidate B's position: If $a_A < a_B$, a higher a_A increases $\bar{l}(a_A, a_B)$, thereby increasing A's probability of winning, $\pi_A(a_A, a_B) = \frac{1}{2}(1 + \bar{l}(a_A, a_B))$. For $a_A > a_B$, a decrease in a_A reduces $\bar{l}(a_A, a_B)$, which again implies a higher probability of winning, $\pi_A(a_A, a_B) = \frac{1}{2}(1 - \bar{l}(a_A, a_B))$. Therefore, outside the green area shown in Figure 2 there exists no interval or point where A wins with certainty.

We now consider B's best response correspondence. For the time being we ignore the constraints $-1 \leq \bar{l}(a_A, a_B) \leq 1$. The proof of Part (ii) establishes that these constraints are not binding, given the assumptions of the Proposition.

For $a_B < a_A$, B's probability of winning the election (its objective) is $\frac{1}{2}(1 + \bar{l}(a_A, a_B))$ and for $a_B > a_A$, it is $\frac{1}{2}(1 - \bar{l}(a_A, a_B))$, so in both cases a necessary condition for a local extreme point is $\frac{\partial \bar{l}(a_A, a_B)}{\partial a_B} = 0$. Using equation (7) and rearranging the derivative, this necessary condition implies

$$0 = \Delta v - \frac{(s - a_A)^2}{p_A} + \frac{(s - a_B)^2}{p_B} - \frac{2(s - a_B)(a_A - a_B)}{p_B}. \tag{30}$$

This equation is a quadratic in a_B with roots

$$a_B^{1,2}(a_A) = a_A \pm \sqrt{\frac{\Delta p}{p_A}(s - a_A)^2 + p_B \Delta v}, \text{ with } a_B^1 > a_A > a_B^2. \tag{31}$$

We now confirm that both of these roots are *strict* local maxima. Again using (7), we obtain the second derivative

$$\frac{\partial^2 \bar{l}(a_A, a_B)}{\partial a_B^2} = -\frac{\Delta p(s - a_A)^2 + p_A p_B \Delta v}{p_A p_B (a_A - a_B)^3}.$$

The sign of this expression determines the curvature of $\bar{l}(a_A, a_B)$ with respect to a_B. The numerator on the right side is positive. At the larger root, $a_B = a_B^1(a_A) > a_A$; here the right side is positive, the extreme point minimizes $\bar{l}$ and thus maximizes B's probability of winning. Similarly, at the smaller root, $a_B = a_B^2(a_A) < a_A$, so the extreme point maximizes $\bar{l}$ and thus maximizes B's probability of winning.

Therefore, both of the roots in equation (31) are strict local maxima. Candidate B's best response to a_A is the local maximum that gives B the higher winning probability. Passing through the discontinuity point (as a_A increases) of B's best response correspondence, B switches from a policy that is perceived by the voter as more rightist ($a_B^1(a_A) > a_A$) to one that is more leftist ($a_B^2(a_A) < a_A$). This switch enables B to appeal to the voter under more extreme realizations of l.

To compute the point of discontinuity, we equalize B's winning probability for a_B^1, $\frac{1}{2}(1 - \bar{l}(a_A, a_B^1(a_A)))$, with B's winning probability for a_B^2, $\frac{1}{2}(1 + \bar{l}(a_A, a_B^2(a_A)))$, to obtain the following indifference condition:

$$\bar{l}(a_A, a_B^1(a_A)) + \bar{l}(a_A, a_B^2(a_A)) = 0. \tag{32}$$

Using equations (7) and (31), this yields the location of the discontinuity point:

$$a_A^* = (1 - p_B)s. \tag{33}$$

Note that because equation (32) has a unique solution, for every $a_A \neq a_A^*$ one of the two local maxima gives B a strictly higher probability of winning than the other: for every $a_A \neq a_A^*$, B's best response correspondence is single-valued.

Part ii. The conclusion that A wins with certainty if B leads is an immediate consequence of Part i: A merely has to match B's choice sufficiently closely, i.e., she chooses any action in her best response correspondence associated with a_B.

We now turn to the case where A leads, maintaining the assumption that the constraints $-1 \leq \bar{l}(a_A, a_B) \leq 1$ are not binding; we confirm at the end of this step that Inequality (9) is necessary and sufficient for that assumption to hold.

We need to show that if A leads, she chooses a_A^*, the discontinuity point in B's best response correspondence. At this point (by construction) B is indifferent between a_B^1 and a_B^2. We want to show that a_A^* indeed maximizes A's winning probability when A leads. Recall that when $a_A < a_A^*$, as the leader, A becomes the leftist candidate (because B chooses $a_B^1(a_A) > a_A$ as the follower), so that $\pi_A(a_A, a_B^1(a_A)) = \frac{1}{2}(1 + \bar{l}(a_A, a_B^1(a_A)))$. Computing $\bar{l}(a_A, a_B^1(a_A))$ (using equation (7)), we obtain (after rearranging):

$$\bar{l}(a_A, a_B^1(a_A)) = \frac{a_A - (1 - p_B)s}{p_B} + \frac{1}{p_B}\sqrt{\frac{\Delta p}{p_A}(s - a_A)^2 + p_B \Delta v}. \tag{34}$$

We need to verify that $\bar{l}(a_A, a_B^1(a_A))$ is strictly increasing in a_A for $a_A < a_A^*$. To this end, we compute the derivative with respect to a_A to obtain:

$$\frac{d\bar{l}(a_A, a_B^1(a_A))}{da_A} = \frac{(p_A - p_B)(a_A - s) + \sqrt{p_A \Delta p (s - a_A)^2 + p_A^2 p_B \Delta v}}{p_B \sqrt{p_A \Delta p (s - a_A)^2 + p_A^2 p_B \Delta v}}.$$

The denominator on the right side of this equality is positive, so the expression is positive if and only if the numerator is positive. The numerator is always positive if $a_A \geq s$. For $a_A < s$, notice that the numerator is strictly larger than

$$(s - a_A) \left(\sqrt{p_A \Delta p} - \Delta p \right),$$

which we obtain by setting $\Delta v = 0$ (since the numerator increases in Δv). This expression is greater than zero (for $a_A < s$) if $\sqrt{p_A \Delta p} > \Delta p$. This inequality holds because $p_A > p_B$. Therefore, we conclude that $\bar{l}(a_A, a_B^1(a_A))$ is strictly increasing in a_A for $a_A < a_A^*$.

To complete the proof, one can follow similar steps as above to verify that $\frac{d\bar{l}(a_A, a_B^2(a_A))}{da_A} < 0$ when $a_A > a_A^*$, i.e., when A is the rightist candidate (not shown). So in this range, $\pi_A(a_A, a_B^2(a_A))$ decreases in a_A.

As the leader, the optimal policy for candidate A is thus a_A^*, the point where B is indifferent between choosing $a_B^1(a_A)$ or $a_B^2(a_A)$ as the follower. At this point, B is indifferent between these responses: they result in an identical winning probability for B. Therefore, A is also indifferent between these responses at $a_A = a_A^*$. Hence, conditional on leading, a_A^* maximizes A's payoff.

We now confirm that the constraints $-1 \leq \bar{l}(a_A, a_B) \leq 1$ are not binding. Recall that for $a_B > a_A$, B's probability of winning is $\frac{1}{2}(1 - \bar{l}(a_A, a_B))$. Here, B wins with positive probability if and only if $\bar{l}(a_A, a_B) < 1$. Using the definitions $a_A^* \equiv (1 - p_B)s$ and equation (31) for a_B^1 in equation (7) and simplifying, we obtain

$$\bar{l}(a_A^*, a_B^1(a_A^*)) = \frac{1}{\sqrt{p_B}} \sqrt{\frac{p_B \Delta p}{p_A} s^2 + \Delta v}. \tag{35}$$

This expression is strictly smaller than 1 if and only if

$$\Delta v < p_B \left(1 - \frac{\Delta p}{p_A} s^2 \right).$$

This reproduces Inequality (9) in the main text. The right-hand side is positive for all $p_A > p_B > 0$ and $s \in [-1, 1]$, so there always exists a positive Δv that satisfies Inequality (9). The calculations for a_B^2 are similar.

Finally, we confirm that even as the leader, A has a higher probability of winning than B, despite the fact that B obtains the second-mover advantage. Consider the case where, when A leads with a_A^*, B follows with $a_B = a_B^1$, so that $a_B > a_A$ in equilibrium. (The gist of the argument when B chooses a_B^2 is the same.) Here, B's winning probability is $\pi_B = \frac{1}{2}(1 - \bar{l}(a_A^*, a_B^1(a_A^*)) < 1/2$, where the inequality uses equation (35), which implies that $\bar{l}(a_A^*, a_B^1(a_A^*)) > 0$.

To summarize, we have shown that $\bar{l}(a_A^*, a_B^1(a_A^*)) < 1$ and $\bar{l}(a_A^*, a_B^2(a_A^*)) > -1$, so when A leads and chooses $a_A = a_A^*$, B has positive probability of winning. Because a_A^* maximizes A's probability of winning, it minimizes B's probability of winning. Therefore, B has a strictly higher (and thus positive) probability of winning for any other choice of a_A. Consequently, for all a_A, B's optimal response guarantees B a positive probability of winning: $\bar{l}(a_A, a_B^1(a_A)) < 1$ and/or $\bar{l}(a_A, a_B^2(a_A)) > -1$.

Part iii. The proof of Part ii establishes that A and B's best response correspondences are disjoint. Thus, there exists no pure strategy simultaneous move Nash equilibrium.

□

Proof. (Remark 1.) We confirm the first sentence of the Remark by checking that all of the inequalities used to establish Proposition 1 continue to hold when $\Delta p = 0$. We establish the second sentence in the Remark using equation (33), evaluated at $p_B = 1$, to obtain $a_A^* = 0$. □

Proof of Proposition 2. Much of the proof follows from specializing the proof of Proposition 1, setting $\Delta v = 0$. We discuss only those features that are different, including the outcome when one candidate locates at point s.

Part i. We can use the same argument as in Proposition 1 to establish that no point other than $a_A = a_B = s$ can be a potential pure strategy Nash equilibrium: for any $a_A = a_B \neq s$, A wins with probability 1, so B has an incentive to deviate, whereas for $a_B \neq s$ and $a_A \neq a_B$, A has an incentive to deviate (if she does not win with probability 1) and match a_B (which assures that A wins). Hence, in what follows, we only need to check when $a_A = a_B = s$ is an equilibrium.

For analyzing deviations from a (potential) equilibrium, the following observation is useful: If the rival locates at s, a candidate may choose a different location, the same location, or choose a location arbitrarily close to s ("shading"); in the latter case, for $s > 0$, shading from below (by choosing $s - \epsilon$, with ϵ arbitrarily close to zero) leads to a higher winning probability than shading from above, so we only consider the former (arguments for $s < 0$ are qualitatively the same and, thus, not shown here). Considering $a_B = s$ and $a_A = s - \epsilon$, we find (using equation (7)): $\bar{l}(s-\epsilon, s) = s - \frac{\epsilon}{2p_A}$. Since $a_A < a_B$, this leads to a winning probability for candidate A of $\pi_A = \frac{1}{2}(1+\bar{l}) = \frac{1}{2}(1+s-\frac{\epsilon}{2p_A})$ (for any $\epsilon > 0$ sufficiently small). If A matches (rather than shades) B's action, she obtains a winning probability of α, so that A does not shade B's action for any $\alpha \geq \frac{1}{2}(1+s-\frac{\epsilon}{2p_A})$. Considering $a_A = s$ and $a_B = s-\epsilon$, we find (using equation (7)): $\bar{l}(s, s-\epsilon) = s - \frac{\epsilon}{2p_B}$. Since $a_B < a_A$, this leads to a winning probability for candidate B of $\pi_B = \frac{1}{2}(1+\bar{l}) = \frac{1}{2}(1+s-\frac{\epsilon}{2p_B})$. If B matches (rather than shades) A's action, she obtains a winning probability of $1-\alpha$, so that B does not shade A's action for $1-\alpha \geq \frac{1}{2}(1+s-\frac{\epsilon}{2p_B})$, which never holds when $s > 0$ and ϵ is sufficiently small (given our assumption that $\alpha \geq 1/2$); B thus always shades when $s > 0$.

The action pair $a_A = a_B = s$ is a pure strategy Nash equilibrium if and only if neither candidate wants to defect from that position; this condition holds if and only if both of the above inequalities are satisfied, implying

$$\frac{1}{2}\big(1+s-\frac{\epsilon}{2p_A}\big) \leq \alpha \leq \frac{1}{2}\big(1-s+\frac{\epsilon}{2p_B}\big)\ \forall \epsilon > 0.$$

This pair of inequalities requires $\alpha = 1/2$ and $s = 0$.

Part ii. To start, we calculate the payoffs in a sequential equilibrium where candidate A leads with $a_A = a_A^*$ and B follows with $a_B = a_B^1(a_A^*)$ (following with $a_B^2(a_A^*)$ leads to identical payoffs). In what follows, unless otherwise stated, we only consider the case where $s \geq 0$ (arguments for the opposite case are qualitatively the same). Using equation (7), we find (after simplifying): $\bar{l}(a_A^*, a_B^1(a_A^*)) = s\sqrt{\frac{\Delta p}{p_A}}$. Since $a_B > a_A$, we have $\pi_B = \frac{1}{2}(1-\bar{l})$. This yields:

$$\pi_B^{follow} \equiv \frac{1}{2}\left(1-|s|\sqrt{\frac{\Delta p}{p_A}}\right). \tag{36}$$

The absolute value takes care of the case where $s < 0$. A's payoff is $\pi_A = 1 - \pi_B$.

First consider the case where $s \neq 0$, i.e., $s > 0$. If A chooses $a_A = s$ (instead of a_A^*), which is the only potentially profitable deviation, B always shades (see part (i)). A's resulting winning probability is $\pi_A = \frac{1}{2}(1-s+\frac{\epsilon}{2p_B})$, which is smaller than 1/2 for ϵ sufficiently small (given $s > 0$). Without the deviation, A's winning probability is (using equation (36)): $\pi_A = \frac{1}{2}\left(1+|s|\sqrt{\frac{\Delta p}{p_A}}\right) > \frac{1}{2}$. Deviating to $a_A = s$ is thus not profitable for candidate A. (A deviation to s by candidate B is not profitable by construction: $a_B^1(a_A^*)$ is B's best reply.) We also observe that B's winning probability (see equation (36)) is positive since $\Delta p < p_A$.

Now consider the case where $s = 0$, which yields $a_A^* = a_B^1(a_A^*) = a_B^2(a_A^*) = 0$ in a sequential equilibrium where A leads. Hence, $\pi_A = \alpha$ and $\pi_B = 1 - \alpha$. This is only an equilibrium if $\alpha = 1/2$. For $\alpha > 1/2$, B responds to $a_A^* = 0$ by shading (see part (i)), which leads to winning probabilities of $\pi_B = \frac{1}{2}(1 - \frac{\epsilon}{2p_B})$, and $\pi_A = \frac{1}{2}(1 + \frac{\epsilon}{2p_B})$. The latter is at least as large as 1/2. If A instead leads with some $a_A \neq 0$, B responds optimally, choosing $a_B^{1,2}(a_A)$, and places itself in between s and a_A. It is easy to verify that this leads to a winning probability of A that is strictly smaller than 1/2. Hence, A strictly prefers to lead with $a_A^* = 0$ also in this case, and B's winning probability is again positive (equal to 1/2 in the limit $\epsilon \to 0$).

Part iii. If B leads with any action $a_B \neq s$, A wins with certainty by choosing a sufficiently similar position. Therefore, B's best choice as leader is $a_B = s$. Focusing once more on the case where $s \geq 0$ (see above), for any $\alpha \geq \frac{1}{2}(1 + s)$, A has no incentive to shade B's action (see part (i)). Then the winning probabilities are $\pi_A = \alpha$ and $\pi_B = 1 - \alpha$. For any $\alpha < \frac{1}{2}(1 + s - \frac{\epsilon}{2p_A})$, there is an ϵ sufficiently small so that A prefers $a_A = s - \epsilon$ over $a_A = s$. Hence, A shades B's action for all $\alpha < \frac{1}{2}(1 + s)$ (see part (i)). Then B's winning probability is $\pi_B = \frac{1}{2}(1 - s + \frac{\epsilon}{2p_A})$, which is at least as large as $\frac{1}{2}(1 - s)$. In the limit ($\epsilon \to 0$), we thus obtain

$$\pi_B^{lead} \equiv \begin{cases} 1 - \alpha, & \text{if } \alpha \geq \frac{1+|s|}{2} \\ \frac{1-|s|}{2}, & \text{if } \frac{1}{2} \leq \alpha < \frac{1+|s|}{2}, \end{cases}$$

a lower bound for B's winning probability. (The absolute value once more takes care of the case where $s < 0$.) This can be written more concisely as:

$$\pi_B^{lead} = \min\{1 - \alpha, \frac{1 - |s|}{2}\}. \tag{37}$$

Candidate B's winning probability is thus positive, unless $\alpha = 1$.

Part iv. Comparing equations (36) and (37), using the fact that $\sqrt{\Delta p / p_A} < 1$, we find that

$$\pi_B^{follow} = \frac{1}{2}\left(1 - |s|\sqrt{\frac{\Delta p}{p_A}}\right) > \frac{1 - |s|}{2} \geq \pi_B^{lead}.$$

□

B Continuous-time framework and general results

Appendix B.1 provides a formal introduction of the general framework of Karp et al. (2025), which is the basis for the analysis of our electoral competition game in continuous time (Section 4). An intuitive summary is provided in Section 3.1 in the main text. Appendix B.2 formally characterizes SPNE in continuous-time games with a second-mover advantage, including our central Theorem 1 (Section 3.3 in the main text gives an intuitive summary of these results). Appendix B.3 contains the proofs.

B.1 General continuous-time setup

Each of two players, $i \in \{1, 2\}$, can move at most once in the time interval $[0, 1)$. Player i has a default action, ω_i, which we can think of as "waiting". The agent can also select an action a_i from a compact subset of $\mathbb{R}^n$ (e.g., an interval), A_i^-. Player i's full action set is thus $A_i = A_i^- \bigcup \omega_i$. If player i moves at $t_i \in [0, 1)$, selecting a_i, that action remains fixed after t_i. If i never moves, we set $t_i \equiv 1$ and $a_i \equiv \omega_i$. Note, however, that time $t = 1$ is *not* part of the game.

A *decision node* is a time $t \in [0, 1)$, combined with all information about the history, h, at t. At any node, either one player or neither player has moved; the game is over once both have moved. The history at t is thus either $h = \emptyset$ (nobody has moved), or player j has moved at $t_j \leq t$, implementing action a_j. In the latter case, the history is $h = [t_j, (\omega_i, a_j)]$, where (ω_i, a_j) indicates that player i maintains the default action ω_i, and player j "jumps" to action $a_j \in A_j^-$ at t_j. If both players moved at different points in time and player j moved first, then $h = [(t_j, (\omega_i, a_j)), (t_i, (a_i, a_j))]$. If both players move simultaneously at $t' \leq t$, then $h = [t', (a_1, a_2)]$. This differs from the history obtained if player i follows j's move immediately after observing it. In that case, players move sequentially, but at the same time t', and $h = [(t', (\omega_i, a_j)), (t', (a_i, a_j))]$. A *strategy*, denoted $f_i(t, h)$, instructs a player who has not previously moved whether to move and what action to take. We assume that players use pure strategies regarding their timing decisions. Section 3.2 explains why they do not mix over actions.

A "discontinuity point" in f_i is a decision node where either the player switches between the default action ω_i and some $a_i \in A_i^-$, or where i switches discontinuously between two different actions in A_i^- (such subgame at t can only be reached if i deviated from her own strategy before t and did not move, thus not implementing the previously planned action, before the discontinuity point). We adopt:

Assumption 2. For every history h of the game and $i = 1, 2$, $f_i(t, h)$ contains at most a finite number of discontinuity points and is right-continuous in t.

For a subgame beginning at t at which neither player has previously moved, we define the time $t_i^m(t) \in [t, 1]$ where player i plans to move according to his strategy $f(t, \emptyset)$, conditional on the rival not having previously moved, and an associated action $a_i^m(t) \in A_i$ that the player implements at $t_i^m(t)$. If i plans to wait at all $t \in [0, 1)$, provided that the other player also does not move, then $t_i^m(t) = 1$ and $a_i^m(t) = \omega_i$. Players who follow their strategies move simultaneously at $t \in [0, 1)$ if and only if $t_i^m(t) = t_j^m(t) = t$. In combination, the functions $t_i^m(t)$ and $a_i^m(t)$ describe the "leader's part" of player i's strategy.

For a subgame at $t \in [0, 1)$ where j has already moved, i's strategy $f_i(t, h)$ consists of functions $t_i^s(t|t_j, a_j) \in [t, 1]$ and $a_i^s(t|t_j, a_j) \in A_i$, determining the follower's response and time of response. The superscript s indicates that i is the second-mover. If i follows j's move immediately, then $t_i^s(t_j|t_j, a_j) = t_j$. If i plans not to make any move after observing j's move, then $t_i^s(t|t_j, a_j) = 1$ and $a_i^s(t|t_j, a_j) = \omega_i$. The functions $t_i^s(t|t_j, a_j)$ and $a_i^s(t|t_j, a_j)$ thus describe the "follower's part" of player i's strategy.

Assumption 2 implies that $t_i^m(t)$, $a_i^m(t)$, $t_i^s(t|t_j, a_j)$, and $a_i^s(t|t_j, a_j)$, are right-continuous and have a finite number of discontinuities in t. Therefore, there can be no sequence of subgames over which the leader-follower roles switch infinitely many times. This rules out a particular type of preemption, and is used in the proof of Theorem 1.

Assumption 2 does not specify how player i's strategy as a follower varies with t_j and a_j, the timing and action choice of the leader. We thus adopt:

Assumption 3. The functions $t_i^s(t|t, a_j^m(t))$ and $a_i^s(t|t, a_j^m(t))$ are right-continuous in t over any interval where player j plans to lead (i.e., where $t_j^m(t) = t$ holds).

Let player i's *payoff* be $\Pi_i(t_i, a_i, t_j, a_j)$. If neither player moves in $[0, 1)$, then player i obtains a fixed endgame payoff $E_i \equiv \Pi_i(1, \omega_i, 1, \omega_j)$.

Assumption 4. $\Pi_i(t_i, a_i, t_j, a_j)$ is *bounded* and *continuous*.

Continuity of players' payoffs in their timing and action choices seems natural in many applications where players' action sets are infinite.

B.2 General results for games with second-mover advantage

We first give a formal definition of a strict second-mover advantage:

Definition 1. *Player i has a* strict second-mover advantage *at $t \in (0, 1)$ if there exists some $\delta > 0$ such that $F_i(t) > \sup_{t' \in [t-\delta, t]} L_i(t')$, and at $t = 0$ if $F_i(0) > L_i(0)$.*

We now define three additional pieces of notation. Let

$$\tau_i \equiv \inf\{t \in [0,1) : L_i(t') < E_i \text{ for all } t' \geq t\} \text{ (“latest time for } i \text{ to lead”)}, \tag{38}$$

$$\bar{t}_i \equiv \operatorname*{argmax}_{t\in[0,1)} L_i(t), \text{ (“best time for } i \text{ to lead”)}, \tag{39}$$

$$\hat{t}_i \equiv \sup\{t \in [0, \bar{t}_j) : L_i(t) \geq F_i(\bar{t}_j)\} \text{ (“latest time for } i \text{ to preempt } j\text{”)}. \tag{40}$$

If there is no $t \in [0,1)$ such that $L_i(t') < E_i$ for all $t' \geq t$, we set $\tau_i = 1$.[52] We further assume that the maximum in Definition (39) exists and is unique.[53] The value $\hat{t}_i$ is the upper boundary of an interval where i prefers to preempt j by leading prior to j's optimal time to lead, $\bar{t}_j$, rather than wait for j to lead (see Figure 4B in Section 3.3 for illustration). If no such time exists, we set $\hat{t}_i = 0$.

We now collect conditions used in Theorem 1:

Condition 1. (Conditions relevant for Theorem 1)

(i) At every $t \in [\bar{t}_2, \tau_2)$, player 1 has a strict second-mover advantage and $L_2(t)$ is strictly decreasing over this interval; in addition, $\tau_2 > \tau_1$.

(iia) $\sup_{t\in[0,\bar{t}_2)} L_1(t) < F_1(\bar{t}_2)$.

(iib) $\sup_{t\in[0,\bar{t}_2)} L_1(t) > F_1(\bar{t}_2)$, and $\sup_{t\in[0,\bar{t}_1)} L_2(t) < F_2(\bar{t}_1)$.

(iii) At every $t \in [\bar{t}_1, \hat{t}_1)$, player 2 has a strict second-mover advantage, and $L_1(t)$ is strictly decreasing over this interval.

Condition 1.iia implies that player 1 does not want to preempt player 2 by leading prior to $\bar{t}_2$, 2's optimal time to lead. The first part of Condition 1.iib states that player 1 would like to preempt player 2 by leading before $\bar{t}_2$. The second part of this condition states that player 2 is willing to accommodate this preemption. Condition 1.iii mirrors Condition 1.i, with the roles of the two players reversed. The proof of Theorem 1 uses the observation that under Conditions 1.iib and 1.iii, the subgames in the interval $t \in [\bar{t}_1, \bar{t}_2)$ are isomorphic to those in the interval $t \in [\bar{t}_2, 1)$, with the roles of the players reversed.

The following condition assures *uniqueness* of the SPNE:[54]

[52] This is for instance the case if $L_i(t') \geq E_i$ for all $t \in [0,1)$.

[53] We thus rule out the (knife-edge) case where $L_i(t)$ has a downwards discontinuity exactly at the point where it attains its highest value.

[54] If this condition does not hold, then (as in Section 3.1) we assume that both players anticipate which maximizer is selected. Then the SPNE is unique conditional on that choice.

Condition 2. (Conditions for uniqueness in Theorem 1)

(i) For any $t \in [0, 1)$, $t_j \leq t$, and $a_j \in A_j^-$, the maximizers of equation (10) are unique.

(ii) For any $t \in [0, 1)$, the maximizer of equation (11) is unique.

If one player has previously moved, the other agent faces a standard decision problem. In Theorem 1, it is understood that a follower responds optimally to any prior move of the other player; and a player who leads at time t uses her best leader's action, $a_i^L(t)$ (Section 3.1).

Theorem 1. *Suppose that Assumptions 1–4, as well as Condition 1.i hold. Consider subgames starting at time $t \in [0, 1)$, such that nobody has previously moved:*

(i) (See Figure 4 in Section 3.3.) In subgames starting at any $t \in [\bar{t}_2, 1)$, there is a SPNE: player 1 waits at decision nodes $t \in [\bar{t}_2, 1)$; player 2 leads at nodes $t \in [\bar{t}_2, \tau_2)$, and waits at $t \in [\tau_2, 1)$.

(ii) (See panel A in Figure 4.) If Condition 1.iia holds, there is also a SPNE for subgames starting at $[0, \bar{t}_2)$: both players wait at nodes $t \in [0, \bar{t}_2)$. (Players' behavior at later decision nodes is as under (i).)

(iii) (See panel B in Figure 4.) If Conditions 1.iib and 1.iii hold, there is a SPNE for subgames starting at $[0, \bar{t}_2)$, with the strategies: player 1 leads at decision nodes $t \in [\bar{t}_1, \hat{t}_1)$ and waits at $t \in [0, \bar{t}_1) \bigcup [\hat{t}_1, \bar{t}_2)$; player 2 waits at decision nodes $t \in [0, \bar{t}_2)$. (Players' behavior at later decision nodes is as under (i).)

If, additionally, Condition 2 holds, then this is the ***unique*** *pure strategy SPNE.*

Section 3.3 in the main text provides an intuition for Theorem 1.

B.2.1 Extension: Non-monotonicity of $L_i(t)$

Condition 1(i), and thus Theorem 1, requires that over particular intervals, $L_i(t)$ is strictly decreasing. Here we show how to restore uniqueness in one case where this monotonicity fails. The top graphs in Figure 9 show two different types of non-monotonic functions $L_i(t)$. In panel A, $L_i(t)$ has an upwards jump, while in panel B it has an increasing segment. For both panels, between t' and t'' this player prefers to wait rather than lead, conditional on the other player also waiting. The functions $\bar{L}_i(t)$, shown at the bottom, are obtained by "ironing" (or "flattening") $L_i(t)$: between

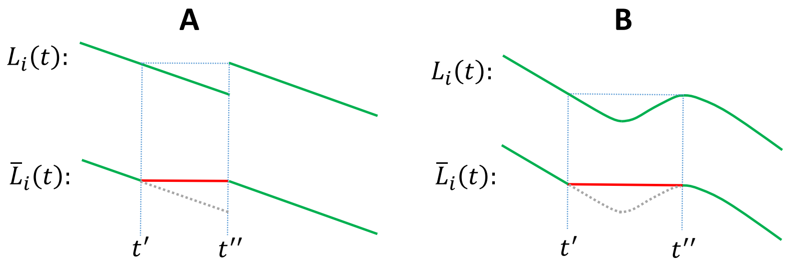

Figure 9: $L_i(t)$ non-monotonic; panel A (top part): $L_i(t)$ has an upwards discontinuity at t''; panel B (top): $L_i(t)$ has an increasing segment; bottom part of each panel: "modified leader's payoff function" $\bar{L}_i(t)$ (player i waits between t' and t'')

t' and t'', $\bar{L}_i(t)$ is constant, equal to the higher value at the discontinuity point (panel A), or at the local maximum (panel B).

The functions $L_i(t)$ and $\bar{L}_i(t)$ have different interpretations. By definition, $L_i(t)$ is the maximal payoff of player i, conditional on leading at time t, regardless of whether this is optimal for player i at t. The function $\bar{L}_i(t)$, on the other hand, incorporates the fact that between t' and t'', i prefers to wait, conditional on j also waiting. The red segment in each panel indicates this interval. Outside of the interval (t', t''), $\bar{L}_i(t)$ has the same interpretation as $L_i(t)$.

When the monotonicity of $L_i(t)$ stated in Condition 1(i) fails over (t', t''), a sufficient alternative condition for uniqueness is that j does not want to lead over this interval given that the alternative is to wait until t'' and enjoy the second-mover advantage: j does not want to preempt i. This sufficient condition is

Condition 3. (Condition relevant for Corollary 2)
$\sup_{t \in [t', t'']} L_j(t) < F_j(t'')$.

This alternative to Condition 1.i turns out to be useful for Section 4, so we state:[55]

Corollary 2. *Suppose that Condition 1.i fails, so that a non-monotonicity in $L_i(t)$ gives rise to the interval $[t', t'']$ over which the "ironed" function $\bar{L}_i(t)$ is constant. Moreover, Condition 3 is satisfied. The unique SPNE is as described in Theorem 1, except that both agents wait over the interval $[t', t'']$.*

[55]The proof of this corollary closely follows the proof of Theorem 1, except that over $[t', t'']$ player i prefers to wait, knowing that j will not preempt it. If the function $L_i(t)$ (rather than the ironed function $\bar{L}_i(t)$) were constant over $[t', t'']$, then there is a (trivial) multiplicity of equilibria: over some interval i is indifferent between leading and waiting.

B.3 Proofs of general results for continuous-time games

We establish two lemmas before proving Theorem 1. Lemma 1 establishes right continuity of the functions $F_j(t)$ and $L_i(t)$ (defined in Section 3.1 in the main text), and Lemma 2 provides conditions over an interval, ensuring that the players' leader-follower role does not switch.

Lemma 1. *Suppose that Assumptions 2–4 hold. Then*

(i) at any $t \in [0,1)$ where $a_i^L(t)$ is continuous, $F_j(t)$ is also continuous;

(ii) at any $t \in [0,1)$ where $a_j^{s*}(t|t, a_i^L(t))$ and $t_j^{s*}(t|t, a_i^L(t))$ are continuous, $L_i(t)$ is also continuous; and

(iii) $L_i(t)$ and $F_i(t)$ are right-continuous in t.

Proof of Lemma 1. Part (i): Due to Assumption 4 and given that (by the definition of L_i and F_j) player i leads at t and player j plays her optimal response to i's move, a discontinuity in the follower's payoff, $F_j(t)$, can only arise at t if the leader's action choice $a_i^L(t)$ changes discontinuously at t. Without a discontinuous change in $a_i^L(t)$, the follower's behavior (timing and/or action choice) may still change discontinuously at t, but only if the follower is indifferent between these types of responses. Hence, a discontinuous change in the follower's response cannot lead to a discontinuity in $F_j(t)$, if $a_i^L(t)$ is continuous at t.

Part (ii): Similarly as in Part (i), as long as $a_j^{s*}(t|t, a_i^L(t))$ and $t_j^{s*}(t|t, a_i^L(t))$ change continuously with t, a discontinuity in the leader's payoff, $L_i(t)$, cannot arise due to Assumption 4. This is because, as long as the follower's response does not change discontinuously, the leader has to be indifferent between two actions at t, in order to be willing to change her action discontinuously at this time. Hence, although $a_i^L(t)$ may be discontinuous at some t, this discontinuity does not lead to a discontinuity in $L_i(t)$, provided that $a_j^{s*}(t|t, a_i^L(t))$ and $t_j^{s*}(t|t, a_i^L(t))$ are continuous at t.

Part (iii): Given statements (i) and (ii) in the Lemma, it follows from Assumptions 2 and 3 that $L_i(t)$ and $F_j(t)$ are right-continuous: if there is a discontinuity in the leader's or in the follower's behavior (or both) at some t, then the "new" behavior (to the right of the discontinuity point in the respective player's strategy) is adopted at t, i.e., there is a first decision node where this happens. This implies right-continuity of the payoff functions $L_i(t)$ and $F_i(t)$. This holds also in cases where both the leader's and the follower's behavior changes discontinuously at the same t (so neither case (i) nor case (ii) applies).

It remains to be shown that this behavior is not in conflict with players' optimization problems (10) and (11). In other words, we need to show that the assumption of right-continuity of the players' strategies does not in itself restrict players in a way that conflicts with their respective payoff-maximizing behavior, subject to maintaining the timing of the leader's move and holding the order of players' moves fixed. To this end, note that by their definition, the functions $a_i^L(t)$ as well as $a_j^{s*}(t|t, a_i^L(t))$ and $t_j^{s*}(t|t, a_i^L(t))$, are mutually optimized, for the given order of moves and subject to player i leading at time t. Hence, preemption by player j is ruled out by assumption.

We need only show that a player looses nothing by switching to a new type of behavior at a first decision node at some t, instead of immediately after time t. (The latter case would violate right-continuity of players' strategies.) To confirm this claim, consider some interval where players' behavior changes continuously with time, following a discontinuous change in the leader's or the follower's behavior (or both) at t. Moving backwards in time, the same kind of behavior as inside of this interval can also be adopted at the decision node *at* time t, without a discontinuous change in either player's payoff (thanks to Assumption 4). Hence, although there might exist another "mutually optimized behavior" in which players behave differently at the decision node at t (for example by extending their type of behavior from immediately before t onto the decision node at t, or by selecting another type of mutually optimized behavior if such exists), it must co-exist with the mutually optimized behavior that is selected by imposing right-continuity on the players' strategies (Assumptions 2 and 3), i.e., the type of behavior shortly after t. Hence, there is no strictly profitable deviation at any t that is ruled out by imposing Assumptions 2 and 3, conditional on maintaining the order of players' moves and player i leading at t. □

Lemma 2. *Suppose that Assumptions 1–4 (see Section 3.2 and Appendix B.1) hold and there is an interval $0 \leq t_l < t_h < 1$ such that: player i has a strict second-mover advantage at every $t \in (t_l, t_h]$, and $L_j(t)$ is strictly decreasing at every $t \in [t_l, t_h]$. Suppose further that there exists a unique SPNE in the subgame that starts at t_h, such that player j leads and player i waits at t_h. Then player j also leads and player i waits in the unique SPNE at all decision nodes $t \in [t_l, t_h)$. In particular, the identity of the leader cannot switch at any t in the interval $[t_l, t_h]$.*

Proof. By Assumption 1, there are no SPNE with simultaneous moves: all equilibria involve either sequential moves or waiting. As j leads at t_h and $L_j(t)$ is strictly decreasing over $[t_l, t_h]$, both players waiting over the entire interval is not an equilibrium, since moving immediately is a profitable deviation for player j. Therefore, any SPNE involves sequential moves.

Figure 10 provides a graphical aide for the proof, which proceeds by contradiction. Suppose to the contrary of the statement in Lemma 2, that there are decision nodes $t \in [t_l, t_h)$ where j does not lead. If player i also waits at those nodes, then player j is strictly better off by deviating and leading at t, because $L_j(t)$ is strictly decreasing at every $t \in [t_l, t_h]$. If player i instead leads at those decision nodes, then let t' be the largest t (decision node) where players "switch" their roles between leading and following – see Figure 10. (Thus, i waits and j leads over $[t', t_h]$.) But then there exists an interval of decision nodes prior to t' where waiting until t' produces a strictly higher payoff for player i than leading during this interval, due to her strict second-mover advantage (see Definition 1). Moreover, if i deviates from the proposed strategy and waits over this interval, then j's best response is to lead in this interval. Thus, the switching strategy shown in Figure 10 cannot be part of any SPNE under the stated conditions. $\square$

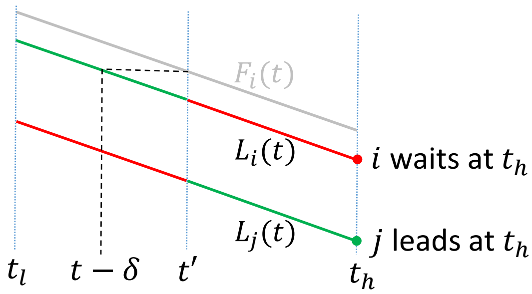

Figure 10: $L_i(t)$ green/red: player i leads/waits at t; the case shown is *incompatible* with SPNE if player i has strict second-mover advantage and $L_j(t)$ strictly decreasing

Figure 10 illustrates Lemma 2.[56] The red (respectively, green) segment indicates that the player plans to wait (respectively, lead) over that interval. The proof of Lemma 2 shows that the situation in Figure 10 leads to a contradiction, and thus cannot be part of any SPNE under the stated conditions.

Proof of Theorem 1. By Assumption 1, there are no SPNE with simultaneous moves: all equilibria involve either sequential moves or waiting. We first demonstrate that the behavior described in the theorem is an equilibrium, and then that it is unique, provided that Condition 2 is satisfied.

Part i: We start with the latest subgames, and work backwards in time.

Subames at $t \in [\tau_2, 1)$. This step is relevant only if $\tau_2 < 1$.

[56]For simplicity, the figure shows L_i as monotonically decreasing as well, but neither the statement of the lemma nor the proof use this feature.

We first show that both players waiting at all times $t \in [\tau_2, 1)$ is a SPNE in this part of the game. Because $\tau_2 > \tau_1$, waiting is player 1's best response if player 2 waits for $t \geq \tau_2$. For player 2, due to the definition of τ_i and by Lemma 1, it holds that $L_2(t) \leq E_2$ for all $t \geq \tau_2$. Therefore, waiting is also a best response for player 2 if player 1 waits for $t \geq \tau_2$, so that both payers waiting at all times $t \in [\tau_2, 1)$ is a SPNE.

We now show that this SPNE is unique, provided that Condition 2 holds. Given that there are no equilibria with simultaneous moves, we need only show that there is also no equilibrium in any subgame beginning at $t \in [\tau_2, 1)$ such that players move sequentially. We demonstrate this by contradiction. Suppose, contrary to our claim, that there exists a SPNE such that player i leads at some $t \in (\tau_2, 1)$. (We treat $t = \tau_2$ separately below.) Over this interval $E_i > L_i(t)$ by the definition of τ_i and the assumption $\tau_2 > \tau_1$. Here, the only rationale for player i $(i = 1, 2)$ to move at t is to preempt a later move by player j. Thus, for such a SPNE to exist, subgames over which one player leads and the other follows would have to alternate along the time axis. If they alternated a finite number of times, there would be a final interval over which a particular player, e.g. i, leads and j waits. However, over this interval, waiting is a profitable deviation for i. Therefore, in subgames over which the leader-follower roles alternate, they must do so an infinite number of times. However, that behavior contradicts Assumption 2, which requires that there are a finite number of discontinuity points in each player's strategy. Therefore, in the unique pure strategy SPNE for all $t \in (\tau_2, 1)$, both agents wait.

Subgame at $t = \tau_2$: We showed above that $L_2(\tau_2) \leq E_2$. If $L_2(\tau_2) = E_2$, then player 2 is indifferent between leading or waiting at τ_2. However, we have shown above that for all $t \in (\tau_2, 1)$, both agents wait. Therefore, by Assumption 2, each player must also wait at τ_2.

Subgames at $t \in [\bar{t}_2, \tau_2)$: We first show that the strategies of the two players stated in the theorem are a SPNE. Since player 1 has a strict second-mover advantage (Condition 1.i), there is no profitable deviation for player 1, who expects player 2 to move immediately at any t (decision node) in this interval. There is also no profitable deviation for player 2, because player 1 is waiting and $L_2(t)$ is decreasing in this range. (Thus, a delay lowers player 2's payoff).

It remains to be shown that these strategies constitute a *unique* pure strategy SPNE for subgames in this interval under Condition 2. Assumption 1 guarantees that there are no equilibria with simultaneous moves. Furthermore, there is no SPNE in which both players wait at any $t \in [\bar{t}_2, \tau_2)$, because $L_2(t) > E_2$ holds for any t in this interval. Therefore, it remains to be shown that there cannot exist any sequential move SPNE other than the one stated in the theorem.

In particular, we show that there does not exist any SPNE where player 1 leads in any of these subgames. We rule out the existence of such equilibria by contradiction. Suppose, contrary to our claim, that there exists a SPNE where player 1 leads in some subgame beginning at a time $t \in (\max\{\tau_1, \bar{t}_2\}, \tau_2)$, a non-empty set. Because player 1 has a strict second-mover advantage at any t in this range and $L_1(t) < E_1$, player 1's plan to lead at t can be sustained in a SPNE, only if this player fears that player 2 will otherwise lead at a strictly later point in time, after a delay of at least δ units of time (for some $\delta > 0$). However, such a period of delay is incompatible with $L_2(t)$ being strictly decreasing in this range – a contradiction. It then follows that at any $t \in (\max\{\tau_1, \bar{t}_2\}, \tau_2)$, player 2 leads and player 1 waits. Thus, we can take any t in this range as the "starting value" t_h, to initiate the procedure of working backwards in time until $\bar{t}_2$, applying Lemma 2. It follows that the proposed SPNE is unique.

Part ii The assumptions in Parts *i* and *ii* of the theorem differ only by the inclusion of Condition 1.iia to Part *ii*. That condition puts added structure only on payoffs prior to $\bar{t}_2$. It therefore does not alter (relative to Part *i*) the equilibrium for $t \in [\bar{t}_2, 1)$. The proof of Part *i* already confirms the characteristics of the equilibrium over that interval. Therefore, to verify Part *ii* of the Theorem we need only confirm that in the SPNE both players wait at subgames $t \in [0, \bar{t}_2)$. We first show that waiting over this interval is a SPNE and then we show that it is unique, given Condition 2.

By Condition 1.iia, player 1 does not benefit from preempting player 2, if player 2 plans to lead at $\bar{t}_2$, but not before. Player 2 also has no profitable deviation from the waiting strategy, because leading at $\bar{t}_2$ is more profitable than leading at any other time. Thus, the pair of waiting strategies for $t \in [0, \bar{t}_2)$ is a SPNE.

We now establish uniqueness. By Condition 1.iia, the only reason that player 1 might want to lead over this interval is to preempt her rival from leading at a later point, but prior to $\bar{t}_2$. The only reason that player 2 might want to lead at such a point is to preempt player 1 from leading at a later point, also prior to $\bar{t}_2$. Thus, a strategy involving either player leading at $t \in [0, \bar{t}_2)$ requires that there be a sequence of subgames over which the leader-follower role switches. An argument that parallels that given above for the interval $(\tau_2, 1)$ implies that this sequence must be infinite. However, an infinite sequence of switches implies that strategies are discontinuous at infinitely many points, contradicting Assumption 2. Thus, the pair of waiting strategies for $t \in [0, \bar{t}_2)$ is the unique SPNE. Given these strategies, the unique outcome of the game is for player 2 to lead at $\bar{t}_2$ and for player 1 to follow.

Part iii As with Part *ii*, the assumptions of Part *iii* differ from those of Part *i* only in placing added structure on the payoffs prior to $\bar{t}_2$. This added structure does not alter the equilbrium at subgames $t \geq \bar{t}_2$. Therefore the equilibrium for $t \geq \bar{t}_2$ described in Part *i* remains an equilibrium. We need only determine the equilibrium for $t < \bar{t}_2$.

Subgames at $t \in [\bar{t}_1, \bar{t}_2)$: Given that a unique SPNE has already been identified for the subgame beginning at $\bar{t}_2$, where player 2 leads, let us denote the equilibrium payoffs of the players in that subgame by $E_2' \equiv L_2(\bar{t}_2)$, and $E_1' \equiv F_1(\bar{t}_2)$. From the perspective of earlier subgames, these values are fixed. A move by any player before $\bar{t}_2$ effectively ends the game; the subgame at $\bar{t}_2$ can be reached only if nobody moves earlier.

Now compare the subgames starting at values of $t \in [\bar{t}_2, 1)$ with those starting at $t \in [\bar{t}_1, \bar{t}_2)$. In both cases, there is an ideal time for one agent to move, and also a latest time that they are willing to move given that the other agent follows a waiting strategy beyond that time. In between these two points $L_i(t)$ is strictly decreasing for this agent and the other agent has a strict second-mover advantage.

Therefore, the game played over $[\bar{t}_1, \bar{t}_2)$, is isomorphic to the game over the interval $[\bar{t}_2, 1)$. The roles of the two players are reversed, $\hat{t}_1$ in the former game replaces τ_2 in the latter game, and E_i' $(i = 1, 2)$ in the former game replaces E_i in the latter.[57] The same set of assumptions applies in each of these ranges (except for the reversal of the identities of the two players). In particular, player 2 has a strict second-mover advantage in the range $[\bar{t}_1, \hat{t})$, and player 1's payoff from leading is strictly decreasing in this range. Our earlier proof for the range $[\bar{t}_2, 1)$ can thus be applied to identify a (unique) SPNE for the range $[\bar{t}_1, \bar{t}_2)$.

Subgames at $t \in [0, \bar{t}_1)$: In the same way, it is easy to see that the game in $[0, \bar{t}_1)$, under Condition 1.iib is isomorphic to the game in the range $[0, \bar{t}_2)$ under Condition1.iia. It thus follows from the same reasoning as in Part *ii* that the unique SPNE is for both players to wait at every $t \in [0, \bar{t}_1)$. □

C Proofs for Sections 4 and 5

Proof of Proposition 4. Inspection of the payoffs and the strategies confirms that Assumptions 2 – 4 of Theorem 1 are satisfied. Our parametric assumptions below equation (14) (either (i) $\Delta v > 0 \wedge \Delta p \geq 0$, or (ii) $\Delta v = 0 \wedge \Delta p > 0 \wedge s \neq 0$) together with Propositions 1 and 2 imply that there is no pure strategy equilibrium in the

[57] Note that, since $\bar{t}_2$ is the optimal time for player 2 to lead, there is no $t \in [0, \bar{t}_2)$ such that $L_2(t) \geq E_2'$.

static game. By Proposition 3 there thus exists no simultaneous move equilibria in the dynamic game either, so Assumption 1 is also satisfied.

Conditions 1.i – (iia) and 2 are also satisfied by inspection. The strict monotonicity of $L_A(t)$ required by Condition 1.i is satisfied if $\overline{\pi}_A \leq \pi_A^L$ but not if $\overline{\pi}_A > \pi_A^L$. In the latter case, inequality (20) enables us to use Corollary 2 to modify the equilibrium strategy from Theorem 1 for the interval $(t_A^\sharp, t_A^{crit})$. Inequality (19) ensures that $\overline{\pi}_B$ is not so large that B would want to lead at the point beyond which A prefers not to follow (i.e., to wait until $t = 1$). This condition implies that the definition of τ_B in this game is indeed the same as in Theorem 1.

If inequality (19) fails, then beginning at t_A^{crit} there is a range where both candidates have a first-mover advantage. The conclusion that B has a first-mover advantage follows directly from $L_B(t_A^{crit}) > 0$, and the fact that $t_A^{crit} > t_B^{crit}$ (so B's payoff as follower is zero in this range). Candidate A has a first-mover advantage due to our assumption $\overline{\pi}_A \geq \overline{\pi}_B$, and also $F_A(t) = 0$ because $t \geq t_A^{crit}$.

When both players have a first-mover advantage, Theorem 1 does not apply, and it is easy to see that either of the players may lead in subgames within this range. Hence, the identity of the leader at $t = t_A^{crit}$ is not uniquely determined. However, for subgames beginning at any t between t_B^{crit} and t_A^{crit}, we can uniquely identify A as the leader. Shortly before t_A^{crit}, A would rather lead than wait to enjoy her first-mover advantage. If A waits until t_A^{crit} and leads at that time, her payoff is lower than if she had led earlier (because $L_A(t)$ is decreasing); if A follows at t_A^{crit} her payoff is zero. Thus, A chooses to lead during an interval before t_A^{crit}. Over this interval, B receives a negative payoff by leading, so B waits. Reasoning backwards in time, A leads at all times within this range. We can then apply the arguments of Theorem 1 for earlier subgames. □

Proof of Proposition 5. For the case where $c_A > c_B$, note first that there is no SPNE that does not involve a move by B, because by following a move of A taking place before t_B^{crit}, or by leading before τ_B, B obtains a strictly positive payoff (and one of these options is always available to this candidate). The key observation to establish the respective claim in the proposition (for $c_A > c_B$ and under assumption (28)) is that A's leader's profit at $t = 0$ exceeds the highest payoff that she may obtain in any later subgame that entails a policy announcement by B (as leader or as follower). If B leads at $\bar{t}_B$, A obtains the payoff $\frac{1}{2}\left(1 - \omega_A + \sqrt{\Delta v}\right)$. This is smaller than A's payoff from leading at $t = 0$: $\frac{1}{2}\left(1 + \sqrt{\Delta v}\right)$ for any $\omega_A \neq 0$. If A leads at t_B^{crit} where $L_A(t)$ attains a local maximum (because B stays out as follower from t_B^{crit} onwards), it is straight-forward to verify that her leader's payoff is smaller than at $t = 0$: leading at t_B^{crit}, her payoff is $1 - c_A t_B^{crit}$. Comparing this with her payoff from leading at $t = 0$:

$\frac{1}{2}(1+\sqrt{\Delta v})$, using equation (26), we find that the latter is larger than the former if $\Delta v < 1$. This holds by assumption (9) from Section 2, that we maintain for this variant of the game. We will argue further below that (unless $\min\{\bar{t}_B, t_B^{crit}\} \geq 1$) B may either lead at $\bar{t}_B$, where its leader's payoff attains its maximum, or at t_B^{crit}. But in the latter case, A's payoff from following is identical to its leader's payoff at the same calendar time. Therefore, there exists no later subgame in which A can achieve a higher payoff than from leading immediately at $t = 0$. And if $\min\{\bar{t}_B, t_B^{crit}\} \geq 1$, the relevant point of comparison for candidate A is her payoff if B leads just before $t = 1$. In this case, assumption (28) becomes relevant that assures that leading at the beginning of the game still yields a higher payoff to candidate A than the payoff she obtains if she does not lead at any $t \in [0, 1)$, and B leads just before the end of the game.

To complete the argument for the case where assumption (28) holds and $c_A > c_B$, we analyze when there exists a SPNE. We distinguish the following cases:

Case: $\bar{t}_B < \min\{t_B^{crit}, 1\}$*:* This is the case illustrated in Figure 8. Here, we can directly apply Theorem 1 to identify a unique pure strategy SPNE for the game. As is easy to verify, Condition 1 (see Appendix B.2) is fulfilled; which cases are relevant will become clear below. Recall that $\tau_A = 0$, and $\tau_B > 0$. Hence, A is more patient in this dimension. We confirm that for $t \geq \bar{t}_B$, A has a strict second-mover advantage and $L_B(t)$ is strictly decreasing in this range. Hence, B leads in any subgame at $t \in [\bar{t}_B, \tau_B)$ and waits thereafter (if $\tau_B < 1$), while A waits at all t in this range. In subgames at $t \in [\hat{t}_A, \bar{t}_B)$, both players wait, and in subgames at $t \in [0, \hat{t}_A)$, A leads. In the intervals where both players wait, no player has an incentive to preempt their rival.

Case: $t_B^{crit} < \min\{\bar{t}_B, 1\}$*:* The main difference to the previous case is that there is now a range of subgames: $t \in (t_B^{crit}, \min\{\bar{t}_B, 1\})$ where $L_A(t) = F_A(t)$, and $L_B(t)$ is increasing, with $F_B(t) = 0$ (first-mover advantage for B). In this range, A leads: as $L_A(t)$ is decreasing, A leads rather earlier than later, and A does not wait. If A were to wait, B would also wait (because $L_B(t)$ is increasing), but then A preempts B. Furthermore, there is a range preceding t_B^{crit} where A has a strict second-mover advantage, $L_A(t)$ decreases, while B has a first-mover advantage and $L_B(t)$ increases with t. In this range, A does not lead because she expects a higher payoff if both players wait until t_B^{crit} (see above). However, given that A waits, B also prefers to wait until t_B^{crit} because $L_B(t)$ is increasing. Existence of a SPNE in pure strategies now requires that we relax the right-continuity assumption on players' strategies (Assumption 2, Appendix B.1). The only sensible outcome is where B leads at t_B^{crit} (and waits thereafter) while A waits at t_B^{crit} (and leads thereafter). Otherwise, if A were to lead at t_B^{crit}, B has an incentive to preempt this move, but at the latest possible instant. But because there is no "latest instant" before t_B^{crit}, this strategy is not well defined. Then in all subgames at $t \in [\hat{t}_A, t_B^{crit})$, both players wait, while A leads and B waits at all $t \in [0, \hat{t}_A)$.

Case: $\min\{\bar{t}_B, t_B^{crit}\} \geq 1$*:* The situation is qualitatively the same as in the previous case at times before t_B^{crit}, which is now replaced by time $t = 1$. However, $t = 1$ is not part of the game, so B cannot lead at $t = 1$ even when the right-continuity requirement is relaxed. Thus, strictly speaking, we cannot identify a SPNE for subgames after $\hat{t}_A$. But candidate B will seek to lead as shortly before $t = 1$ as possible. Therefore, an upper boundary on their leader's payoff is $L_B(1)$. Irrespective of the indeterminacy at these later subgames, it continues to hold that candidate A leads in all subgames before $\hat{t}_A$, so that the result stated in the proposition is confirmed.

The other case where $c_A < c_B$ is trivial: in this case, candidate B cannot achieve a positive payoff from leading at any $t > 0$. The only possibility for this player to lead without incurring a loss is at $t = 0$. Here, right-continuity serves as a tie-breaker and we conclude that B waits at all $t \in [0, 1)$. Given this strategy, A also waits at all $t \in [0, 1)$ and wins the election with probability 1. Hence, A achieves her maximum payoff and neither player announces a policy platform at any time. □

D Online Appendix (for online publication only)

D.1 Basic electoral competition game, case $\alpha < 1/2$

We show here the parts of the proof of Proposition 2 that need to be modified to include the case $\alpha < 1/2$. The results stay qualitatively the same. The modifications concern only part *ii* of the proof.

Case: $s \neq 0$ Suppose that $\alpha \leq (1-s)/2$, so that (as shown in the proof of Part (i)) B has no strict incentive to shade A's action (by choosing a slightly lower position). Hence, leading with $a_A = s$ gives A a payoff of α. If A were to lead with any $a_A^* < a_A < s$, B would pick $a_B^2(a_A) = a_A - \sqrt{\frac{\Delta p}{p_A}(s-a_A)^2}$ (see the proof of Proposition 1). A would thus become the more rightist candidate. As A is also the more competent candidate, a voter with $l \geq a_A$ would then always prefer candidate A over B, and A's payoff would thus be at least $(1-a_A)/2$ which is larger than $(1-s)/2$, by $a_A < s$. If, instead, $1/2 > \alpha > (1-s)/2$, then B's optimal response to $a_A = s$ is to shade that policy. As shown in Part (i), B wins with probability $\frac{1}{2}(1+s-\frac{\epsilon}{2p_B})$ if she shades with $a_B = s - \epsilon$, which means A's probability of winning is then $\frac{1}{2}(1-s+\frac{\epsilon}{2p_B})$. However, if A would lead with $a_A^* < a_A < s - \frac{\epsilon}{2P_A}$, by the same reasoning as above, she would get a higher payoff. Hence, leading with $a_A = s$ can never be more profitable than leading with $a_A = a_A^*$.

Case: $s = 0$: If $\alpha < \frac{1}{2}$ then it is a best response for B to follow with $a_B = 0$ when A leads with $a_A^* = 0$. If A would instead lead with $a_A = \epsilon > 0$, then B's best response correspondence has this candidate follow with $a_B = \epsilon(1 - \sqrt{\frac{\Delta p}{p_A}})$. Candidate A then wins with probability

$$\frac{1}{2}\left(1 - \epsilon\frac{p_B - p_A\left(1-\sqrt{\frac{\Delta p}{p_A}}\right)^2}{2p_Ap_B\sqrt{\frac{\Delta p}{p_A}}}\right).$$

Candidate A's probability of winning is thus higher if she shades 0 than if she chooses $a_A = 0$. It is easy to see that the same is true if A chooses $a_A = -\epsilon$ instead.

D.2 Basic electoral competition game, case $\Delta v < 0, \Delta p > 0$

Whereas the main text assumes that one candidate is unambiguously stronger, here we consider the case where candidate A enjoys a competence advantage, while B dominates in valence.[58] Unlike the case where one candidate dominates in both characteristics, in this mixed case, the incentives to match or to differentiate away from the

[58]The case where $\Delta v > 0$ and $\Delta p < 0$ is obtained by swapping the indices of the candidates.

opponent's policy platform depend on the location in the policy space. The reason is that the competence effect Δq changes with the distance from s. The game no longer has the structure of an anti-coordination game, because it is sometimes the one, and sometimes the other candidate who prefers to match or to differentiate away from the other candidate's action choice.

This is easiest to see along the diagonal where $a_A = a_B$. Here, it holds that $\Delta q|_{\Delta a=0}$ increases with the distance between a and s (see equation (3)), with $\Delta q|_{\Delta a=0} = 0$ for $a = s$. Intuitively, in order to benefit from her competence advantage, candidate A needs to announce a policy platform that is sufficiently different from the status quo. Otherwise, competence hardly matters. The valence advantage of candidate B, by contrast, is constant: it does not depend on candidates' actions. Therefore, along the diagonal, for values of $a = a_A = a_B$, B wins for sure if a is close to s; if a is sufficiently far away from s, by contrast, A wins the election.

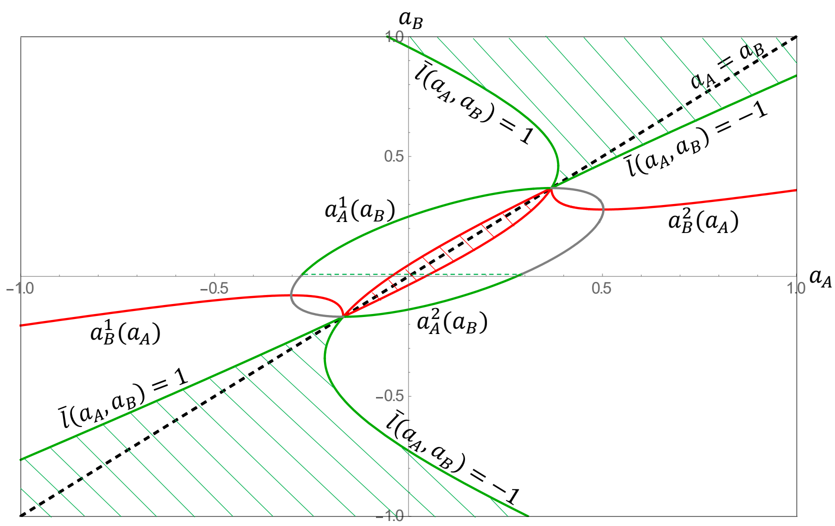

Figure 11: Best response correspondences, for $p_A = 0.9$, $p_B = 0.4$, $s = 0.1$, $\Delta v = -0.1$; candidate A: green; candidate B: red

By continuity, this logic extends to locations near the diagonal. Holding Δa fixed at some small (positive or negative) value, when varying a, there is again an interval around s where B wins, whereas outside of this interval, A wins (see Figure 11). Hence, for values of a_A and a_B sufficiently far away from s, the incentives in the game resemble those in the case where $\Delta v > 0$ and $\Delta p > 0$: A prefers to match B's platform, whereas B prefers to differentiate away from a_A. By contrast, for values of a_A near s, B has an incentive to match A's platform, because the valence advantage then assures that B wins the election. A then has an incentive to differentiate away from a_B to benefit from her competence advantage. In Figure 11, the combinations of a_A

and a_B where B wins for sure (around the diagonal and near s) are indicated by the red striped area. The green striped areas around the diagonal further away from s are the combinations for which A wins for sure. For intermediate values of a_B, A's best response is either smaller or larger than a_B, with a discontinuous switch between these values $(a_A^1(a_B), a_A^2(a_B))$, as illustrated by the thin dashed line (green) in Figure 11. This resembles our earlier findings (see Section 2) where B's best response correspondence entailed such a switch, but now it is candidate A who switches discontinuously from a rightist ($a_A > a_B$) to a leftist policy as a_B is increased. The switching point in A's best response correspondence is easily calculated (using similar steps as in the proof of Proposition 1), and is located at $a_B = (1 - p_A)s$.

A formal proof is omitted, but a simple thought reveals that also in this mixed case, a pure strategy Nash equilibrium often does not exist under simultaneous moves. This is due to the constant sum property of the electoral competition game: an increase in one candidate's winning probability implies a decrease in the other's winning probability, and vice versa. Therefore, if one candidate's payoff attains a local maximum in the a_A-a_B-space, this is generally a local minimum of the other candidate's payoff function.

In Figure 11, this can be seen at the intersection points of the curves $a_B^1(a_A)$ (red) and $a_A^1(a_B)$ (grey) in the left part of the figure, as well as $a_B^2(a_A)$ (red) and $a_A^2(a_B)$ (grey) on the right. The grey color indicates that this is *not* the best reply chosen by candidate A in the respective range, because a response on the other side of the political spectrum leads to a higher winning probability of this candidate.[59] Additionally, the intersection points of the curves $\bar{l}(a_A, a_B) = -1$ and $\bar{l}(a_A, a_B) = 1$, where the red striped and the green striped areas become tangent, are also not candidates for a pure strategy Nash equilibrium. At these points, at least one candidate will have an incentive to shade the policy of her rival, because this leads to a discontinuous change in candidates' winning probabilities. Exceptions are possible for some specific parameter constellations (see Section 2).

Whether a candidate is relatively better or worse off as the leader or as the follower, depends strongly on the parameters, which can lead to very tedious case distinctions. This mixed case is thus less amenable for an extension to a game that can be analyzed in continuous time. Although this is possible, we omit this case in our dynamic analysis of the electoral competition game.

[59] Results may differ when the parameter values are changed. We do not claim any generality here and content ourselves with these intuitive explanations.

D.3 Welfare analysis

We conduct a partial welfare analysis based on the sequential SPNE identified in Section 4.[60] We content ourselves with some basic observations. We assume that the candidates and their characteristics are exogenously given. Even a social planner cannot affect candidates' valence and competence. Our goal is to analyze how (in)efficient their endogenously chosen locations in the policy space are, compared to a hypothetical case where a planner dictates these choices.[61]

We define welfare as follows:

$$W(a_A, a_B) = \int_{-1}^{1} U_l(w, a_w)dl, \tag{41}$$

where $w = A$ if $\Delta U_l > 0$, $w = B$ if $\Delta U_l < 0$, and $w \in \{A, B\}$ if $\Delta U_l = 0$. U_l and ΔU_l are given by equations (2) resp. (4). For given actions a_A and a_B, the planner does not interfere with the median voter's choice of a candidate since the voter chooses optimally. Candidates' utility is neglected, including their entry costs.[62]

Note, that the definition of welfare in equation (41) only considers the median voter, who is decisive for the outcome of the election. Hence, we neglect here the preferences of the other voters. This would necessitate additional assumptions about their "ideal policies", and how these relate to the median voter's preference. The above definition of welfare is precise if all voters share identical preferences, and it is still a good approximation if the "dispersion" of the other voters' preferences around the median voter's ideal policy is small, compared to the size of the interval over which the median voter's bliss point is distributed, i.e, $[-1, 1]$.

As a *benchmark* case, we first analyze how the planner would position a candidate, say, A, if only one candidate were available. Instead of the above expression for welfare, we thus maximize $\int_{-1}^{1} U_l(A, a_A)dl$. Solving this problem is straight-forward and yields:

$$a_A^o = (1 - p_A)s \Leftrightarrow x_A^o = 0.$$

This result is unsurprising: if only one candidate were available, the policy platform that maximizes welfare is the center of the policy space. If both candidates are equally competent, or if $s = 0$, in a sequential equilibrium, the leader (A) thus positions itself

[60] A comprehensive welfare analysis is hard to achieve, due to the analytical complexity.

[61] We only consider the case where candidates enter sequentially at time zero, see Proposition 4. Welfare is thus the same as in the static electoral competition game (Section 2) under sequential moves, with A leading.

[62] If candidates are citizens, their fraction of the population is negligible.

in the same way as a planner would position a candidate if there were only one. The sequential equilibrium is then at least as efficient as the benchmark case where only candidate (A) exists, and this candidate's location is optimal.[63] In fact, the sequential equilibrium is strictly more efficient, because the weaker candidate positions itself at a different location in the policy space and is elected with a strictly positive probability (Proposition 1), so it must contribute to the (expected) welfare.[64]

However, the more relevant comparison is with the case where two candidates exist, and their positions are optimized. To analyze this case, suppose $a_A > a_B$. Then equation (41) can be rewritten as

$$W(a_A, a_B) = \int_{-1}^{\bar{l}(a_A,a_B)} U_l(B, a_B)dl + \int_{\bar{l}(a_A,a_B)}^{1} U_l(A, a_A)dl, \tag{42}$$

provided that $-1 < \bar{l}(a_A, a_B) < 1$, where $\bar{l}$ is given by equation (7). Evaluating these integrals is straight-forward, but the resulting algebraic expressions (not shown) are too complex to lend themselves to a general analysis. We content ourselves with a simple numerical example.

Consider the following parameter values: $s = 0.2, p_A = 0.9, p_B = 0.8, v_A = 0.15, v_B = 0.1$. For a given set of parameter values, we can analyze equation (42) numerically. We find the following optimized action choices: $a_A \approx 0.45$, and $a_B \approx -0.37$. This yields $\bar{l}(a_A, a_B) \approx -0.036$ and $W \approx -0.023$. The planner thus positions the stronger candidate A to the right of the status quo policy, and the weaker one to the left of the political center. For comparison, in the sequential SPNE where the stronger candidate leads, we find: $a_A = a_A^* = 0.04$, and $a_B = a_B^2(a_A) \approx -0.17$.[65] This yields $\bar{l}(a_A, a_B) \approx -0.26$ and $W \approx -0.26$. In the benchmark case where only candidate A is available, and the planner optimizes a_A, we find $a_A^o = 0.02$ and $W \approx -0.37$. If candidates are not too heterogeneous,[66] the existence of the second candidate thus permits to achieve a substantially higher welfare, because each candidate can then "specialize" on one part of the policy space (left or right). For the given parameter values, however, candidates' positions in the policy space are too moderate in equilibrium. The planner would force them to adopt much more extreme positions, so that for many realizations

[63] Note that, due to continuity, the same welfare comparison still holds true if candidates are almost equally competent, or if s is near zero.

[64] The only case where this is not true is when $\Delta v = 0$ and $s = 0$. As shown in Proposition 2, in that case B follows by choosing $a_B = a_A$ making the voter indifferent between both candidates.

[65] Candidate B is indifferent between $a_B^1(a_A)$ and $a_B^2(a_A)$, so we assume she chooses the latter, which leads to a higher expected welfare.

[66] If candidates are very heterogeneous, so that one candidate clearly dominates the other one, the existence of the weaker candidate does not contribute much to the welfare optimum.

of the median voter's bliss point, one of the candidates is located nearer to the voter's ideal policy.

In the limit case where $p_A = p_B = 1$ and $\Delta v = 0$, it is easy to verify that the social optimum entails $a_A = x_A = 0.5$ and $a_B = x_B = -0.5$, with $\bar{l} = 0$.[67] The planner then specializes the candidates' positions so as to maximally "cover" the policy space. For these parameter values, our static electoral competition model collapses to a standard Downsian framework (Downs 1957) without the competence or valence feature. For this setup, it is well-known that the equilibrium entails $x_A = x_B = 0$ (Median voter theorem). In our model, it is straight-forward to check that as the parameters approach these extreme values, the sequential equilibrium with A leading converges to the same outcome. Both candidates then position themselves in the center of the policy space. The planner, by contrast, would force them to specialize.

D.4 Discussion of right-continuity assumption

Although we believe to have demonstrated the applicability of our continuous time framework for a fruitful analysis of specific games, our continuous time machinery is obviously not a "magic bullet". We build on a set of general assumptions that are often useful, but can in some cases also limit the analysis. Specifically, our right-continuity assumption on players' strategies (Assumption 2 in Appendix B.1) helps to simplify the definition of strategies, and how they map into outcomes and thus payoffs. Without right-continuity, it is not clear if a player who discontinuously switches to a new type of behavior at time t does so *at* the decision node t, or if the new behavior sets in immediately after t. Hence, if both players have a discontinuity point in their strategies at t, without additional information, it is not clear if they will move simultaneously or sequentially, or in what order. Right-continuity clarifies this, as it requires that there is a well-defined first decision node where a new type of behavior sets in. We find that this assumption is natural, and it also serves as a tie-breaker when a player is indifferent between two qualitatively different types of behavior at a certain time t.[68]

However, as is true with any assumption, it can also limit the analysis in some cases. As we have seen in the variant with an incumbent and a challenger (Section 5), the right-continuity requirement for players' strategies can sometimes rule out the existence of a SPNE (in a strict sense). Also the assumption that the time frame of the game $[0, 1)$ is an open interval on the right can have the same consequence. However, we believe that these limitations are less severe than they might appear. Right-continuity

[67] Of course, due to symmetry, the indices of the candidates can be swapped.

[68] We used it as a tie-breaker for example in the proofs of Theorem 1 and Proposition 4.

is only a limitation if there is a specific point in time when a player wants to move, and the same kind of behavior is not optimal for this player immediately before or after this time. In the proof of Proposition 5, we argued that the problem arises (for some parameter constellations) because there are two discontinuities in candidates' behavior that effectively happen at the same point in time: candidate B wants to move as late as possible, but before candidate A switches to a new kind of behavior. So there is a single decision node where B wants to lead. If he fails to do so (due to a mistake or a deviation), immediately after this time, candidate A starts to lead and thus switches to a qualitatively new behavior. This type of situation requires an exceptional degree of coordination among the players: equilibrium behavior requires that the clocks are perfectly synchronized, because the delay between these two discontinuities in players' behavior is of measure zero. Such precision is merely a theoretical possibility. In reality, one might expect the player who wants to preempt (here: candidate B) to lead shortly before this critical point in time, to rule out coordination failures among the players. But in this case, right-continuity of players' strategies can be restored.[69]

Also the problem related with the openness of the time interval $[0, 1)$ on the right can easily be addressed, without changing the overall structure of the game in a significant way. Just suppose that the fixed costs (e.g., for campaigning) "level off" shortly before the election, so that there is an interval where these costs stay constant or decline again shortly before $t = 1$ (when the election takes place). The length of this interval can be arbitrarily short, yet, the technical issue (arising for some parameter constellations) described in the proof of Proposition 5 disappears: candidate B may then lead at every time within this interval, once more restoring right-continuity of his strategy. This modification of the game would resolve also the existence problem of a SPNE underlying the statement in Remark 2 (Section 5).

D.5 Discussion of SPNE in mixed strategies

The following discussion complements Theorem 1 (in Appendix B.2) and Corollary 1 (in Section 3.3). While these results focus on pure strategies, here we argue that this is without loss of generality because under suitable conditions, there exist no mixed strategy equilibria. We focus on the case where Proposition 3 (Section 3.2) applies, i.e., the static game does not have a pure strategy Nash equilibrium. However, similar arguments apply also for games where Assumption 1 is satisfied for other reasons.

We assume that the *timing* of player i's move under some mixed strategy can be

[69] Formalizing this argument is beyond the scope of this paper.

described by a right-continuous probability distribution function (see Hendricks et al., 1988). Conditional on moving at decision node t such that nobody has previously moved (once a player has moved, the other player solves a simple optimization problem), a player may randomize over *actions*. We do not impose restrictions on how the latter randomization is done, and how it may change across times (decision nodes t).[70]

Right-continuity of the probability distribution that describes a player's (possible) randomization over the timing of his move, conditional on the other player not having previously moved, implies that the integral of the probability distribution function does not include a mass point at time t (see Footnote 7 in Hendricks et al., 1988).[71] Therefore, if player i has a mass point at t, his distribution function is continuous over an interval $[t, t + \Delta)$ for some $\Delta > 0$. In other words, if there is a mass point at t, then there cannot be another mass point "immediately after" t.

With this, we can expand the scope of Proposition 3 (Section 3.2) as follows:

Proposition 3'. *If player j moves at t with a strictly positive probability and randomizes over actions (over which i is not indifferent), then i strictly prefers to wait at t.*

Proof: Fix a probability $p > 0$ with which j moves at decision node t (where this player has a mass point in their timing distribution function). Fix a conditional probability distribution over j's action choices (conditional on moving at t). Because i is not indifferent across (all) these actions, there is a strictly positive gain for i if i can condition their response on j's actual choice, rather than moving simultaneously with j at t.

On the other hand, by waiting at t, there is a potential cost for i: if j does *not* move (which happens with probability $1 - p$), then i cannot implement some different action immediately after t, due to right-continuity (Assumption 2) that we still impose on pure strategies. However, this cost is arbitrarily small, because for any $\delta > 0$, i can still move earlier than at $t + \delta$, if j does not move at t (if j does move, then i can respond immediately without any delay, in line with right-continuity). Hence, no matter how j's strategy looks like after t, the loss for i can be made arbitrarily small, because in this short interval after t, j does not have another mass point in their timing distribution. But the gain from waiting at t is strictly positive. Hence, the statement is true for any mixed strategy of j that conforms with our general assumptions. □

The implication of Proposition 3' is that there does not exist a SPNE that entails simultaneous moves that occur at any $t \in [0, 1)$ (on or off the equilibrium path) with

[70] We acknowledge the possibility of measurability issues and other complexities related with mixed strategies; see Ewerhart (2025) and the references cited therein.

[71] These authors define the probability distribution function G with the convention $\int_{t_0}^{t_1} f(v)dG_\alpha(v) = \lim_{t\uparrow t_1} \int_{t_0}^{t} f(v)dG_\alpha(v)$.

a strictly positive probability. In other words, in any SPNE, at most one player may attach a positive probability mass to moving (exactly) at time t. Since (by Proposition 3), we already know that there are no SPNE where players only randomize over actions, the subsequent discussion focuses on the possibility that players randomize in such a way that simultaneous moves occur with a probability of zero at any $t \in [0, 1)$ such that nobody has previously moved (once a player has moved, the other player follows optimally in any SPNE). Our goal is to argue that the main results of Theorem 1 are robust, such that also this type of mixing cannot occur in equilibrium.

Before we come to the details, the following observation is useful: Considering only (potential) mixed strategy equilibria where the probability that both players move simultaneously at any t is zero, *if* a player moves at some t, she becomes the leader with probability 1. Therefore, this player necessarily must choose their optimal leader's action (conditional on moving) at t in any SPNE. This permits us to describe the payoffs for such (potential) equilibria via our usual functions $L_i(t)$ and $F_j(t)$ for the leader resp. follower. In other words, players then only randomize over the timing of their moves, but not over their action choices. Additionally, in order to be willing to randomize over the timing of their move, each player i must be indifferent between moving and not at any t that is in the support of their timing distribution function.

We now lay out our arguments why allowing for mixed strategies does not alter the main conclusions of Theorem 1. In other words, the SPNE identified there (under the respective conditions) is unique also when allowing for mixed strategies. We will follow the steps of the proof of Theorem 1, but only discuss how the possibility of randomization may affect the argumentation.

Simultaneous moves: We already noted that by Proposition 3', there does not exist a SPNE that entails simultaneous moves by the players at any t (occurring with a strictly positive probability).

Subgames at $t \in (\tau_2, 1)$: To rule out mixed strategies such that the probability of moving simultaneously at any such t is zero, we assume that $F_i(t) > L_i(t)$ for all t in this interval holds for at least one player. (In the proof of Theorem 1, we used Assumption 2 to rule out equilibria where players switch infinitely many times between leading and following. A strict second-mover advantage is an alternative way to rule this out. See Footnote 25 in Section 3.3.) Then there also does not exist a SPNE in mixed strategies such that the probability of moving simultaneously at any t in this range is zero: in such a case, a player who moves chooses their optimal leader's action (because she is sure to become the leader). This amounts to the payoffs $L_i(t)$, resp.

$F_j(t)$, as argued above. Since $F_i(t) > L_i(t)$ holds for at least one player, $L_i(t) < E_i$ implies that player i is always better off waiting (both if the other player moves first, or if both end up waiting until $t = 1$).

Decision node $t = \tau_2$: Maintaining Assumption 2 (right-continuity for pure strategies) implies that player 2 waits also at τ_2: as player 2 waits at all $t \in (\tau_2, 1)$, a pure strategy, this player must wait also at $t = \tau_2$ to satisfy right-continuity.

Subgames at $t \in [\bar{t}_2, \tau_2)$: Fix a point $t \in (\max\{\tau_1, \bar{t}_2\}, \tau_2)$ near τ_2; here: $L_1(t) < E_1$. Recall that there cannot be any mass points at an identical t for both players. Furthermore, there cannot be a mass point of only player 1: this player is better off waiting at all times or if player 2 leads in this interval. But because $L_2(t) > E_2$, player 2 will lead with probability 1 at any such t. Therefore, we can take such t as the starting point for our procedure of working backwards in time. As explained in the proof of Theorem 1, from any such t where player 2 leads with certainty, because $L_2(t)$ is decreasing and player 1 has a strict second-mover advantage, we can always identify an interval of strictly positive length up until t in which player 1 waits and therefore player 2 leads in every subgame in this interval. Recursively, we find that the same conclusion holds over the entire interval $[\bar{t}_2, \tau_2)$. Because the preference for player 2 to lead earlier rather than later is strict, and also player 1's preference to wait (for player 2 to lead in the next instant, if she did not move yet) is strict, there cannot be any randomization by either of the players about the timing of their move.

Subgames at $t \in [0, \bar{t}_2)$: We argued in the proof of Theorem 1 that either the preference for both players to wait at all t prior to $\bar{t}_2$ is strict, or that there is a range of subgames that is the "mirror picture" of the later ranges (see above), with the roles of the two players (and the respective conditions that need to be satisfied over parts of these ranges) reversed. Therefore, we rule out mixing over the timing of players' moves and over action choices by using arguments that parallel those for decision nodes after $\bar{t}_2$ and thus do not repeat these arguments here.

References (for Online Appendix):

Ewerhart, Christian (2025) Games with continuous payoff functions and the problem of measurability, Working Paper, No. 467, University of Zurich, Department of Economics, Zurich, https://doi.org/10.5167/uzh-276544