Sparse Recovery with Time-Frequency Structured Random Matrices

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Compressive sensing predicts that sparse signals can be recovered from only a small number of linear measurements. Several recovery algorithms apply, most notably ℓ_1-minimization and greedy algorithms.

Suitable constructions of measurement matrices A are usually based on randomness. Simple choices are either Bernoulli random matrices or Gaussian random matrices A, where in particular, all entries are stochastically independent. A typical result in compressive sensing states that an s-sparse vector $x \in \mathbb{C}^N$ can be recovered exactly (and stably) from $y = Ax$ with high probability on the random draw of A using ℓ_1-minimization provided $m \geq Cs\ln(N/s)$.

Despite the optimality of Bernoulli and Gaussian random matrices in this context, they are of limited use for practical purposes since they do not possess any structure. In this talk we will consider a particular structured measurement matrix that arises from time-frequency analysis. Let T denote the translation operator, and M the modulation operator on $\mathbb{C}^n$, defined by

$$(T^k g)_q = g_{q-k \mod n} \quad \text{and} \quad (M^\ell g)_q = e^{2\pi i \ell q/n} g_q.$$

The operators $\pi(\lambda) = M^\ell T^k$, $\lambda = (k, \ell)$, are called time-frequency shifts. The matrix $\Psi_g \in \mathbb{C}^{n \times n^2}$ whose columns are the vectors $\pi(\lambda)g$, $\lambda \in \mathbb{Z}_n \times \mathbb{Z}_n$, of a Gabor system, is referred to as a Gabor synthesis matrix. Here, we choose the vector g at random by taking all entries to be independent and uniformly distributed on the complex torus $\{z \in \mathbb{C}, |z| = 1\}$. Then Ψ_g becomes a structured random matrix.

We show that a (fixed) s-sparse vector $x \in \mathbb{C}^{n^2}$ can be recovered from $y = \Psi_g \in \mathbb{C}^n$ with probability at least $1 - \varepsilon$ via ℓ_1-minimization provided

$$n \geq Cs\log(n/\varepsilon).$$

Further, we give an estimate of the restricted isometry constants of Ψ_g. This implies uniform and stable s-sparse recovery using ℓ_1-minimization and other recovery algorithms.

Potential applications of compressive sensing with time-frequency structured measurement matrices include radar, wireless communications and sonar.