Abstract

Compressed Sensing with Radar Applications

In radar, n antenna elements $a_1, \dots, a_n \in [0, A]^2$ mounted on an aperture $[0, A]^2$ emit an isotropic electromagnetic wave of wavelength $\lambda > 0$. The spatial part of this wave emitted in $a_k = (\xi, \eta, 0)$ and recorded in a point $r_\ell = (x, y, z_0)$ at distance z_0 can be approximated by

$$G(a_k, r_j) := \frac{\exp(i\omega z_0)}{4\pi z_0} \exp\left(i\omega \frac{(x-\xi)^2 + (y-\eta)^2}{2z_0}\right),$$

where $\omega := 2\pi/\lambda$ denotes the wavenumber. If we discretize the target space into a grid of resolution cells of meshsize $d > 0$, we may assume that possible targets at distance z_0 are located in a discrete set of points $\{r_\ell\}_{\ell=1,\dots,N} \subset [-L, L]^2 \times \{z_0\}$, where $L > 0$ is the size of the target domain. Set $v(a_k, a_j) := (G(a_k, r_\ell)^* G(a_j, r_\ell)^*)_{\ell=1,\dots,N} \in \mathbb{C}^N$. The sensing matrix for the inverse scattering problem is then given by

$$A := (v(a_k, a_j)^*)_{k,j=1,\dots,n} \in \mathbb{C}^{n^2 \times N}$$

and the inverse problem is to recover a vector of reflectivities $x \in \mathbb{C}^N$ from the knowledge of $y = Ax \in \mathbb{C}^{n^2}$. In many situations, the number of resolution cells N is much larger than the actual number of targets occupying them, meaning that $\|x\|_0 := |\{j \in \{1, \dots, N\} | x_j \neq 0\}| = s \ll N$, or in other words, that the targets are sparse in the number of resolution cells. Therefore, compressed sensing techniques are applicable. In this talk, we will show that if we choose the antenna elements $a_1, \dots, a_n$ uniformly at random in the aperture $[0, A]^2$, then a fixed target scene $x \in \mathbb{C}^N$ which is s-sparse can be recovered from $y = Ax$ by basis pursuit with probability at least $1 - \epsilon$ provided

$$n^2 \geq Cs \log^2(cN/\epsilon), \qquad c, C > 0 \text{ universal constants.}$$

We will furthermore show that the recovery is robust if the measurements are corrupted by noise and we will present numerical results validating the theoretically obtained bounds. The results are joint work with Holger Rauhut and Thomas Strohmer.