An Observation on Real Division Algebras

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In [1] a short and elementary proof is given for the well-known fact that there are no associative real division algebras of dimension 3. Without claiming originality, we present here an even shorter but still elementary proof for the following more general observation.

Observation. Non-associative real division algebras of odd dimension > 1 do not exist.

Proof. Suppose on the contrary that A is such an algebra and write \( L_u : A \rightarrow A, v \mapsto L_u(v) = uv \), for the left multiplication by \( u \in A \), which, thanks to our hypothesis, is a bijective linear transformation unless \( u = 0 \). Hence, given linearly independent vectors \( x, y \in A \), \( p(t) = \det L_{x+ty} \) \((t \in \mathbb{R})\) is a real polynomial of odd degree having no real roots, a contradiction. Notice that we do not assume A to contain an identity element.

References