TIGHT p-FUSION FRAMES AND ROBUSTNESS AGAINST SUBSPACE ERASURES

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In modern signal processing, basis-like systems are applied to derive stable and redundant signal representations. Frames are basis-like systems that span a vector space but allow for linear dependency, that can be used to reduce noise, find sparse representations, or obtain other desirable features unavailable with orthonormal bases. Fusion frames enable signal decompositions into weighted linear subspace components, and tight fusion frames provide a direct reconstruction formula. If all the subspaces are equiangular, then fusion frames are maximally robust against two subspace erasures.

We first verify that the maximal number of mutually equiangular subspaces in $\mathbb{R}^d$ is $\binom{d+1}{2}$. We circumvent this limitation by extending the concept of fusion frames and by changing the measure for robustness against subspace erasures. Our main contribution is the introduction of the notion of tight p-fusion frames, a sharpening of the notion of tight fusion frames, that is closely related to the classical notions of designs and cubature formulas in Grassmann spaces. We analyze tight p-fusion frames with methods from harmonic analysis in the Grassmannians. For subspaces of equal dimension, we give several characterizations of tight p-fusion frames; the most practical one involves only the principal angles between subspaces and certain multivariate Jacobi polynomials.

We verify the existence of tight p-fusion frames for any integer $p \geq 1$. Moreover, we present general constructions of tight p-fusion frames. Finally, we verify that tight p-fusion frames are robust against up to p subspace erasures. More precisely, by using list decoding, we show that tight p-fusion frames, which are also cubatures of strength 4, allow for signal reconstruction even if the projections onto up to p subspaces are erased. Here, the output in list decoding is a list of potential signals of size bounded by $2p!$, one of which is the correct one. Further analysis reduces this list to only two elements.

(This is a joint work with Christine Bachoc, Univ. Bordeaux)

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