New and improved Johnson-Lindenstrauss embeddings via the Restricted Isometry Property

Felix Krahmer

Hausdorff Center for Mathematics, Universität Bonn

The Johnson-Lindenstrauss (JL) Lemma states that any set of p points in high dimensional Euclidean space can be embedded into $O(\delta^{-2}\log(p))$ dimensions, without distorting the distance between any two points by more than a factor between $1-\delta$ and $1+\delta$. We establish a new connection between the JL Lemma and the Restricted Isometry Property (RIP), a well-known concept in the theory of sparse recovery often used for showing the success of ℓ_1-minimization.

Consider an $m \times N$ matrix satisfying the (k, δ_k)-RIP with randomized column signs and an arbitrary set E of $O(e^k)$ points in $\mathbb{R}^N$. We show that with high probability, such a matrix with randomized column signs maps E into $\mathbb{R}^m$ without distorting the distance between any two points by more than a factor of $1 \pm 4\delta_k$. Consequently, matrices satisfying the Restricted Isometry of optimal order provide optimal Johnson-Lindenstrauss embeddings up to a logarithmic factor in N. Moreover, our results yield the best known bounds on the necessary embedding dimension m for a wide class of structured random matrices. In particular, for partial Fourier and partial Hadamard matrices, our method optimizes the dependence of m on the distortion δ : We improve the recent bound $m = O(\delta^{-4}\log(p)\log^4(N))$ of Ailon and Liberty (2010) to $m = O(\delta^{-2}\log(p)\log^4(N))$, which is optimal up to the logarithmic factors in N. Our results also have a direct application in the area of compressed sensing for redundant dictionaries.

This is joint work with Rachel Ward.