Compactly supported shearlets in $L^2(\mathbb{R}^3)$

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Many important problem classes are governed by anisotropic features such as singularities concentrated on lower dimensional embedded manifolds. To analyze the ability of representation systems to reliably capture and sparsely represent anisotropic structures, Donoho introduced the model situation of so-called cartoon-like images, i.e., two-dimensional functions which are C^2 smooth apart from a C^2 discontinuity curve. In the past years, it was shown that curvelets, contourlets, and shearlets all have the ability to essentially optimal sparsely approximate cartoon-like images measured by the L_2-error of the (best) n-term approximation. Traditionally, this type of results has only been available for band-limited generators, but recently Kutyniok and Lim showed that optimal sparsity also holds for spatial compactly supported shearlet generators under weak moment conditions.

In this talk we introduce three-dimensional cartoon-like images, i.e., functions of three variables which are piecewise C^2 smooth with discontinuities on a C^2 surface, and consider sparse approximations of such. We first derive the optimal rate of approximation which is achievable by exploiting information theoretic arguments. Then we introduce three-dimensional pyramid-adapted shearlet systems with compactly supported generators and prove that such shearlet systems indeed deliver essentially optimal sparse approximations of three-dimensional cartoon-like images. Finally, we even extend this result to the situation of piecewise C^2 discontinuity surfaces , and again derive essential optimal sparsity of the constructed shearlet frames.

This is joint with G. Kutyniok and W.-Q Lim (University of Osnabrück).