Sparse Recovery with Fusion Frames

Abstract

Fusion frames are generalization of frames that provide a richer description of signal spaces. Instead of frame vectors, we have a collection of subspaces $W_j \subset \mathbb{R}^M$, $j = 1, \ldots, N$, where M is the dimension of the space. Then it holds that

$$A\|x\|_2^2 \leq \sum_{j=1}^{N} v_j^2 \|P_j x\|_2^2 \leq B\|x\|_2^2$$

for some universal fusion frame bounds $0 < A \leq B < \infty$ and for all $x \in \mathbb{R}^M$, where P_j denotes the orthogonal projection onto the subspace W_j. Weight coefficients v_j will be taken 1 in our talk. The generalization to fusion frames allows us to capture interactions between frame vectors to form specific subspaces that are not possible in classical frame theory. In this model, a sparse signal has energy in very few of the subspaces of the fusion frame, although it does not need to be sparse within each of the subspaces it occupies. This sparsity model is captured using a mixed ℓ_1/ℓ_2 norm for fusion frames.

In this talk, we provide a result on conditioning of random submatrices for fusion frames. we show that as in classical nonuniform recovery setup, sparse signal in a fusion frame can be sampled using very few random projections and exactly reconstructed using a convex optimization that minimizes this mixed ℓ_1/ℓ_2 norm. Our work exploits the property of fusion frame which is the angle between frame subspaces. If $\alpha(W_i, W_j)$ denotes the angle between subspaces W_i and W_j and $\lambda := \max_{i \neq j} cos(\alpha(W_i, W_j))$ then number of measurements m needs to be order of

$$m = \mathcal{O}[(1 + \lambda s)\ln(N)]$$

in order to recover s sparse vector with high probability. Here, as λ gets smaller, number of measurements decrease.