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Vicinity Respecting Homomorphisms for Abstracting System Requirements

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Abstract. This paper is concerned with the problem of structuring system and software requirements on an abstract conceptual level. Channel/Agency Petri nets are taken as a formal model. They allow to represent functional aspects as well as data aspects of the requirements in a graphical way. Vicinity respecting homomorphisms are presented as a means to refine and abstract these nets. They preserve paths, i.e., dependencies between computational elements. Further, it is shown that they preserve important structural properties of nets, such as S- and T-components, traps and siphons. These structural objects are helpful to gain better understanding of the whole system. For example, S-components give the main streams of choice while T-components give the main streams of concurrency. Moreover, these objects have important interpretations for marked Petri nets and can therefore be used for the analysis of system models at more concrete levels.
0 Introduction

A nontrivial task in the design of large and complex systems is to organize the requirements into a coherent structure. Usually, this organization is a gradual process which involves refinement and abstraction at different conceptual levels of the system. In this paper we take Channel/Agency Petri nets [22] to model systems and propose vicinity-respecting homomorphisms as a means to refine and abstract these nets.

Petri nets (see e.g. [21]) are special bipartite graphs. The two different types of nodes are called places and transitions. Places are considered as passive elements that allow to represent data while transitions are considered as active elements that allow to model activities or functions of system components. Directed edges between places and transitions reflect the flow and transformation of data and control. Thus a path formed by consecutive edges represents causal dependencies between its first and its last element. With these two types of elements, Petri nets combine a data-oriented view with a process-oriented view of the system.

For large and complex systems it is necessary to have models for different levels of abstraction and notions for refinement and abstraction to formulate relations between these models. Channel/Agency Petri nets are a Petri net model where all elements of a net are labeled by informal descriptions. They have been proposed for the conceptual modeling of the architecture of information systems e.g. in [9,6,14]. As shown in [22] they can be used for different levels of abstraction. On a low level of abstraction containing all details nets can be equipped with markings and a notion of behavior which simulates the behavior of the modeled system. In this way Petri nets can be used as a means for prototyping.

We introduce vicinity-respecting homomorphisms of Petri nets to formalize the refinement and abstraction relations between nets. This encompasses modular techniques because each composition of subsystems may be viewed as an identification of the respective interface elements and thus as a particular abstraction. Vicinity-respecting homomorphisms rely on the graph structure of a net. They are special graph homomorphisms that are able to formalize abstractions including contractions of graphs not only in their breadth but also in their length. Working with graphs or nets, we distinguish the pre- and the post- vicinity of an element $x$: the pre- vicinity $S_r$ includes the preset of $x$ (the set of elements with an arc leading to $x$) and $x$ itself while the post- vicinity $S_x$ includes the post-set together with $x$. In vicinity-respecting homomorphisms it is required that the pre- vicinity of an element is mapped either surjectively onto the pre- vicinity of its image or entirely to its image and likewise for post- vicinities.

Figure 1 shows four models of a sender/receiver system, representing four levels of abstraction. As usual, places are represented by circles and transitions are represented by squares. The interrelation between these models is formally given by mappings between the respective elements. For example: the place message in channel in the second model is refined to the place message in channel A and to the subsystem containing the place message in channel B and the transition message transmission and the place message in channel B end in the first model. These mappings are homomorphisms between the nets.

The definition of vicinity-respecting homomorphisms is based on the local vicinities of elements. This concept suffices to preserve important global structural properties like connectedness. If two elements of a net are connected by a path, then the respective system components are in a causal dependency relation. Because they preserve paths, vicinity respecting homomorphisms not only respect dependency but also its complementary relation independency.

Petri nets not only allow to combine data- and function-oriented views of a system. They also allow to concentrate on either aspect. The data aspect including nondeterministic choice is reflected by $S$- components, i.e. by subnets where every transition has at most one input place and at most one output place, i.e. where only places are branched. For example, in the first net of Figure 1 we have an $S$-component for the sender containing the places idle and message produced and the transitions produce message, cancel message, send message is channel A and send message in channel B. Similarly, the receiver is represented by an $S$-component. Likewise, $T$-components represent an activity-oriented view, where only transitions are branched. Petri nets that are covered by $S$- and $T$-components allow for a compositional interpretation of these two aspects. We show that vicinity respecting net homomorphisms preserve coverings by $S$- and $T$-components. As a consequence, they respect the notions of choice (a forward branching place) and of synchronization (a backward branching transition).

This paper gathers, generalizes and deepens results obtained in [8,17]. In section 1 we investigate homomorphisms of arbitrary graphs. Amongst other results it is shown that for arbitrary surjective graph homomorphisms the image of a path of the source graph is a path of the target graph while the converse direction - a path of the target graph is necessarily the image of some path of the source graph - does not necessarily hold but holds for vicinity respecting homomorphisms. Section 2 is a section which introduces Petri nets and transfers the notion of vicinity respecting homomorphisms to them. In
section 3 we show that vicinity respecting homomorphisms respect coverings by $S$- and T-components of Petri nets and draw consequences for Petri nets composition. Siphons and traps are concepts known from Petri net theory that allow for an analysis of the data contained in sets of places. Section 4 proves that vicinity respecting homomorphisms preserve siphons and traps. Finally, section 5 concludes this paper with final remarks and related works.

### 1 Graph Homomorphisms

Petri nets are special graphs. Vicinity respecting homomorphisms will be defined for arbitrary graphs in this section.

Figure 2 shows a model of a sender/receiver system on the left hand side and a coarser view of the same system on the right hand side. The left model can be viewed as a refinement of the right model. The interrelation between the graphs is given by a mapping, depicted by arrows between the vertices of the graphs. This mapping is a particular graph homomorphism. As we shall see, in this example dependencies between vertices of the source graph are strongly related to dependencies between vertices of the target graph.

We start with a formal introduction of graphs and related concepts. We consider only finite directed graphs without multiple edges and without loops.
Definition 1.1
A graph is a pair \((X, F)\) where \(X\) is a finite set and \(F \subseteq X \times X\). The set \(X\) is the set of vertices and \(F\) is the set of edges of the graph. A loop is an edge \((x, x)\) where \(x \in X\). The graph is said to be loop-free if no edge is a loop.

The classical notion of graph homomorphism [19] respects edges in the sense that the images of connected vertices are again connected. Since we also consider contractions of loop-free graphs, where two connected vertices are mapped to one vertex without a loop, a slightly more liberal definition will be employed; we allow the images of connected vertices to be either connected or identical.

Definition 1.2
Let \((X, F)\) and \((X', F')\) be graphs. A mapping \(\varphi : X \to X'\) is a graph homomorphism (denoted by \(\varphi : (X, F) \to (X', F')\)) if, for every edge \((x, y) \in F\), either there is an edge \((\varphi(x), \varphi(y)) \in F'\) or \(\varphi(z) = \varphi(y)\).

To describe the environment of an element we shall use the notions of pre- and post-sets and related notions of pre- and post-vicinities.

Notation 1.3
Given a graph \((X, F)\), we denote by \(x^*\) the pre-set of a vertex \(x \in X\), which is defined by \(x^* = \{ y \in X | (y, x) \in F \}\).

Similarly, the post-set \(x^+\) is defined by \(x^+ = \{ y \in X | (x, y) \in F \}\).

The pre-vicinity of \(x\) is defined by \(x^0 = \{ x \} \cup x^*\).

The post-vicinity of \(x\) is defined by \(x^0 = \{ x \} \cup x^+\).

In Figure 3, \(a = 0 \, @a = \{ a \}\) and \(a^0 = \{ a, b \}\). It is immediate to note that \(x^0 = a^0\) if and only if there is a loop \((x, x) \in F\). Using the notion of vicinities, homomorphisms can be phrased as: the image of the pre-vicinity is included in the pre-vicinity of the image and the image of the post-vicinity is included in the post-vicinity of the image.

Proposition 1.4
Let \((X, F)\) and \((X', F')\) be graphs. A mapping \(\varphi : X \to X'\) is a graph homomorphism if and only if, for all \(x \in X\), we have \(\varphi(x^0) \subseteq \varphi(x)^0\).

Note that \(\varphi(x^0) \subseteq \varphi(x)^0\) does not hold for arbitrary graph homomorphisms because in case of contractions elements of the pre-set of a vertex \(x\) can be mapped to \(\varphi(z)\). The same holds for post-sets.

Definition 1.5
A sequence \(x_1, x_2, \ldots, x_n\) of vertices of a graph is a path if there exist edges \((x_1, x_2), \ldots, (x_{n-1}, x_n)\) of the graph.

A graph is strongly connected if for any two vertices \(x\) and \(y\) there exists a path \(x \ldots y\).

We allow a single element to be a path as a technical facility. Since consecutive vertices of a graph can be mapped onto a single element without a loop the sequence of images of some path elements is not necessarily a path of the target graph. So we define for loop-free graphs the image of a path to ignore stuttering of vertices.

Definition 1.6
Let \((X, F)\) and \((X', F')\) be loop-free graphs and let \(\varphi : (X, F) \to (X', F')\) be a graph homomorphism. The image of a path \(x_1, \ldots, x_n\) of \((X, F)\) is inductively defined by

\[
\varphi(x_1, \ldots, x_n) =
\begin{cases}
\varphi(x_1) & \text{if } m = 1 \\
\varphi(x_1, \ldots, x_{m-1}) & \text{if } m > 1 \text{ and } \varphi(x_{m-1}) = \varphi(x_m) \\
\varphi(x_1, \ldots, x_{m-1}) \varphi(x_m) & \text{if } m > 1 \text{ and } \varphi(x_{m-1}) \neq \varphi(x_m)
\end{cases}
\]

Using Definition 1.6, the image of the path \(ace\) in Figure 3 is \(dan\). Graph homomorphisms do not preserve edges but they preserve paths between vertices, as shown in the next lemma.
Lemma 1.7

Let \((X, F)\) and \((X', F')\) be loop-free graphs and let \(\varphi : (X, F) \to (X', F')\) be a graph homomorphism. If \(x_1 \ldots x_n\) is a path of \((X, F)\) then \(\varphi(x_1 \ldots x_n)\) is a path of \((X', F')\) leading from \(\varphi(x_1)\) to \(\varphi(x_n)\).

Proof: We proceed by induction on \(n\).
If \(n = 1\) then \(\varphi(x_1)\) is a path of \((X', F')\).
Let \(n > 1\) and assume that \(\varphi(x_1 \ldots x_{n-1})\) is a path leading from \(\varphi(x_1)\) to \(\varphi(x_{n-1})\). We have \((x_{n-1}, x_n) \in F\) by the definition of a path. By the homomorphism property, we can distinguish two cases:

1. \(\varphi(x_{n-1}), \varphi(x_n) \in F'\). Then \(\varphi(x_1 \ldots x_n) = \varphi(x_1 \ldots x_{n-1}) \varphi(x_n)\) is a path of \((X', F')\) leading from \(\varphi(x_1)\) to \(\varphi(x_n)\).

2. \(\varphi(x_{n-1}) = \varphi(x_n)\). Then \(\varphi(x_1 \ldots x_n) = \varphi(x_1 \ldots x_{n-1})\). By assumption, this is a path leading from \(\varphi(x_1)\) to \(\varphi(x_{n-1})\). Since \(\varphi(x_{n-1}) = \varphi(x_n)\), this path leads to \(\varphi(x_n)\).

Surjectivity is a first condition when graph homomorphisms are used for abstractions. The following corollary states that surjective graph homomorphisms preserve strong connectivity of graphs.

Corollary 1.8

Let \((X, F)\) and \((X', F')\) be loop-free graphs and let \(\varphi : (X, F) \to (X', F')\) be a surjective graph homomorphism. If \((X, F)\) is strongly connected then \((X', F')\) is also strongly connected.

Proof: Let \(x', y' \in X'\). Since \(\varphi\) is surjective there are \(x, y \in X\) such that \(\varphi(x) = x'\) and \(\varphi(y) = y'\). There is a path leading from \(x\) to \(y\) because \((X, F)\) is strongly connected. Using Lemma 1.7, some path \(x'_{n-1}\) leads from \(x'\) to \(y'\).

Surjectivity concerns vertices only. An additional requirement is that every edge of a target graph reflects a connection between respective vertices of the source graph. We call such a homomorphism a quotient.

Definition 1.9

Let \((X, F)\) and \((X', F')\) be loop-free graphs. A graph homomorphism \(\varphi : (X, F) \to (X', F')\) is called quotient if:

1. the mapping \(\varphi : X \to X'\) is surjective, and
2. for every edge \((x', y') \in F'\) there exists an edge \((x, y) \in F\) such that \(\varphi(x) = x'\) and \(\varphi(y) = y'\).

The name "quotient" is justified because for quotients, target graphs are determined up to renaming by the equivalence classes of vertices that are mapped onto the same vertex (see [9]). Therefore, we can represent quotients graphically by solely depicting equivalence classes as shown in Figure 4. The edges of the target graph are the equivalence classes of the edges of the source graph. There is a vertex connecting two edges of the target graph if and only if there is at least one edge connecting elements of the respective sets of vertices in the source graph.

When thinking of \((X', F')\) as an abstraction of \((X, F)\), dependencies between nodes of \(X'\) that are represented through paths, have to mirror dependencies already present in \(X\). Therefore, since dependencies between vertices are modeled by paths connecting the vertices, we look for a converse of Lemma 1.7. For quotients, this lemma has a weak converse: every path of the
target graph with at most two vertices is the image of a path of the source graph. The same does not necessarily hold for longer paths, as shown in Figure 5. The target graph has a path $\varphi(a) \varphi(b) \varphi(f)$ which is not the image of a path of the source graph. What is wrong with this homomorphism? The post-vicinity of $b$ is the set $\{b, d\}$. The post-vicinity of the image of $b$ contains three vertices, namely $\varphi(b), \varphi(d)$ and $\varphi(f)$. So the image of the post-vicinity of $b$ is properly included in the post-vicinity of the image. We say that the post-vicinity is not respected.

We strengthen Proposition 1.4 and define homomorphisms that respect vicinities of vertices. Since we still allow contractions we also have to consider 'inner' vertices where the entire post-vicinity of a vertex is mapped to one element, i.e. to the image of the vertex.

**Definition 1.10**

Let $(X, F)$ and $(X', F')$ be loop-free graphs.

A graph homomorphism $\varphi: (X, F) \rightarrow (X', F')$ is called pre-vicinity respecting if, for every $x \in X$, either $\varphi(\varphi^0 x) = \varphi(\varphi(x))$ or $\varphi(\varphi^0 x) = \varphi(\varphi(x))$.

Similarly, $\varphi$ is called post-vicinity respecting if, for every $x \in X$, either $\varphi(x^0) = \varphi(\varphi(x))^0$ or $\varphi(x^0) = \varphi(x)$.

A graph homomorphism is called vicinity respecting if it is pre-vicinity respecting and post-vicinity respecting.

In the next theorem we show that for surjective post-vicinity respecting homomorphisms of strongly connected graphs there is a converse of Lemma 1.7. By symmetry, the same holds for pre-vicinity respecting homomorphisms.

**Theorem 1.1**

Let $(X, F)$ and $(X', F')$ be loop-free graphs such that $(X, F)$ is strongly connected and let $\varphi: (X, F) \rightarrow (X', F')$ be a surjective post-vicinity respecting graph homomorphism. Let $x_1 \ldots x_n$ be a path of $(X', F')$ such that for all $i, 1 \leq i < n$, $x_i \neq x_{i+1}$ (no stuttering). Then there is a path $z_1 \ldots z_n$ of $(X, F)$ such that $\varphi(z_1 \ldots z_n) = x_1 \ldots x_n$.

**Proof:** We proceed by induction on $n$.

Let $m = 1$. Since $\varphi$ is surjective, some $x_1 \in X$ satisfies $\varphi(x_1) = z_1$. The path consisting of $z_1$ satisfies the required property.

Let $m > 1$. Assume that $z_1 \ldots z_n$ is a path of $(X, F)$ such that $\varphi(z_1 \ldots z_n) = x_1 \ldots x_{m-1}$. Since $\varphi$ is surjective, some $y \in X$ satisfies $\varphi(y) = z_m$. Since the graph $(X, F)$ is strongly connected, it contains a path leading from $z_n$ to $y$. Consider the first vertex $x_{i+1}$ in this path that is not mapped to $z_m \in X'$. Such a vertex exists because the last vertex $y$ in the path is mapped to $z_m$ and $x_{i+1} \neq z_m$ by the assumption. By definition, the predecessor $x_i$ of $x_{i+1}$ is mapped to $x_m$ and so are all vertices in the path $x_{i+1} \ldots x_n$.

We have $x_{i+1} \in z_n^0$. Since $\varphi(x_{i+1}) \neq \varphi(z_m)$ we obtain $\varphi(x_{i+1}) \neq \varphi(z_m)$. Using the definition of a post-vicinity respecting homomorphism we conclude that $\varphi(x_{i+1}) = (\varphi(z_m))^0$. Therefore, since $x_n \in (x_n^0)^0 = (\varphi(z_m))^0$, some vertex $x \in z_n^0$ is mapped to $z_m$. This vertex $x$ cannot be $x_1$ itself, hence it is in $z_n$.

Now the path $x_{i+1} \ldots z_m \ldots x_n$ combined from the paths $x_{i+1} \ldots x_n$, $x_{i+1} \ldots z_{m-1}$, and $z_m \ldots x_n$ is mapped to $x_{i+1} \ldots x_n$, which concludes the proof.

The example in Figure 6 shows that the previous theorem is necessary that the source graph $(X, F)$ is strongly connected. This graph homomorphism $\varphi$
is a vicinity respecting quotient. The target graph has a path $\varphi(c) \varphi(d) \varphi(c)$ which is not the image of a path of the source graph. There exist simpler examples to demonstrate that not all vicinity respecting homomorphisms satisfy a converse of Lemma 1.7. It is easy to find an example where the source graph has only two edges. We have chosen a slightly more involved example to demonstrate that neither recurring weak connectedness nor nonempty pre-c or post-sets for all vertices constitute sufficient conditions.

**Corollary 1.12**

Let $(X, F)$ and $(X', F')$ be loop-free graphs such that $(X, F)$ is strongly connected and let $\varphi: (X, F) \rightarrow (X', F')$ be a surjective and post vicinity preserving graph homomorphism. Then $\varphi$ is a quotient.

**Proof:** If $|X'| \leq 1$, then $F' = \emptyset$ and we are finished. So assume $|X'| > 1$.

Let $(z', y') \in F'$. By Theorem 1.11 there exists a path $x_1 \ldots x_n$ of $X$ with $\varphi(x_1) = z'$ and $\varphi(x_n) = y'$. Let $z$ be the last element of the path with $\varphi(x_i) = z'$. Then $(x_i, x_{i+1}) \in F$ and $\varphi(x_{i+1}) = y'$, which was to prove.

The following result, that will be used later, is much weaker than 1.12 but it holds for arbitrary surjective mappings.

**Lemma 1.13**

Let $(X, F)$ and $(X', F')$ be loop-free graphs such that $(X, F)$ is strongly connected and $|X| > 1$. If $\varphi: (X, F) \rightarrow (X', F')$ is a surjective mapping, then we have for every $z' \in X'$:

1. there exists an arc $(x, y) \in F$ with $\varphi(x) = z'$ and $\varphi(y) \neq z'$;
2. there exists an arc $(x, y) \in F$ with $\varphi(x) = z'$ and $\varphi(y) \neq z'$.

**Proof:** We show only the first part, the second one being similar.

Let $y'$ be an element of $X'$ distinct from $z'$ (which is possible because $|X'| > 1$). Since $\varphi$ is surjective there are $\xi, \eta \in X$ with $\varphi(\xi) = z'$ and $\varphi(\eta) = y'$. Since $(X, F)$ is strongly connected, there exists a path $x_1 \ldots x_n$ of $X$ with $x_1 = \xi$ and $x_n = \eta$. Let $i$ be the least index such that $\varphi(x_i) = z'$ and $\varphi(x_{i+1}) \neq z'$. With $x(x, y) = (x_i, x_{i+1})$ we are finished.

Concentrating on different elements which are mapped onto the same image instead of comparing source graph and target graph leads to another aspect of vicinity respecting homomorphisms in the case of quotients.

**Proposition 1.14**

Let $(X, F^*)$ and $(X', F')$ be loop-free graphs and let $\varphi: (X, F^*) \rightarrow (X', F')$ be a quotient. $\varphi$ is vicinity respecting if and only if for all $x, y \in X$ satisfying $\varphi(x) = \varphi(y)$:

1. $\varphi(\varphi(x)) = \varphi(\varphi(y))$ or $\varphi(\varphi(y)) = \varphi(\varphi(x))$;
2. $\varphi(x^\varphi) = \varphi(y^\varphi)$ or $\varphi(y^\varphi) = \varphi(x^\varphi)$.

**Proof:** It is immediate that Definition 1.10 implies 1. and 2. We only show that 1. implies pre- vicinity respecting; showing that 2. implies post- vicinity respecting is similar.

Let $x \in X$ such that $\varphi(x^\varphi) \neq \varphi(x)$. Let $z' \notin \varphi(x)$. Then there exist $y, z \in X$ such that $z \in z' \vee (z, y) = z'$ and $\varphi(y) = \varphi(x)$, because $\varphi$ is a quotient. By 1, we obtain that $\varphi(y^\varphi) = \varphi(x^\varphi)$. Since $z \in \emptyset^\varphi$, some element in $\emptyset^\varphi$ is mapped to $z'$.

Since $z$ was chosen arbitrarily in $\emptyset^\varphi$ we finally obtain $\varphi(x^\varphi) = \emptyset^\varphi(z)$.

In the proof of Proposition 1.14, we showed that for any element $z' \in \emptyset^\varphi(x)$ there exists an element $z \in \emptyset^\varphi(z)$ with $\varphi(z) = z'$. From this fact, we deduce immediately the following technical corollary that will be used later.

**Corollary 1.15**

Let $(X, F^*)$ and $(X', F')$ be loop-free graphs and let $\varphi: (X, F^*) \rightarrow (X', F')$ be a vicinity respecting quotient. Then we have for every $z \in X$:

1. if $\varphi(x) \neq \varphi(x)$ then $\varphi(z^\varphi) \leq \varphi(x)$;
2. if $\varphi(x^\varphi) \neq \varphi(x)$ then $\varphi(z^\varphi) \leq \varphi(x^\varphi)$.

**2 Net homomorphisms**

A net can be seen as a loop-free graph $(X, F)$ where the set $X$ of vertices is partitioned into a set $S$ of places and a set $T$ of transitions such that $F$ may not relate two places or two transitions. Formally:
Definition 2.1

A triple $N = (S, T; F)$ is called a Petri net or a net if

1. $S$ and $T$ are disjoint sets;
2. $F \subseteq (S \times T) \cup (T \times S)$.

The set $X = S \cup T$ is the set of elements or nodes of the net.

This definition allows to consider nets with isolated elements, i.e. elements with empty pre- and post-sets, in contrast to the definition given in [2] (a net, as defined here, is called pre-net in [2]). Graphically places are represented by circles, transitions are represented by squares and the flow relation $F$ is represented by arrows between the elements. Note that we do not consider markings and behavioural notions but concentrate on the structure of net models.

We use the following conventions: indices and primes used to denote a net $N$ are carried over to all parts of $N$. For example, speaking of a net $N_i$, we implicitly understand $N'_i = (S'_i, T'_i; F'_i)$ and $X'_i = S'_i \cup T'_i$.

A consequence of Definition 2.1 is that the pre-set of the post-set of a place are sets of transitions, and the pre-set and the post-set of a transition are sets of places.

The transitions of a net model the active subsystems, i.e. functions, operators, transformers etc. They are only connected to places which model passive subsystems, i.e. data, messages, conditions etc. On a conceptual level, it is not always obvious to classify a subsystem active or passive. The decision to model it by a place or by a transition is based on the interaction of the subsystem with its vicinity. As an example, consider a channel that is connected to functional units that send and receive data through the channel. Then the channel has to be modeled by a place. In contrast, if the channel is connected to data that have to be sent on one side and to already received data on the other side then the channel is modeled by a transition. As we shall see, a transition may represent a subsystem that is modeled by a net containing places and transitions on a lower level of abstraction. The same holds respectively for places.

An arrow in a Petri net either leads from a place to a transition or from a transition to a place. In the first case the place is interpreted as a pre-requisite (pre-condition, input) for the transition which can be consumed by the action modeled by the transition. In the second case the place is interpreted as a post-requisite (post-condition, output) for the transition which can be produced by the action modeled by the transition. In this sense, arrows are used to denote two different types of relations between the elements of a Petri net.

Homomorphisms of Petri nets are particular graph homomorphisms that additionally respect the type of relation between the elements given by arrows. Since we again allow contractions, places can be mapped to transitions and transitions can be mapped to places. However, if two connected elements are not mapped to the same element of the target net, then the place of the two has to be mapped to a place and the transition has to be mapped to a transition.

So Definition 1.2 becomes for Petri nets:

Definition 2.2

Let $N, N'$ be nets. A mapping $\varphi : X \rightarrow X'$ is called net homomorphism, denoted by $\varphi : N \rightarrow N'$, if for every edge $(x, y) \in E$ holds:

1. if $(x, y) \in E \cap (S \times T)$ then either $(\varphi(x), \varphi(y)) \in E' \cap (S' \times T')$ or $\varphi(x) = \varphi(y)$ and $\varphi(z) = \varphi(y)$;
2. if $(x, y) \in E \cap (T \times S)$ then either $(\varphi(x), \varphi(y)) \in E' \cap (T' \times S')$ or $\varphi(x) = \varphi(y)$.

This definition is equivalent to the one given in [12] or [11]. Note that there also are similar but slightly different notions in the literature, see e.g. [1]. A consequence of our definition is that a transition is allowed to be mapped to a place only if all elements of its pre-vicinity and post-vicinity are mapped to the same place, and vice versa.

Lemma 2.3

Let $\psi : N \rightarrow N'$ be a net homomorphism. Then:

1. if a transition $t \in T$ is mapped to a place $s'$ then $\psi(t) \cup \psi(s) = \{s'\};$
2. if a place $s \in S$ is mapped to a transition $t'$ then $\psi(s) \cup \psi(t') = \{t\}.$

Proof: We only show part 1, part 2 being similar.

Let $t \in T, s' \in S'$ such that $\varphi(s') = s$. Then for each place $s \in \psi(t)$ we have $(s, t) \in E \cap S \times T$ and $(\varphi(s), \varphi(t)) \notin S' \times T'$ and therefore $\varphi(s) = s, \varphi(t)$. Likewise, each place $s \in \psi(t)$ satisfies $\varphi(s) = s, \varphi(t)$. Since $\psi(t) \cup \psi(s) = \psi(t') \cup \varphi(t')$ and $\varphi(s) = s$ we obtain the result.
Corollary 2.4
Let \( \varphi : N \rightarrow N' \) be a net homomorphism and let \( (x, y) \in F \) such that \( \varphi(x) \neq \varphi(y) \). Then \( \varphi(x) \in S' \) if and only if \( x \in S \) and \( \varphi(y) \in S' \) if and only if \( y \in S \).

For Petri nets the vicinity respecting homomorphism definition can be split into two notions: homomorphisms that respect the vicinity of places and homomorphisms that respect the vicinity of transitions.

Definition 2.5
Let \( \varphi : N \rightarrow N' \) be a net homomorphism.
1. \( \varphi \) is \( S \)-vicinity respecting if, for every \( x \in S \):
   (a) \( \varphi(\theta x) = \theta (\varphi(x)) \) or \( \varphi(\theta x) = \{\varphi(x)\} \)
   (b) \( \varphi(x^0) = \{\varphi(x^0)\} \) or \( \varphi(x^0) = \{\varphi(x)\} \).
2. \( \varphi \) is \( T \)-vicinity respecting if, for every \( x \in T \):
   (a) \( \varphi(\theta x) = \theta (\varphi(x)) \) or \( \varphi(\theta x) = \{\varphi(x)\} \)
   (b) \( \varphi(x^0) = \{\varphi(x^0)\} \) or \( \varphi(x^0) = \{\varphi(x)\} \).
3. \( \varphi \) is vicinity respecting if it is both \( S \)-vicinity respecting and \( T \)-vicinity respecting.

A subnet of a net is generated by its elements and preserves the flow relation between its elements. We will be interested in subnets that are connected to the remaining part only via places or only via transitions.

Notation 2.6
The pre-set (post-set) notions of 1.3 are generalized to sets \( A \subseteq X \) of elements
\[ A = \bigcup_{x \in A} \{x\}, \quad A^* = \bigcup_{x \in A} x^* \]

Definition 2.7
Let \( N \) be a net. \( X_1 \subseteq X \) generates the subnet
\[ N_1 = (S \cap X_1, T \cap X_1, F \cap (X_1 \times X_1)). \]

\( N_1 \) is called transition-bordered if \( \varphi_1 \cup \varphi_1^* \subseteq \varphi_1 \).
\( N_1 \) is called place-bordered if \( \varphi_1 \cup \varphi_1^* \subseteq \varphi_1^* \).

A single transition of a net constitutes a transition-bordered subnet. Similarly, a place constitutes a place-bordered subnet. Figure 7 shows three nets. The net in the middle of the figure is a subnet of the net on the left-hand side. This subnet is generated by its set of elements \( \{a, b, c, d, f, h, i, m, o\} \).

It is transition-bordered because, for its places \( a, d, f \) and \( ra \), it contains all transitions in the pre- and the post-set. Similarly, the set on the right-hand side is place-bordered.

Net homomorphisms allow to map places to transitions and vice versa. Nevertheless, the role of ‘active’ and ‘passive’ components of a net are preserved in the following sense. The refinement of a transition-bordered subnet is a transition-bordered subnet, i.e., the reverse image of the elements of a transition-bordered subnet generates a transition-bordered subnet of the source net. Similarly, the set of elements of the source net that are mapped to some place-bordered subnet of the target net constitute a place-bordered subnet of the source net. The following results have been proved in [14] in a topological framework.

Proposition 2.8
Let \( \varphi : N \rightarrow N' \) be a net homomorphism.
1. If \( N_i' \) is a transition-bordered subnet of \( N_i \) then \( \{z \in X | \varphi(z) \in X_i'\} \) generates a transition-bordered subnet of \( N \).
2. If \( N_i' \) is a place-bordered subnet of \( N_i' \) then \( \{z \in X | \varphi(z) \in X_i'\} \) generates a place-bordered subnet of \( N \).

Proof: We show only part 1, part 2 being similar.
Let \( (x, y) \in F \). Assume that \( \varphi(x) \notin N_i' \) and \( \varphi(y) \notin N_i' \). Since \( N_i' \) is a transition-bordered subnet of \( N_i' \), \( \varphi(x) \) is a place and \( \varphi(y) \) is a transition. By Corollary 2.4, \( x \) is a place and \( y \) is a transition. It is similarly shown that \( \varphi(x) \notin N_i' \) and \( \varphi(y) \notin N_i' \) implies that \( x \) is a transition and \( y \) is a place. The result follows by the definition of a transition-bordered subnet.
3 Transformation of S- and T-components

Recall from the introduction that an S-component of a net yields a data-oriented view of a part of the system. An S-component can contain nondeterministic choices that are modeled by branching places, i.e., by places with more than one output transitions. It does however not contain aspects of concurrency, whence its transitions are not branched. Similarly, T-components concentrate on functional aspects. They do not contain branching places. Formally S-components and T-components are particular subnets.

Definition 3.1
A strongly connected transition-bordered subnet $N_i$ of a net $N$ is called S-component of $N$ if, for every $t \in T_i$, $|t \cap S_i| \leq 1 \land |t \cap S_i| \leq 1$ (pre-sets and post-sets are taken with respect to $N$).
A net $N$ is said to be covered by $S$-components if there exists a family of S-components $(N_i), i \in I$, such that for every $x \in X$ there exists an $i \in I$ such that $x \in X_i$.

Definition 3.2
A strongly connected place-bordered subnet $N_i$ of a net $N$ is called T-component of $N$ if, for every $s \in S_i$, $|s \cap T_i| \leq 1 \land |s \cap T_i| \leq 1$ (pre-sets and post-sets are taken with respect to $N$).
A net $N$ is said to be covered by $T$-components if there exists a family of T-components $(N_i), i \in I$, such that for every $x \in X$ there exists an $i \in I$ such that $x \in X_i$.

The net shown in Figure 7 is covered by S- and by T-components.

Definition 3.3
Let $\varphi : N \to N'$ be a net homomorphism and let $N_i$ be a subnet of $N$.
The net $(\varphi(X_i) \cap S', \varphi(X_i) \cap T'; \{ \langle \varphi(x), \varphi(y) \rangle \mid \langle x, y \rangle \in F_i \land \varphi(x) \neq \varphi(y) \})$ is called the net image of $N_i$ by $\varphi$. It is denoted by $\varphi(N_i)$.
By $\varphi_{N_i} : X_i \to \varphi(X_i)$ we denote the restriction of $\varphi$ to $X_i$, with the range of $\varphi$ restricted to $\varphi(X_i)$; $\varphi_{N_i}$ is surjective by definition.

Note that $\varphi(N_i)$, the net image of $N_i$, is not necessarily a subnet of the target net $N'$. Figure 8 gives an example.

Using Definition 3.3 we get immediately the following results:

Proposition 3.4
If $\varphi : N \to N'$ is a net homomorphism and $N_i$ is a subnet of $N$ then $\varphi_{N_i} : N_i \to \varphi(N_i)$ is a quotient.

Corollary 3.5
A net homomorphism $\varphi : N \to N'$ is a quotient if and only if $N' = \varphi(N)$ and in this case $\varphi = \varphi_N$.

S-vicinity respecting net homomorphisms map a strongly connected transition-bordered subnet either onto a single element or onto a strongly connected transition-bordered subnet.

Proposition 3.6
Let $\varphi : N \to N'$ be an S-vicinity respecting net homomorphism and let $N_i$ be a strongly connected transition-bordered subnet of $N$. Define $N_i' = \varphi(N_i)$. Then:
1. $N_i'$ is a subnet of $N'$;
2. If $|X_i'| \geq 1$ then $N_i'$ is a transition-bordered subnet of $N'$;
3. $\varphi_{N_i} : N_i \to N_i'$ is S-vicinity respecting.
The example in Figure 9 shows that being strongly connected is a necessary prerequisite for Proposition 3.6. In Figure 8 we gave an example of a strongly connected subnet which is not a transition-bordered subnet. Its image by the S-vicinity respecting quotient is not a subnet of the target net.

An S-component is a special strongly connected transition-bordered subnet. For respecting coverings by S-components, stronger hypotheses have to be assumed. Let us continue considering the S-vicinity respecting quotient shown in Figure 10.

The net N on the left hand side is covered by S-components. The net homomorphism φ is an S-vicinity respecting quotient. However, the target net is not covered by S-components. Observe that the restriction of φ to any S-component is not T-vicinity respecting. Consider the S-component N_t containing b. The image of N_t is the entire target net. We have φ(N_t) = {a, b} but φ(N_t) = {u, v} ≠ φ(N_t) = {u, v, w}. The net image of N_t is not an S-component of the target net.

In Figure 11, the quotients restricted to any S-component are T-vicinity respecting. Remember that quotients can be simply drawn by depicting the equivalence classes of elements that are identified by the quotient, as shown for arbitrary graphs in section 1.

**Proposition 3.7**

Let φ: N → N' be an S-vicinity respecting net homomorphism and let N_t be an S-component of N. Define N'_t = φ(N_t) and suppose that φ(N_t) → N'_t is T-vicinity respecting. If |X'_t| > 1 then N'_t is an S-component of N'.
Figure 10: An $S$-vicinity respecting quotient which is not $T$-vicinity respecting if restricted to an $S$-component.

Proof: Assume $|X_1| > 1$. Since $N_1$ is an $S$-component, it is a transition-bordered subnet. Hence, by Proposition 3.6, $N_1^T$ is a transition-bordered subnet of $N$.

It remains to prove: every $y' \in T_1$ satisfies $|y' \cap S_1| \leq 1$ and $|y' \cap S_1| \leq 1$.

Let $y' \in T_1^T$. We show only: $|y' \cap S_1| \leq 1$ (similarly). Since $\varphi_{N_1}$ is surjective we can find an arc $(x, y) \in F_1$ with $\varphi(x) \neq y'$ and $\varphi(y) = y'$ (Lemma 1.13). Since $\varphi_{N_1}$ is a $T$-vicinity respecting quotient, Corollary 1.15 implies $|S_1 \cap ^*y'| \leq |S_1 \cap ^*y|$ and $|S_1 \cap ^*y| \leq 1$ because $N_1$ is an $S$-component.

From Proposition 3.7 we deduce:

**Theorem 3.8**

Let $N$ be a net which is covered by a family $(N_i)_i \in I$ of $S$-components. Let $\varphi: N \rightarrow N'$ be an $S$-vicinity respecting quotient such that, for all $i \in I$, $\varphi_i: N_i \rightarrow \varphi(N_i)$ is $T$-vicinity respecting. Then $N'$ is covered by $S$-components.

Proof: In Proposition 3.7 we have shown that, given the assumptions above, every $S$-component of $N$ is either mapped to an $S$-component of $N'$ or to an element of $N'$.

Let $x' \in X'$. If $x'$ is an isolated element then it is a trivial $S$-component. So assume that $x'$ is not isolated. Then we can find a $y' \in X'$ such that $(x', y') \in F'$ or $(y', x') \in F'$.

Since $\varphi$ is a quotient, there are $x, y \in T$ with $\varphi(\{x, y\}) = \{x', y'\}$ and either $(x, y) \in F$ or $(y, x) \in F$. $N$ is covered by $S$-components and hence we can find an $i \in I$ such that $x \in S_i$ and $y \in T_i$. $|X_i| > 1$ since $x'$ and $y'$ are distinct elements of $\varphi(N_i)$. Thus $\varphi(N_i)$ is an $S$-component of $N'$.

By Proposition 3.6 (3) $'\varphi$ is $S$-vicinity respecting' implies for all $i \in I$: $'\varphi_{N_i}$ is $S$-vicinity respecting'. So all the $\varphi_{N_i}$ have to be both $S$- and $T$-vicinity respecting. However, this alone does not imply that $\varphi$ is $S$-vicinity respecting and is not sufficient for $N'$ to be covered by $S$-components as is shown in Figure 12. For the $S$-components $N_i$ of the net $N$ of Figure 12 $\varphi_{N_i}$ is $S$- and $T$-vicinity respecting. However, $\varphi$ is not $S$-vicinity respecting and $N'$ is not covered by $S$-components.

Theorem 3.8 implies that, given a family of $S$-components which cover the source net, a respective covering of the target net is obtained by the images of the $S$-components which are not mapped to single non-isolated places.
The choice of a covering family of $S$-components is decisive. In the example of Figure 13, the quotient is vicinity respecting. Its restriction to either the $S$-component $N_1$ which contains $a_1$, $a_2$ or to the $S$-component $N_2$ which contains $a'_1$, $a'_2$ is T-vicinity respecting. Taking the $S$-components $N_3$ and $N_4$ as a cover of $N$, with $a_1$, $a'_2$ belonging to $N_3$ and $a'_1$, $a_2$ belonging to $N_4$, the restriction of $\varphi$ to any of these $S$-components is not T-vicinity respecting. This example points out that the choice of an abstraction and the choice of an S-component covering are not independent.

By duality we get:

**Corollary 3.9**

Let $\varphi: N \rightarrow N'$ be a T-vicinity respecting net homomorphism and let $N_1$ be a strongly connected place-bordered subset of $N$. Define $N'_1 = \varphi(N_1)$. Then:

1. $N'_1$ is a subset of $N'$;
2. If $|X'| > 1$ then $N'_1$ is a place-bordered subset of $N'$;
3. $\varphi_{N_1}: N_1 \rightarrow N'_1$ is T-vicinity respecting.

The dual version of Theorem 3.8 reads as follows:

**Theorem 3.10**

Let $N$ be a net which is covered by a family $(N_i)_{i \in I}$ of $T$-components. Let $\varphi: N \rightarrow N'$ be a T-vicinity respecting quotient such that, for all $i \in I$, $\varphi_{N_i}: N_i \rightarrow \varphi(N_i)$ is T-vicinity respecting. Then $N'$ is covered by $T$-components.

The net homomorphisms depicted in Figure 11 are vicinity respecting. Their restrictions to any $S$-component or $T$-component are also vicinity respecting. Hence their net images are covered by $S$- and $T$-components.

A particular case of Theorem 3.8 is the composition of $S$-components; the source net $N$ is the disjoint union of a family of $S$-components and the mapping, restricted to each of these $S$-components, is injective (and hence a fortiori T-vicinity respecting).

We can reformulate our result as a property of net homomorphisms as follows:

For every place $a$ of an $S$-component $N_i$ of a net $N$ the entire vicinity belongs to the $S$-component as well by definition. Therefore the natural injection $\psi_i: N_i \rightarrow N$ is S-vicinity respecting but not necessarily surjective.

A covering by $S$-components $N_i (i \in I)$ can be expressed by a set of net homomorphisms $\psi_i (i \in I)$ such that each element of $N$ is in $\psi_i(N_i)$ for at least one $i$. Using the disjoint union of the $S$-components $(\bigcup N_i)$, the net homomorphisms $\psi_i$ induce a quotient $\psi$ from $\bigcup N_i$ to $N$. 
Now Theorem 3.8 reads as follows. Given

- a family $(N_i), i \in I$ of strongly connected sets with $|I| \leq 1, |I^*| \leq 1$ for all transitions $l$ (S-components),

- $S$-vicinity respecting injective net morphisms $\psi_i: N_i \rightarrow N(i \in I)$ such that the induced mapping $\psi$ is a quotient (i.e., $N$ is covered by the $N_i$),

- an $S$-vicinity respecting quotient $\varphi: N \rightarrow N'$ such that $\varphi_{N_i}$ is $T$-vicinity respecting for all $i \in I$,

we can find injective $S$-vicinity respecting mappings $\psi_i: \varphi(N_i) \rightarrow N'$ such that the induced mapping $\psi: \bigcup \varphi(N_i) \rightarrow N'$ is surjective (i.e., $N'$ is covered by the $\varphi(N_i)$).

An example is shown in Figure 14. Again, by duality we can use the same formalism to capture the composition of $T$-components.

4 Siphons, Traps and Free-Choice Property

We turn now to the structural concepts called siphons and traps. As the names suggest, a trap is a set of places where any output transition is also an input transition of one of the places while any input transition is also an output transition for the places of a siphon.

Definition 4.1

A siphon of a net is a nonempty set of places $A$ satisfying $A \subseteq A^*$. A trap of a net is a nonempty set of places $A$ satisfying $A^* \subseteq A$.

Figure 14: An example of the commutativity of composition and coarsening of nets
Theorem 4.7: If \( N \) is a graph, then \( N \) is a strongly connected graph if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in \( N \).

Proposition 4.2: If \( N \) is a graph and \( \mathcal{P} \) is a partition of \( N \) into non-empty subsets, then \( \mathcal{P} \) is a strongly connected partition if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).

We begin with a preliminary result that the proof follows from the above propositions.

Let \( N = (V, E) \) be a graph. Then every partition of \( V \) is a strongly connected partition if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).

Proof: Let \( \mathcal{P} \) be a partition of \( V \). Then \( \mathcal{P} \) is a strongly connected partition if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).

By Proposition 4.2, \( \mathcal{P} \) is a strongly connected partition if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).

Since \( \mathcal{P} \) is a partition of \( V \), every node is in exactly one component of \( \mathcal{P} \).

Therefore, \( \mathcal{P} \) is a strongly connected partition if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).

Hence, \( \mathcal{P} \) is a strongly connected partition if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).

Thus, \( \mathcal{P} \) is a strongly connected partition if and only if for every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).

Figure 4.7: The set \( \{a, b, c\} \) is a strongly connected partition of \( N \) if and only if every pair of nodes \( u, v \) in \( N \), there is a path from \( u \) to \( v \) and a path from \( v \) to \( u \) in each component of \( \mathcal{P} \).
Figure 16: The trap \{a, b\} is mapped onto a single element; its image is not a trap of the target net.

Figure 17: The trap \{a, b\} is not strongly connected; its image is not a trap of the target net.

Figure 18: This quotient is S-vicinity respecting only, the target net is not free choice.

Figure 19: This quotient is T-vicinity respecting only, the target net is not free choice.

Proof: We show that for any two places \(s_1', s_2'\) of \(S'\) we have: \(s_1' \cap s_2' = \emptyset\) or \(s_1' = s_2'\). We proceed indirectly. Let \(s_1'\) and \(s_2'\) be places of \(S'\) such that \(s_1' \cap s_2' \neq \emptyset\) and \(s_1' \neq s_2'\). Without loss of generality assume that there is a transition \(t_1' \in s_2'\) with \(t_1' \not\in s_1'\). Let \(t_1' \in s_1' \cap s_2'\). Since \(\varphi\) is a quotient we find \((s_1, t_1) \in F\) with \(\varphi(s_1) = s_1'\) and \(\varphi(t_1) = t_1'\). Since \(\varphi\) is S-vicinity respecting we have \(\varphi(s_1') = \varphi(s_2') = \varphi(t_1) = t_1'\). Hence there is a transition \(t_2' \in s_2'\) satisfying \(\varphi(t_2') = t_2\). Since \(\varphi\) is S-vicinity respecting, \(\varphi^{-1}(t_1) = \varphi^{-1}(t_1)\). Hence there is a place \(s_1 \in s_1'\) with \(\varphi(s_1) = s_1'\). Since \(\varphi\) is vicinity respecting and since \(t_2' \not\in s_1'\) we get \(t_1 \not\in \varphi(s_1')\) — a contradiction to the free-choice property of \(N\).

Figure 18 and 19 show that for the previous theorem S-vicinity respecting and T-vicinity respecting alone are not sufficient.

5 Conclusion

Structuring software requirements is a gradual process which involves refinement/abstraction between different conceptual levels. Abstractions should bear formal relations with refinements because otherwise the analysis of some abstraction will be of no help for the induced refinement. In section 2 we
argued that vicinity respecting homomorphisms give a possible and - to some extent - satisfactory solution to these requirements for graph-based models of distributed systems. They provide a method to perform graphical abstraction/refinement such that every element is either glued together with its vicinity or its vicinity is the vicinity of its image.

The vicinity respecting concept is a local notion because its definition only uses local vicinities. However, it has global consequences since it preserves paths and, consequently, connectedness properties. For Petri nets, vicinity respecting homomorphisms preserve moreover important structural properties such as $S$- and $T$-invariants, siphons and traps and the free-choice property - concepts that are also defined employing local vicinities of single elements.

Our starting point was the graph homomorphism notion [19], which is slightly relaxed, and the petri net morphism notion as presented in [20, 12]. We are concerned with the gradual structuring of software requirements. After sufficient steps of refinement and completion a Channel/Agency Petri Net is constructed that contains all relevant details. At this level, a marking can be added to the net. It fixes the resources which are factually available. Now knowledge about $S$- and $T$-components, traps and siphons can be used while simulating and analyzing the marked net with computer tools [21].

Other concepts for refinement and abstraction of Petri nets (see [3] for an overview) and of morphisms [24, 18] have been proposed in the literature. However, all these approaches are concerned with marked Petri nets and aim at results involving the behaviour given by the token game. In contrast, we are concerned with the preliminary task of structuring software requirements down to a working system and aim at structure preservation. Generally, abstraction in our sense is more general than behaviour preserving abstraction. However, structure influences behaviour. The transition refinement considered in [13] turns out to induce a vicinity respecting homomorphism from the refined net to the coarser net.

Structuring software requirements is an emerging issue in software engineering [15]. Many methodologies use at some point graphical representations of their systems. However, most of them rely on a language approach or an algebraic approach to formally describe the system and to define abstraction/refinement mechanisms, see [23], [16], [4] for examples. An originality of the Petri Net approach is to use graphs not only as a friendly visual support but also as a formal mathematical model.

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References


