Vicinity Respecting Homomorphisms for Abstracting System Requirements

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Abstract. This paper is concerned with structuring system requirements on an abstract conceptual level. Channel/Agency Petri nets are taken as a formal model. They allow to represent functional aspects as well as data aspects of the requirements in a graphical way. Vicinity respecting homomorphisms are presented as a means to refine and abstract these nets. They preserve paths, i.e., dependencies between computational elements and they preserve important structural properties of nets, such as S- and T-components, siphons and traps and the free choice property. These properties have important interpretations for marked Petri nets and can therefore be used for the analysis of system models at more concrete levels.

Keywords: Channel/Agency Nets, Homomorphisms, Abstraction.

1 Introduction

A nontrivial task in the design of large and complex systems is to organize the requirements into a coherent structure. Usually, this organization is a gradual process which involves refinement and abstraction between different conceptual levels of the system. In this paper we take Channel/Agency Petri nets [22,24] to model systems and propose vicinity respecting homomorphisms as a means to refine and abstract these nets.

Channel/Agency Petri nets are a Petri net model where all elements of a net are labelled by informal descriptions. They have been proposed for the conceptual modelling of the architecture of information systems e.g. in [1,2,13]. As shown in [23,24] they can be used for different levels of abstraction, in particular in the early phases of system and software engineering. On a low level of abstraction containing all details nets can be equipped with markings and a notion of behavior which simulates the behavior of the modelled system. In this way Petri nets can be used as a means for prototyping.

We introduce vicinity respecting homomorphisms of Petri nets to formalize refinement and abstraction relations between nets. This encompasses modular techniques because each composition of subsystems may be viewed as an identification of the respective interface elements and thus as a particular abstraction. Vicinity respecting homomorphisms rely on the graph structure of a net. They

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are special graph homomorphisms that are able to formalize abstractions including contractions of graphs not only in their breadth but also in their length.

The definition of vicinity respecting homomorphisms is based on the local vicinities of elements. This concept suffices to preserve important global structural properties like connectedness. If two elements of a net are connected by a path then the respective system components are in a causal dependency relation. Because they preserve paths, vicinity respecting homomorphisms not only respect dependency but also its complementary relation independency.

Petri nets not only allow to combine data- and function-oriented views of a system. They also allow to concentrate on either aspect. The data aspect including nondeterministic choice is reflected by S-components. T-components represent an activity-oriented view, where only transitions are branched. Petri nets that are covered by S- and T-components allow for a compositional interpretation of these two aspects. We show that vicinity respecting net homomorphisms preserve coverings by S- and T-components. As a consequence, they respect the notions of choice (a forward branching place) and of synchronization (a backward branching transition).

This paper gathers, generalizes and deepens results obtained in [4,16,6]. In the last years, research concentrated on abstraction techniques for Petri nets on a behavioral level, i.e. morphisms have been defined that preserve occurrence sequences or other behavioral notions. Structural relations between the respective nets appeared as a consequence of behavioral relations. For example, [14] concentrates on abstraction techniques for high-level Petri nets and explicitly distinguishes the advocated behavior-oriented approach from our structure-based work. Our work is different because we concentrate on local structural properties of the relation between nets, i.e., on properties of the homomorphism, and derive global structural properties which have consequences for behavior.

Another line of research considers abstraction and modularity techniques for Petri nets based on graph grammars, see e.g. [8]. Considering relations between Petri nets representing conceptual models, which do not necessarily have a formal behavior, was continued in [18,19]. It is important to notice that this work was not published in the Petri net community but in our intended application domain – hence it points out that there is a demand for structural abstraction techniques of process models.

Recently, there is a renewed interest in construction techniques for (unmarked) Petri nets, applied for requirement analysis [7] and in the context of modular Petri nets [25] that are composed via identification of common nodes.

Today, instead of Petri nets, diagram techniques of the UML play a more important role in practice. However, all these diagram languages are essentially graphs. At least for UML Activity Diagrams, a diagram type closely related to Petri nets, our approach can be applied as well.

Other diagram techniques have a different notion of abstraction; abstraction is a concept visible within a single diagram instead of a relation between diagrams. Examples for such abstractions are aggregation and generalization in data modelling. Also hierarchy notions in process modelling provide abstraction concepts within diagrams. Whereas a Petri net morphism relates two nets, particular morphisms representing abstractions can be depicted in single diagrams. In this paper we concentrate on so-called net quotients where the elements of the more abstract net can be viewed as equivalence sets of the net on the less abstract level. A net quotient is uniquely identified by the less abstract net together with the equivalence classes, which directly leads to a diagram technique comparable to the ones mentioned above.

In Section 2 we investigate homomorphisms of arbitrary graphs. Section 3 introduces Petri nets and transfers the notion of vicinity respecting homomorphisms to them. In Section 4 we show that vicinity respecting homomorphisms respect coverings by S- and T-components of Petri nets and draw consequences for Petri nets composition. Siphons and traps are concepts known from Petri net theory that allow for an analysis of the data contained in sets of places [3]. Section 5 proves that vicinity respecting homomorphisms preserve siphons, traps and the free choice property. Finally, Section 6 concludes this paper.

2 Graph Homomorphisms

Petri nets are special graphs. Vicinity respecting homomorphisms will be defined for arbitrary graphs in this section.

Figure 1 shows a model of a sender/receiver system on the left hand side and a coarser view of the same system on the right hand side. The left model can be viewed as a refinement of the right model. The interrelation between the graphs is given by a mapping which is a particular graph homomorphism. As we shall see, in this example dependencies between vertices of the source graph are strongly related to dependencies between vertices of the target graph. We start with a formal introduction of graphs and related concepts. We consider only finite directed graphs without multiple edges and without loops.

Definition 1. *A* graph *is a pair* (X, F) *where* X *is a finite set (*vertices*) and* $F \subseteq X \times X$ *(*edges*). A* loop *is an edge* (x, x)*. A graph is said to be* loop-free *if no edge is a loop.*

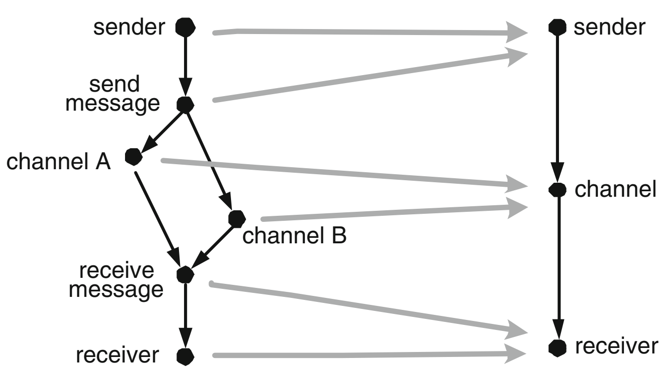

Fig. 1. A graph homomorphism as a mapping

The classical notion of graph homomorphism [20] respects edges in the sense that the images of connected vertices are again connected. Since we also consider contractions of loop-free graphs, where two connected vertices are mapped to one vertex without a loop, a slightly more liberal definition will be employed; we allow the images of connected vertices to be either connected or identical.

Definition 2. *Let (X,F) and (X',F') be graphs. A mapping $\varphi: X \to X'$ is a* graph homomorphism *(denoted by $\varphi: (X,F) \to (X',F')$) if, for every edge $(x,y) \in F$, either $(\varphi(x),\varphi(y)) \in F'$ or $\varphi(x) = \varphi(y)$.*

To describe the environment of an element we shall use the notions of pre- and post-sets and related notions of pre- and post-vicinities.

Definition 3. *Given a graph (X,F) and $x \in X$, we denote by ${}^\bullet x = \{y \in X \mid (y,x) \in F\}$ the pre-set of x and by $x^\bullet = \{y \in X \mid (x,y) \in F\}$ the post-set of x. The pre-vicinity of x is ${}^\odot x = \{x\} \cup {}^\bullet x$, the post-vicinity of x is $x^\odot = \{x\} \cup x^\bullet$.*

$\varphi({}^\bullet x) \subseteq {}^\bullet(\varphi(x))$ does not hold for arbitrary graph homomorphisms because in case of contractions elements of ${}^\bullet x$ can be mapped to $\varphi(x)$, and similarly for post-sets. However, we get:

Lemma 1. *Let (X,F) and (X',F') be graphs and $\varphi: X \to X'$ a mapping.*
The following three conditions are equivalent.

1. *φ is a graph homomorphism.*
2. *for each vertex x in X, $\varphi({}^\odot x) \subseteq {}^\odot(\varphi(x))$.*
3. *for each vertex x in X, $\varphi(x^\odot) \subseteq (\varphi(x))^\odot$.*

Proof: We only show $1 \iff 2$, $1 \iff 3$ being similar.

Assume that $\varphi: X \to X'$ is a graph homomorphism, let x be an element of X and let $y \in {}^\odot x$. If $x = y$ then $\varphi(y) = \varphi(x) \in {}^\odot(\varphi(x))$. If $(y,x) \in F$ then either $\varphi(y) = \varphi(x) \in {}^\odot(\varphi(x))$ or $(\varphi(y),\varphi(x)) \in F'$ since φ is a graph homomorphism and therefore $\varphi(y) \in {}^\odot(\varphi(x))$.

Conversely, assume for each x in X, $\varphi({}^\odot x) \subseteq {}^\odot(\varphi(x))$. Let $(y,x) \in F$. Then $y \in {}^\odot x$ and hence $\varphi(y) \in {}^\odot(\varphi(x))$. So either $\varphi(y) = \varphi(x)$ or $(\varphi(y),\varphi(x)) \in F'$.
□

Definition 4. *A sequence $x_1, x_2 \dots x_n$ $(n \geq 1)$ of vertices of a graph is a* path *if there exist edges $(x_1,x_2),\dots,(x_{n-1},x_n)$ of the graph. A graph is* strongly connected *if for any two vertices x and y there exists a path $x \dots y$.*

We allow a single element to be a path. Since consecutive vertices of a graph can be mapped onto a single element without a loop, the sequence of images of path elements is not necessarily a path of the target graph. So we define, for loop-free graphs, the image of a path to ignore stuttering of vertices.

Definition 5. *Let (X,F), (X',F') be loop-free graphs and $\varphi: (X,F) \to (X',F')$ a graph homomorphism. The image of a path $x_1 \dots x_n$ of (X,F) is defined by*

$$\varphi(x_1 \dots x_n) = \begin{cases} \varphi(x_1) & \text{if } n = 1 \\ \varphi(x_1 \dots x_{n-1}) & \text{if } n > 1 \text{ and } \varphi(x_{n-1}) = \varphi(x_n) \\ \varphi(x_1 \dots x_{n-1})\,\varphi(x_n) & \text{if } n > 1 \text{ and } \varphi(x_{n-1}) \neq \varphi(x_n) \end{cases}$$

Using Definition 5, the image of the path *sender*, *send message*, *channel A* of the left hand graph in Figure 1 is the path *sender channel* of the target graph.

Graph homomorphisms do not preserve edges but they preserve paths:

Lemma 2. *Let* (X, F), (X', F') *be loop-free graphs and let* $\varphi\colon (X, F) \to (X', F')$ *be a graph homomorphism. If* $x_1 \ldots x_n$ *is a path of* (X, F) *then* $\varphi(x_1 \ldots x_n)$ *is a path of* (X', F') *leading from* $\varphi(x_1)$ *to* $\varphi(x_n)$*.*

Proof: We proceed by induction on n.

If $n = 1$ then $\varphi(x_1)$ is a path of (X', F').

Let $n > 1$ and assume that $\varphi(x_1 \ldots x_{n-1})$ is a path leading from $\varphi(x_1)$ to $\varphi(x_{n-1})$. We have $(x_{n-1}, x_n) \in F$ by the definition of a path. By the homomorphism property, we can distinguish two cases:

1. $(\varphi(x_{n-1}), \varphi(x_n)) \in F'$. Then $\varphi(x_1 \ldots x_n) = \varphi(x_1 \ldots x_{n-1})\varphi(x_n)$ is a path of (X', F') leading from $\varphi(x_1)$ to $\varphi(x_n)$.
2. $\varphi(x_{n-1}) = \varphi(x_n)$. Then $\varphi(x_1 \ldots x_n) = \varphi(x_1 \ldots x_{n-1})$. By assumption, this is a path leading from $\varphi(x_1)$ to $\varphi(x_{n-1})$. Since $\varphi(x_{n-1}) = \varphi(x_n)$ this path leads to $\varphi(x_n)$. □

Surjectivity is a first condition when graph homomorphisms are used for abstractions. Surjective graph homomorphisms preserve strong connectivity:

Corollary 1. *Let* (X, F), (X', F') *be loop-free graphs and* $\varphi\colon (X, F) \to (X', F')$ *a surjective graph homomorphism. If* (X, F) *is strongly connected then* (X', F') *is also strongly connected.*

Proof: Let $x', y' \in X'$. Since φ is surjective there are $x, y \in X$ such that $\varphi(x) = x'$ and $\varphi(y) = y'$. There is a path from x to y because (X, F) is strongly connected. Using Lemma 2, some path of (X', F') leads from x' to y'. □

Surjectivity concerns vertices only. An additional requirement is that every edge of a target graph reflects a connection between respective vertices of the source graph. We call such a graph homomorphism a quotient.

Definition 6. *Let* (X, F), (X', F') *be loop-free graphs. A surjective graph homomorphism* $\varphi\colon (X, F) \to (X', F')$ *is called* quotient *if, for every edge* $(x', y') \in F'$*, there exists an edge* $(x, y) \in F$ *such that* $\varphi(x) = x'$ *and* $\varphi(y) = y'$*.*

The graph homomorphism shown in Figure 1 is a quotient. The edges of the target graph are the equivalence classes of the edges of the source graph. There is a vertex connecting two edges of the target graph if and only if there is at least one edge connecting elements of the respective sets of vertices in the source graph.

The name "quotient" is justified because for quotients, target graphs are determined up to renaming by the equivalence classes of vertices that are mapped onto the same vertex (see [5]). Therefore, we can represent quotients graphically by solely depicting equivalence classes. Figure 2 represents the quotient shown in Figure 1 this way.

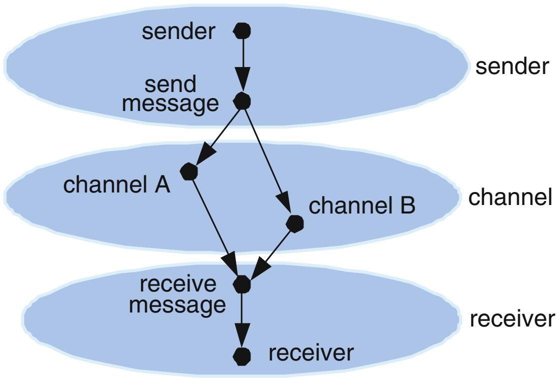

Fig. 2. Another representation of the graph homomorphism of Figure 1

When thinking of a graph (X', F') as of an abstraction of another graph (X, F), dependencies between nodes of X' that are represented through paths have to mirror dependencies already present in X. Therefore we look for a converse of Lemma 2. For quotients, this lemma has a weak converse by the definition of a quotient: every path of the target graph with at most two vertices is the image of a path of the source graph.

The same does not necessarily hold for longer paths, as shown in Figure 3. The target graph has a path $\varphi(a)\,\varphi(b)\,\varphi(f)$ which is not the image of a path of the source graph. What is wrong with this homomorphism? The post-vicinity of b is $\{b, d\}$. The post-vicinity of the image of b contains three vertices, namely $\varphi(b), \varphi(d)$ and $\varphi(f)$. So the image of the post-vicinity of b is properly included in the post-vicinity of the image. We say that the post-vicinity is not respected and define homomorphisms that respect vicinities of vertices:

Definition 7. *Let (X, F), (X', F') be loop-free graphs.*

A graph homomorphism $\varphi\colon (X, F) \to (X', F')$ is called pre-vicinity respecting *if, for every $x \in X$, either $\varphi({}^{\odot}x) = {}^{\odot}(\varphi(x))$ or $\varphi({}^{\odot}x) = \{\varphi(x)\}$.*

φ is called post-vicinity respecting *if, for every $x \in X$, either $\varphi(x^{\odot}) = (\varphi(x))^{\odot}$ or $\varphi(x^{\odot}) = \{\varphi(x)\}$.*

φ is called vicinity respecting *if it is pre-vicinity respecting and post-vicinity respecting.*

The following theorem states that for surjective post-vicinity respecting homomorphisms of strongly connected graphs there is a converse of Lemma 2. By symmetry, the same holds for pre-vicinity respecting graph homomorphisms.

Theorem 1. *Let (X, F), (X', F') be loop-free graphs such that (X, F) is strongly connected, and $\varphi\colon (X, F) \to (X', F')$ a surjective post-vicinity respecting graph homomorphism. For each path $x'_1 \ldots x'_m$ of (X', F') there exists a path $x_1 \ldots x_n$ of (X, F) such that $\varphi(x_1 \ldots x_n) = x'_1 \ldots x'_m$.*

Proof: We proceed by induction on m.

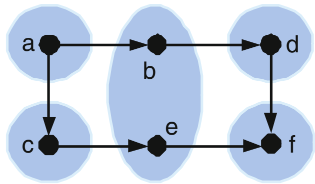

Fig. 3. A path of the target graph is not necessarily image of a source graph path

Let $m = 1$. Since φ is surjective, some $x_1 \in X$ satisfies $\varphi(x_1) = x'_1$. The path consisting of x_1 only satisfies the required property.

Let $m > 1$. Assume that $x_1 \dots x_k$ is a path of (X, F) such that $\varphi(x_1 \dots x_k) = x'_1 \dots x'_{m-1}$. Since φ is surjective, some $y \in X$ satisfies $\varphi(y) = x'_m$. Since the graph (X, F) is strongly connected, it contains a path leading from x_k to y. Consider the first vertex x_{i+1} in this path that is not mapped to x'_{m-1}. Such a vertex exists because the last vertex y in the path is mapped to x'_m and $x'_{m-1} \neq x'_m$ since (X', F') is loop-free. By definition, the predecessor x_i of x_{i+1} is mapped to x'_{m-1} and so are all vertices in the path $x_k \dots x_i$.

We have $x_{i+1} \in x_i^{\odot}$. Since $\varphi(x_{i+1}) \neq \varphi(x_i)$ we obtain $\varphi(x_i^{\odot}) \neq \{\varphi(x_i)\}$. Using the definition of a post-vicinity respecting homomorphism we conclude that $\varphi(x_i^{\odot}) = (\varphi(x_i))^{\odot}$. Therefore, since $x'_m \in {x'_{m-1}}^{\odot} = (\varphi(x_i))^{\odot}$, some vertex $z \in x_i^{\odot}$ is mapped to x'_m. This vertex z cannot be x_i itself, hence it is in $x_i^{\bullet}$.

Now the path $x_1 \dots x_k \dots x_i z$ is mapped to $x'_1 \dots x'_m$. □

The example in Figure 4 shows that in the previous theorem it is necessary that the source graph (X, F) is strongly connected. Moreover, neither requiring weak connectedness nor nonempty pre- or post-sets for all vertices constitute sufficient conditions. This graph homomorphism φ is a vicinity respecting quotient. The target graph has a path $\varphi(c)\,\varphi(d)\,\varphi(c)$ which is not the image of a path of the source graph.

Corollary 2. *Let (X, F), (X', F') be loop-free graphs such that (X, F) is strongly connected and let $\varphi\colon (X, F) \to (X', F')$ be a surjective post-vicinity respecting graph homomorphism. Then φ is a quotient.*

Proof: Let $(x', y') \in F'$. Then $x'\, y'$ is a path. By Theorem 1 there is a path $x_1 \dots x_n$ of X with $\varphi(x_1 \dots x_n) = x'\, y'$. Let x_i be the last element of the path with $\varphi(x_i) = x'$. Then $(x_i, x_{i+1}) \in F$ and $\varphi(x_{i+1}) = y'$, which was to prove. □

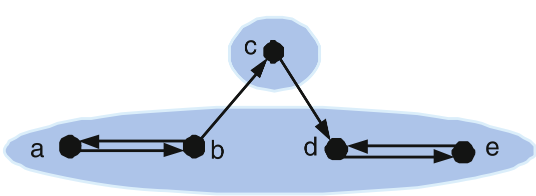

Fig. 4. For non-strongly connected graphs, a path of the target graph by a vicinity respecting quotient is not necessarily the image of a path

The following result, that will be used later, is weaker than Corollary 2 but holds for arbitrary surjective graph homomorphisms.

Lemma 3. *Let (X, F) be a strongly connected graph and let (X', F') be a graph satisfying $|X'| > 1$. If $\varphi: (X, F) \to (X', F')$ is a surjective graph homomorphism, then, for each $x' \in X'$, there are arcs $(y, x_1), (x_2, z) \in F$ with $\varphi(x_1) = x' = \varphi(x_2)$ and $\varphi(y) \neq x' \neq \varphi(z)$.*

Proof: We show only the first part, the second one being similar.
Let y' be an element of X' distinct from x' (which is possible because $|X'| > 1$). Since φ is surjective there are $c, d \in X$ with $\varphi(c) = x'$ and $\varphi(d) = y'$. Since (X, F) is strongly connected, there exists a path $x_1 \dots x_n$ of (X, F) with $x_1 = c$ and $x_n = d$. Let i be the least index such that $\varphi(x_i) = x'$ and $\varphi(x_{i+1}) \neq x'$. With $(x, y) = (x_i, x_{i+1})$ we are finished. □

Concentrating on different elements which are mapped onto the same image instead of comparing source graph and target graph leads to another aspect of vicinity respecting homomorphisms in the case of quotients.

Lemma 4. *Let (X, F), (X', F') be loop-free graphs and $\varphi: (X, F) \to (X', F')$ a quotient. φ is vicinity respecting iff for all $x, y \in X$ satisfying $\varphi(x) = \varphi(y)$:*

1. $\varphi(^{\odot}x) = \{\varphi(x)\}$ *or* $\varphi(^{\odot}y) = \{\varphi(y)\}$ *or* $\varphi(^{\odot}x) = \varphi(^{\odot}y)$;
2. $\varphi(x^{\odot}) = \{\varphi(x)\}$ *or* $\varphi(y^{\odot}) = \{\varphi(y)\}$ *or* $\varphi(x^{\odot}) = \varphi(y^{\odot})$.

Proof: It is immediate that Definition 7 implies 1. and 2. We only show that 1. implies pre-vicinity respecting; showing that 2. implies post-vicinity respecting is similar.

Let $x \in X$ such that $\varphi(^{\odot}x) \neq \{\varphi(x)\}$ and let $z' \in {}^{\bullet}\varphi(x)$. Then there exist $y, z \in X$ such that $z \in {}^{\bullet}y, \varphi(z) = z'$ and $\varphi(y) = \varphi(x)$ because φ is a quotient. By 1. we obtain that $\varphi(^{\odot}x) = \varphi(^{\odot}y)$. Since $z \in {}^{\odot}y$, some element in ${}^{\odot}x$ is mapped to z'. Since z' was chosen arbitrarily in ${}^{\bullet}\varphi(x)$ we finally obtain $\varphi(^{\odot}x) = {}^{\odot}\varphi(x)$. □

In the proof of Lemma 4, we showed that for any element $z' \in {}^{\bullet}\varphi(x)$ there exists an element $z \in {}^{\bullet}x$ with $\varphi(z) = z'$. From this fact, we deduce immediately the following technical corollary that will be used later.

Corollary 3. *Let (X, F), (X', F') be loop-free graphs and $\varphi: (X, F) \to (X, F')$ a vicinity respecting quotient. Then, for every $x \in X$:*

1. *if* $\varphi(^{\odot}x) \neq \{\varphi(x)\}$ *then* $|{}^{\bullet}\varphi(x)| \leq |{}^{\bullet}x|$;
2. *if* $\varphi(x^{\odot}) \neq \{\varphi(x)\}$ *then* $|\varphi(x)^{\bullet}| \leq |x^{\bullet}|$.

3 Net Homomorphisms

A (Petri) net can be seen as a loop-free graph (X, F) where the set X of vertices is partitioned into a set S of *places* and a set T of *transitions* such that the flow relation F must not relate two places or two transitions. Formally:

Definition 8. *A triple $N = (S, T; F)$ is called* net *if S and T are disjoint sets and $F \subseteq (S \times T) \cup (T \times S)$. The set $X = S \cup T$ is the set of* elements *of the net.*

This definition allows to consider nets with isolated elements, i.e. elements with empty pre- and post-sets. Graphically, places are represented by circles, transitions are represented by squares and the flow relation is represented by arrows between elements. We do not consider markings and behavioral notions but concentrate on the structure of net models. However, there are many relations between structure and behavior, whence our concepts have indirect consequences for the behavior of the considered nets as well.

We use the following convention: indices and primes used to denote a net N are carried over to all parts of N. For example, speaking of a net N_i', we implicitly understand $N_i' = (S_i', T_i'; F_i')$ and $X_i' = S_i' \cup T_i'$.

The $\bullet$-notation for pre- and post-sets and the $\odot$-notation for pre- and post-vicinities of vertices of graphs carries over to nets. A consequence of Definition 8 is that the pre-set and the post-set of a place are sets of transitions, and the pre-set and the post-set of a transition are sets of places. We will employ the $\bullet$-notation also for sets of elements as usual: The pre-set of a set of elements is the union of pre-sets of elements of the set, and similar for post-sets.

The transitions of a net model the active subsystems, i.e. functions, operators, transformers etc. They are only connected to places which model passive subsystems, i.e. data, messages, conditions etc. On a conceptual level, it is not always obvious to classify a subsystem active or passive. The decision to model it by a place or by a transition is based on the interaction of the subsystem with its vicinity. As an example, consider a channel that is connected to functional units that send and receive data through the channel. Then the channel has to be modelled by a place. In contrast, if the channel is connected to data to be sent on one side and to already received data on the other side then the channel is modelled by a transition. As we shall see, a transition may represent a subsystem that is modelled by a net containing places and transitions on a finer level of abstraction. The same holds respectively for places.

An arrow in a net either leads from a place to a transition or from a transition to a place. In the first case the place is interpreted as a pre-requisite (pre-condition, input) for the transition which can be consumed by the action modelled by the transition. In the second case the place is interpreted as a post-requisite (post-condition, output) for the transition which can be produced by the action modelled by the transition. In this sense, arrows are used to denote two different types of relations between the elements of a Petri net.

Homomorphisms of Petri nets are particular graph homomorphisms that additionally respect the type of relation between the elements given by arrows [21,11]. Since we again allow contractions, places can be mapped to transitions and transitions can be mapped to places. However, if two connected elements are not mapped to the same element of the target net, then the place of the two has to be mapped to a place and the transition has to be mapped to a transition. So Definition 2 becomes for Petri nets:

Definition 9. *Let N, N' be nets. A mapping $\varphi\colon X \to X'$ is called* net homomorphism*, denoted by $\varphi\colon N \to N'$, if for every edge $(x, y) \in F$ holds:*

1. *if $(x, y) \in F \cap (S \times T)$ then either $(\varphi(x), \varphi(y)) \in F' \cap (S' \times T')$ or $\varphi(x) = \varphi(y)$,*
2. *if $(x, y) \in F \cap (T \times S)$ then either $(\varphi(x), \varphi(y)) \in F' \cap (T' \times S')$ or $\varphi(x) = \varphi(y)$.*

This definition is equivalent to the one given in [10]. Notice that there are various different definitions of Petri net homomorphisms in the literature, for example [17].

A consequence of our definition is that a transition is only allowed to be mapped to a place if all places of its pre-set and its post-set are mapped to the same place, and similarly for places.

Lemma 5. *Let $\varphi\colon N \to N'$ be a net homomorphism. Then:*

1. *If a transition t is mapped to a place s' then $\varphi({}^{\odot}t \cup t^{\odot}) = \{s'\}$.*
2. *If a place s is mapped to a transition t' then $\varphi({}^{\odot}s \cup s^{\odot}) = \{t'\}$.*

Proof: We only show 1., 2. being similar.
Let $t \in T$, $s' \in S'$ such that $\varphi(t) = s'$. Then for each place $s \in {}^{\bullet}t$ we have $(s, t) \in F \cap S \times T$ and $(\varphi(s), \varphi(t)) \notin S' \times T'$ and therefore $\varphi(s) = \varphi(t)$. Likewise, each place $s \in t^{\bullet}$ satisfies $\varphi(s) = \varphi(t)$. Since $\varphi({}^{\odot}t \cup t^{\odot}) = \varphi({}^{\bullet}t) \cup \varphi(\{t\}) \cup \varphi(t^{\bullet})$ and $\varphi(t) = s$ we obtain the result. □

Corollary 4. *Let $\varphi\colon N \to N'$ be a net homomorphism and let $(x, y) \in F$ such that $\varphi(x) \neq \varphi(y)$. Then $\varphi(x) \in S'$ iff $x \in S$ and $\varphi(y) \in S'$ iff $y \in S$.*

For nets the vicinity respecting homomorphism definition can be split into two notions: homomorphisms that respect the vicinity of places and homomorphisms that respect the vicinity of transitions.

Definition 10. *Let $\varphi\colon N \to N'$ be a net homomorphism.*

1. *φ is* S-vicinity respecting *if, for every $x \in S$:*
   (a) $\varphi({}^{\odot}x) = {}^{\odot}(\varphi(x))$ or $\varphi({}^{\odot}x) = \{\varphi(x)\}$ and
   (b) $\varphi(x^{\odot}) = (\varphi(x))^{\odot}$ or $\varphi(x^{\odot}) = \{\varphi(x)\}$.
2. *φ is* T-vicinity respecting *if, for every $x \in T$:*
   (a) $\varphi({}^{\odot}x) = {}^{\odot}(\varphi(x))$ or $\varphi({}^{\odot}x) = \{\varphi(x)\}$ and
   (b) $\varphi(x^{\odot}) = (\varphi(x))^{\odot}$ or $\varphi(x^{\odot}) = \{\varphi(x)\}$.
3. *φ is* vicinity respecting *if it is both S- and T-vicinity respecting*

A subnet of a net is generated by its elements and preserves the flow relation between its elements. We will be interested in subnets that are connected to the remaining part only via places or only via transitions.

Definition 11. *Let N be a net and let X_1 be a subset of X, the set of places and transitions of N. The $\bullet$-notation refers to N in the sequel.*

1. *X_1 generates the* subnet *$N_1 = (S \cap X_1, T \cap X_1; F \cap (X_1 \times X_1))$.*
2. *N_1 is called* transition-bordered *if ${}^{\bullet}S_1 \cup S_1^{\bullet} \subseteq T_1$.*
3. *N_1 is called* place-bordered *if ${}^{\bullet}T_1 \cup T_1^{\bullet} \subseteq S_1$.*

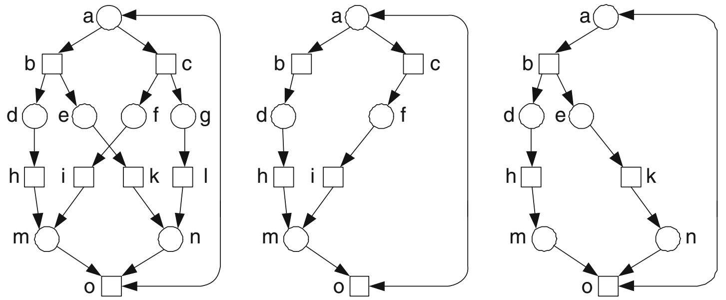

Fig. 5. A net with two subnets

A single transition of a net constitutes a transition-bordered subnet and a single place constitutes a place-bordered subnet. Figure 5 shows three nets. The net in the middle of the figure is a subnet of the net on the left hand side (notice that its set of places is a subset of the set of places of the left net and similar for transitions). This subnet is generated by its set of elements $\{a, b, c, d, f, h, i, m, o\}$. It is a transition-bordered subnet because, for its places a, d, f and m, it contains all transitions in the pre- and post-set of the places. Similarly, the subnet on the right hand side is place-bordered.

Net homomorphism allow to map places to transitions and vice versa. Nevertheless, the role of active and passive components of a net are preserved in the following sense. The refinement of a transition-bordered subnet is a transition-bordered subnet, i.e., the reverse image of the elements of a transition-bordered subnet generates a transition-bordered subnet of the source net. Similarly, the set of elements of the source net that are mapped to some place-bordered subnet of the target net constitute a place-bordered subnet of the source net. The following results were proven in [9] in a topological framework.

Lemma 6. *Let $\varphi\colon N \to N'$ be a net homomorphism.*

1. *If N'_1 is a transition-bordered subnet of N' then $\{x \in X \mid \varphi(x) \in X'_1\}$ generates a transition-bordered subnet of N.*
2. *If N'_1 is a place-bordered subnet of N' then $\{x \in X \mid \varphi(x) \in X'_1\}$ generates a place-bordered subnet of N.*

Proof: We show only 1, 2 being similar.
Let $(x, y) \in F$. Assume that $\varphi(x) \notin X'_1$ and $\varphi(y) \in X'_1$. Since N'_1 is a transition-bordered subnet of N', $\varphi(x)$ is a place and $\varphi(y)$ is a transition. By Corollary 4, x is a place and y is a transition. It is similarly shown that $\varphi(x) \in X'_1$ and $\varphi(y) \notin X'_1$ implies that x is a transition and y is a place. The result follows by the definition of a transition-bordered subnet. □

4 Transformation of S- and T-Components

An S-component of a net yields a data-oriented view of a part of the system. An S-component can contain nondeterministic choices that are modelled by branching places, i.e. by places with more than one output transitions. It does however not contain aspects of concurrency, whence its transitions are not branched [3]. Similarly, T-components concentrate on functional aspects. They do not contain branching places. Formally S-components and T-components are particular subnets. Figure 5 shows a net on the left hand side, one of its S-components in the middle and one of its T-components on the right hand side.

Definition 12. *Let N be a net. The $\bullet$-notation refers to N in the sequel.*

1. *A strongly connected transition-bordered subnet N_1 of N is called* S-component *of N if, for every $t \in T_1$, $|{}^\bullet t \cap S_1| \leq 1 \wedge |t^\bullet \cap S_1| \leq 1$. N is* covered by S-components *if there exists a family of S-components (N_i), $i \in I$, such that for every $x \in X$ there exists an $i \in I$ such that $x \in X_i$.*
2. *A strongly connected place-bordered subnet N_1 of N is called* T-component *of N if, for every $s \in S_1$, $|{}^\bullet s \cap T_1| \leq 1 \wedge |s^\bullet \cap T_1| \leq 1$. N is* covered by T-components *if there exists a family of T-components (N_i), $i \in I$, such that for every $x \in X$ there exists an $i \in I$ such that $x \in X_i$.*

When a net is mapped to another net, then so are its subnets. The following definition provides a notion for the induced mapping of a subnet.

Definition 13. *Let $\varphi\colon N \to N'$ be a net homomorphism and N_1 a subnet of N. The net $(\varphi(X_1) \cap S', \varphi(X_1) \cap T'; \{(\varphi(x), \varphi(y)) \mid (x, y) \in F_1 \wedge \varphi(x) \neq \varphi(y)\})$ is called the net image of N_1 by φ. It is denoted by $\varphi(N_1)$. By $\varphi_{N_1}\colon X_1 \to \varphi(X_1)$ we denote the restriction of φ to X_1, with the range of φ restricted to $\varphi(X_1)$.*

The induced graph homomorphism φ_{N_1} is surjective by definition. Note that $\varphi(N_1)$, the net image of N_1, is not necessarily a subnet of the target net N_1. Figure 6(a) gives an example. The image of the subnet generated by $\{a, b, c, d, e, f\}$ is not a subnet of the target net because the target net has an arrow from $\varphi(b)$ to $\varphi(e)$ whereas the net image of the subnet does not have this arrow.

Definition 13 immediately implies the following result:

Proposition 1. *If $\varphi\colon N \to N'$ is a net homomorphism and N_1 is a subnet of N then $\varphi_{N_1}\colon N_1 \to \varphi(N_1)$ is a quotient.*

Corollary 5. *A net homomorphism $\varphi\colon N \to N'$ is a quotient if and only if $N' = \varphi(N)$ and in this case $\varphi = \varphi_N$.*

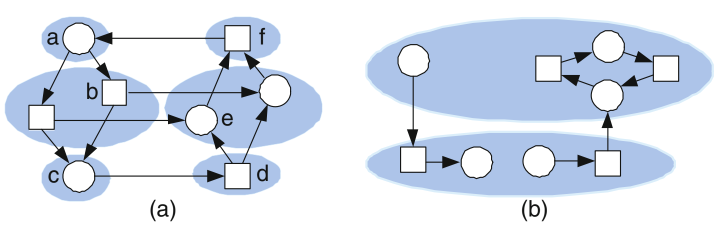

Fig. 6. Net images of subnets are not necessarily subnets

S-vicinity respecting net homomorphisms map a strongly connected transition-bordered subnet either onto a single element or onto a strongly connected transition-bordered subnet:

Theorem 2. *Let* $\varphi: N \to N'$ *be an S-vicinity respecting net homomorphism and let* N_1 *be a strongly connected transition-bordered subnet of* N. *Define* $N'_1 = \varphi(N_1)$. *Then:*

1. N'_1 *is a subnet of* N'.
2. *If* $|X'_1| > 1$ *then* N'_1 *is a transition-bordered subnet of* N'.
3. $\varphi_{N_1}: N_1 \to N'_1$ *is S-vicinity respecting.*

Proof: Assume $|X'_1| > 1$ (otherwise the proposition trivially holds).

1. Obviously $S'_1 \subseteq S', T'_1 \subseteq T'$ and $F'_1 \subseteq F' \cap ((S'_1 \times T'_1) \cup (T'_1 \times S'_1))$.
   Let $x', y' \in X'_1$ such that $(x', y') \in F'$. We show that $(x', y') \in F'_1$.
   Assume that x' is a place. $\varphi_{N_1}: N_1 \to N'_1$ is a surjective net homomorphism. By Lemma 3 we can find an arc $(x, z) \in F_1$ such that $\varphi_{N_1}(x) = x'$ and $\varphi_{N_1}(z) \neq x'$. By Corollary 4, $x \in S_1$. Since φ is S-vicinity respecting and $\varphi(x^{\odot}) \neq \{\varphi(x)\}$ ($\odot$-notation w.r.t. F) there exists some $y \in T_1$ with $y \in x^\bullet$ and $\varphi(y) = y'$ since N_1 is a transition-bordered subnet. Hence $(x, y) \in F_1$ and $(\varphi(x), \varphi(y)) = (x', y') \in F'_1$.
   The case $y' \in S'$ is analogous.
2. We only show $S_1'^{\bullet} \subseteq T'_1$, ${}^\bullet S'_1 \subseteq T'_1$ being similar. Let $x' \in S'_1$, $y' \in T'$ such that $(x', y') \in F'$. Arguing like above, we can find an $x \in S_1$ with $\varphi_{N_1}(x) = x'$ and some $y \in x^\bullet$ with $\varphi(y) = y'$. We have $y \in T_1$ because ${}^\bullet S_1 \cup S_1^\bullet \subseteq T_1$. Hence $y' \in T'_1$, which was to prove.
3. Let $x \in S_1$. We show $\varphi_{N_1}({}^\odot x) = \{\varphi_{N_1}(x)\}$ or $\varphi_{N_1}({}^\odot x) = {}^\odot(\varphi_{N_1}(x))$.
   If $\varphi({}^\odot x) = \{\varphi(x)\}$ then $\varphi_{N_1}(x) = \{\varphi(x)\} = \{\varphi_{N_1}(x)\}$ and we are done.
   Otherwise, since ${}^\odot x \subseteq X_1$ we have $\varphi_{N_1}({}^\odot x) = \varphi({}^\odot x)$. Since φ is S-vicinity respecting, $\varphi({}^\odot x) = {}^\odot(\varphi(x))$. Since N'_1 is a transition-bordered subnet by 2., ${}^\odot\varphi(x) \subseteq X'_1$. Therefore ${}^\odot(\varphi(x)) = {}^\odot(\varphi_{N_1}(x))$. □

The example in Figure 6(b) shows that being strongly connected is a necessary prerequisite for Theorem 2. The image of the left connected subnet by the S-vicinity respecting quotient is not a subnet because of the arc (a, b). In Figure 6(a) we gave an example of a strongly connected subnet which is not transition-bordered. Its image by the S-vicinity respecting quotient is also not a subnet of the target net.

An S-component is in particular a strongly connected transition-bordered subnet. For respecting coverings by S-components, stronger hypotheses have to be assumed. Let us continue considering the S-vicinity respecting quotient shown in Figure 7(a). This net is covered by S-components. The net homomorphism φ is an S-vicinity respecting quotient. However, the target net is not covered by S-components. Observe that the restriction of φ to any S-component is not T-vicinity respecting. Consider the S-component N_1 containing b. The image of N_1 is the entire target net. We have $\varphi_{N_1}(\{a, b\}) \neq \{\varphi_{N_1}(a)\} = \{u\}$ but

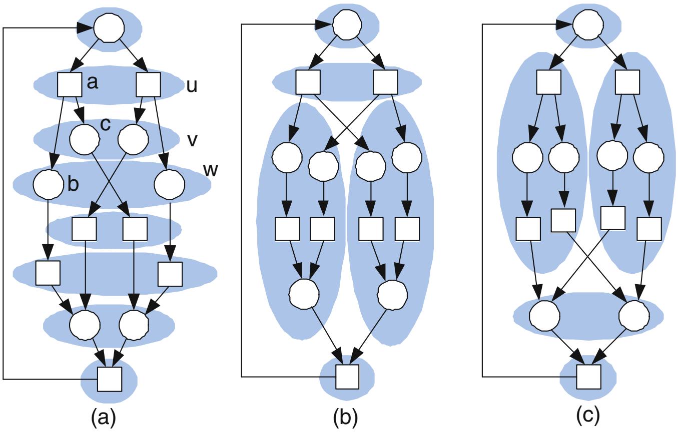

Fig. 7.

$\varphi_{N_1}(\{a,b\}) = \{u,w\} \neq (\varphi_{N_1}(a))^{\odot} = \{u,v,w\}$. The net image of N_1 is not an S-component of the target net.

In Figure 7(b) and 7(c), the quotients restricted to any S-component are T-vicinity respecting.

Proposition 2. *Let $\varphi: N \to N'$ be an S-vicinity respecting net homomorphism and let N_1 be an S-component of N. Define $N'_1 = \varphi(N_1)$ and suppose $\varphi_{N_1}: N_1 \to N'_1$ is T-vicinity respecting. If $|X'_1| > 1$ then N'_1 is an S-component of N'.*

Proof: Assume $|X'_1| > 1$. Since N_1 is an S-component, it is a transition-bordered subnet. Hence, by Theorem 2, N'_1 is a transition-bordered subnet of N'.

It remains to prove: every $y' \in T'_1$ satisfies $|{}^\bullet y' \cap S'_1| \leq 1$ and $|y'^\bullet \cap S'_1| \leq 1$. Let $y' \in T'_1$. We show only: $|{}^\bullet y' \cap S'_1| \leq 1$ ($|y'^\bullet \cap S'_1| \leq 1$ is similar). Since φ_{N_1} is surjective we can find an arc $(x,y) \in F_1$ with $\varphi(x) \neq y'$ and $\varphi(y) = y'$ (Lemma 3). Since φ_{N_1} is a T-vicinity respecting quotient, Corollary 3 implies $|S'_1 \cap {}^\bullet y'| \leq |S_1 \cap {}^\bullet y|$ and $|S_1 \cap {}^\bullet y| \leq 1$ because N_1 is an S-component. □

Theorem 3. *Let N be a net, covered by a family (N_i), $i \in I$ of S-components. Let $\varphi: N \to N'$ be an S-vicinity respecting quotient such that, for all $i \in I$, $\varphi_{N_i}: N_i \to \varphi(N_i)$ is T-vicinity respecting. Then N' is covered by S-components.*

Proof: In Proposition 2 we have shown that, given the assumptions above, every S-component of N is either mapped to an S-component of N' or to an element of N'.

Let $x' \in X'$. If x' is an isolated element then it is a trivial S-component. So assume that x' is not isolated. Then we can find a $y' \in X'$ such that $(x', y') \in F'$ or $(y', x') \in F'$.

Since φ is a quotient, there are $x \in S$, $y \in T$ with $\varphi(\{x,y\}) = \{x', y'\}$ and either $(x,y) \in F$ or $(y,x) \in F$. N is covered by S-components and hence we can

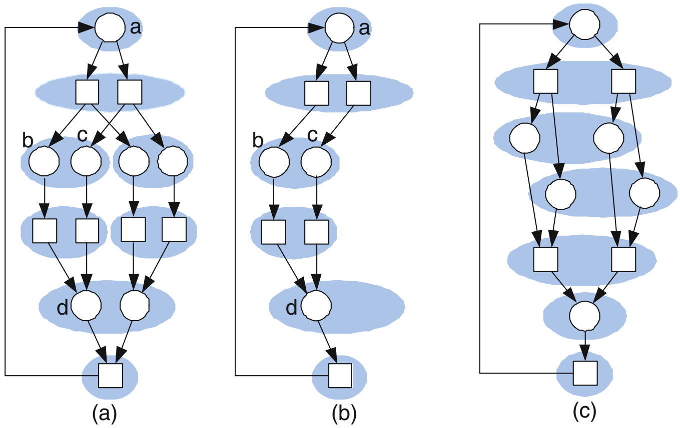

Fig. 8.

find an $i \in I$ such that $x \in S_i$ and $y \in T_i$. $|X_i'| > 1$ since x' and y' are distinct elements of $\varphi(N_i)$. Thus $\varphi(N_i)$ is an S-component of N'. □

By Theorem 2(3), 'φ is S-vicinity respecting' implies for all $i \in I$: 'φ_{N_i} is S-vicinity respecting'. So all the φ_{N_i} have to be both S- and T-vicinity respecting. However, this alone does not imply that φ is S-vicinity respecting and is not sufficient for N' to be covered by S-components as is shown in Figure 8(a). For the S-component N_1 of this net, shown in Figure 8(b), φ_{N_1} is S-and T-vicinity respecting. However, φ is not S-vicinity respecting and N' is not covered by S-components.

Theorem 3 implies that, given a family of S-components which cover the source net, a respective covering of the target net is obtained by the images of the S-components which are not mapped to single non-isolated places.

The choice of a covering family of S-components is decisive. In the example of Figure 8(c), the quotient is vicinity respecting. Its restriction to either the S-component N_1 which contains the respective left places or to the S-component N_2 which contains the respective right places is T-vicinity respecting. Taking the other two possible S-components as a cover of N, the restriction of φ to any of these S-components is not T-vicinity respecting. So the choice of an abstraction and the choice of an S-component covering are not independent.

By duality we get:

Corollary 6. *Let $\varphi: N \to N'$ be a T-vicinity respecting net homomorphism and N_1 a strongly connected place-bordered subnet of N. Define $N_1' = \varphi(N_1)$. Then:*

1. *N_1' is a subnet of N';*
2. *If $|X_1'| > 1$ then N_1' is a place-bordered subnet of N';*
3. *$\varphi_{N_1}: N_1 \to N_1'$ is T-vicinity respecting.*

The dual version of Theorem 3 reads as follows:

Theorem 4. *Let N be a net, covered by a family $(N_i), i \in I$ of T-components. Let $\varphi: N \to N'$ be a T-vicinity respecting quotient such that, for all $i \in I$, $\varphi_{N_i}: N_i \to \varphi(N_i)$ is S-vicinity respecting. Then N' is covered by T-components.*

The net homomorphisms depicted in Figure 7(b) and 7(c) are vicinity respecting. Their restrictions to any S-component or T-component are also vicinity respecting. Hence their net images are covered by S- and T-components.

A particular case of Theorem 3 is the composition of S-components; the source net N is the disjoint union of a family of S-components and the mapping, restricted to each of these S-components, is injective (and hence a fortiori T-vicinity respecting). We can reformulate our result as a property of net homomorphisms as follows: For every place a of an S-component N_1 of a net N the entire vicinity belongs to the S-component as well by definition. Therefore the natural injection $\psi_1: N_1 \to N$ is S-vicinity respecting but not necessarily surjective. A covering by S-components $N_i (i \in I)$ can be expressed by a set of net homomorphisms $\psi_i (i \in I)$ such that each element of N is in $\psi_i(N_i)$ for at least one i. Using the disjoint union of the S-components ($\biguplus N_i$), the net homomorphisms ψ_i induce a quotient ψ from $\biguplus N_i$ to N. Now Theorem 3 reads as follows. Given

- a family (N_i), $i \in I$ of strongly connected nets with $|{}^\bullet t| \leq 1$, $|t^\bullet| \leq 1$ for all transitions t (S-components),
- S-vicinity respecting injective net homomorphisms $\psi_i: N_i \to N (i \in I)$ such that the induced mapping ψ is a quotient (i.e., N is covered by the N_i),
- an S-vicinity respecting quotient $\varphi: N \to N'$ such that φ_{N_i} is T-vicinity respecting for all $i \in I$,

we can find injective S-vicinity respecting mappings $\psi'_i: \varphi(N_i) \to N'$ such that the induced mapping $\psi': \bigcup \varphi(N_i) \to N'$ is surjective (N' covered by the $\varphi(N_i)$).

Again, by duality we can use the same formalism to capture the composition of T-components.

5 Siphons, Traps and Free Choice Property

Definition 14. *Let N be a net.*
A siphon of a net is a nonempty set of places A satisfying ${}^\bullet A \subseteq A^\bullet$.
A trap of a net is a nonempty set of places A satisfying $A^\bullet \subseteq {}^\bullet A$.
A siphon (trap) is minimal if it does not strictly include any other siphon (trap).

For marked Petri nets, siphons and traps are used to deduce behavioral properties of the system [3]. Also at the conceptual level of Channel/Agency nets, they can be used to analyze aspects of the data and information flow in the modelled system. Roughly speaking, if a set of places is a trap then information cannot get completely lost in the component modelled by these places. For the places of a siphon, it is not possible to add information without taking data from the siphon into account.

Minimal siphons and traps are particularly important for the analysis of marked Petri nets. We will show that vicinity respecting net homomorphisms map minimal siphons either onto singletons or onto siphons of the target net, and similarly for minimal traps. We begin with a preliminary result.

Proposition 3. *[3] Let A be a minimal siphon of a net N. Then the subnet generated by $^\bullet A \cup A$ is strongly connected.*

Proof: Let $N_A = (A, {}^\bullet A; F_A)$ be the subnet generated by A. First we observe that every transition t of N_A is an input transition of some place of A by the definition of N_A and also an output transition of some place of A because A is a siphon. Hence, for proving strong connectivity it suffices to show that for every two places $x, y \in A$ there is a path $x \dots y$ in N_A.

Let $y \in A$ and define the set $X = \{z \in A \mid \text{there is a path } z \dots y \text{ in } N_A\}$. We prove that $X = A$. This implies $x \in X$ and, by the definition of X, proves the result we are after.

Let $t \in {}^\bullet X$. Since $X \subseteq A$ and since A is a siphon we have $t \in A^\bullet$. By the definition of X there is a path $t \dots y$ in N_A. So every place in $^\bullet t \cap A$ belongs to X. Therefore $t \in X^\bullet$. So we have $^\bullet X \subseteq X^\bullet$. X is not the empty set because $y \in X$. So X is a siphon included in A. Since A was assumed to be a minimal siphon we conclude that $X = A$. □

Theorem 5. *Let $\varphi\colon N \to N'$ be a surjective S-vicinity respecting net homomorphism. If A is a minimal siphon of N then either $\varphi(A)$ is a single node (place or transition) or $\varphi(A) \cap S'$ is a siphon of N'.*

Proof: Since A is a minimal siphon, $\bullet A \cup A$ generates a strongly connected subnet N_1 by Proposition 3. Hence the subnet N_1' of N' generated by the set of elements $\varphi(^\bullet A) \cup \varphi(A)$ is also strongly connected.

Assume that $\varphi(A)$ contains more than one node. Let x' be a place of N_1' and let $z' \in {}^\bullet x'$. We have to prove that z' has an input place of N_1'.

Since N_1' is strongly connected and since it contains more than one element, it contains a transition $y' \in {}^\bullet x'$. So there exist a place $x \in A$ and a transition $y \in {}^\bullet x$ such that $\varphi(x) = x'$ and $\varphi(y) = y'$. In particular $\varphi(^\odot x) \neq \{\varphi(x)\}$. Since φ is S-vicinity respecting, we obtain $\varphi(^\odot x) = {}^\odot\varphi(x)$. This implies that some $z \in {}^\bullet A$ is mapped to z'. Since A is a siphon, z has an input place of A. So z' has an input place of N_1' which completes the proof. □

By symmetrical arguments, an analogous result holds for traps:

Theorem 6. *Let $\varphi\colon N \to N'$ be a surjective S-vicinity respecting net homomorphism. If A is a minimal trap of N then either $\varphi(A)$ is a single node (place or transition) or $\varphi(A) \cap S'$ is a trap of N'.*

In Figure 9(a), $\{a, b\}$ is a minimal trap that is mapped to a single place which does not constitute a trap. The trap $\{a, b\}$ in Figure 9(b) is not minimal, and the single place of its image does not constitute a trap, too.

We close this section establishing that vicinity respecting quotients respect free choice Petri nets. Important behavioral properties are characterized in terms of traps, siphons for these nets and the class of free choice Petri nets which is

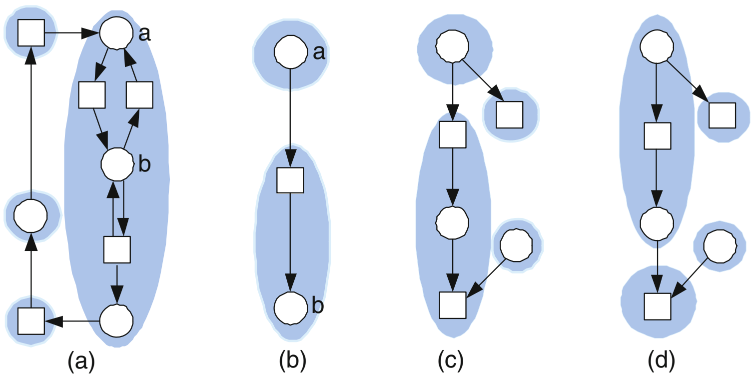

Fig. 9.

covered by S- and T-components is well established [3]. In a free choice net, if two transitions share some input places, then they share all their input places.

Definition 15. *A net N is called free choice if for any two places s_1 and s_2 either $s_1^\bullet \cap s_2^\bullet = \emptyset$ or $s_1^\bullet = s_2^\bullet$.*

Theorem 7. *Let N be a free choice net and $\varphi\colon N \to N'$ be a vicinity respecting quotient. Then N' is free choice as well.*

Proof: We show that for any two places s_1', s_2' of S' we have: ${s_1'}^\bullet \cap {s_2'}^\bullet = \emptyset$ or ${s_1'}^\bullet = {s_2'}^\bullet$.

We proceed indirectly. Let s_1' and s_2' be places of S' such that ${s_1'}^\bullet \cap {s_2'}^\bullet \neq \emptyset$ and ${s_1'}^\bullet \neq {s_2'}^\bullet$. Without loss of generality assume that there is a transition $t_2' \in {s_2'}^\bullet$ with $t_2' \notin {s_1'}^\bullet$. Let $t_1' \in {s_1'}^\bullet \cap {s_2'}^\bullet$. Since φ is a quotient we find $(s_2, t_1) \in F$ with $\varphi(s_2) = s_2'$ and $\varphi(t_1) = t_1'$. Since φ is S-vicinity respecting we have $\varphi(s_2^\odot) = (\varphi(s_2))^\odot$. Hence there is a transition $t_2 \in s_2^\bullet$ satisfying $\varphi(t_2) = t_2'$. Since φ is T-vicinity respecting, $\varphi({}^\odot t_1) = {}^\odot\varphi(t_1)$. Hence there is a place $s_1 \in {}^\bullet t_1$ with $\varphi(s_1) = s_1'$. Since φ is vicinity respecting and since $t_2' \notin {s_1'}^\bullet$ we get $t_2 \notin \varphi(s_1^\bullet)$ - a contradiction to the free-choice property of N. □

Figures 9(c) and 9(d) show that for the previous theorem, S-vicinity respecting and T-vicinity respecting alone are not sufficient.

6 Conclusion

Structuring system requirements is a gradual process which involves refinement/abstraction between different conceptual levels. Abstractions should bear formal relations with refinements because otherwise the analysis of some abstraction will be of no help for the induced refinement. We argued that vicinity respecting homomorphisms give a possible solution to these requirements for graph-based models of distributed systems. They provide a method to perform graphical abstraction/refinement such that every element is either glued together with its vicinity or its vicinity is the vicinity of its image.

The vicinity respecting concept is a local notion because its definition only uses local vicinities. However, it has global consequences since it preserves paths and, consequently, connectedness properties. For Petri nets, vicinity respecting homomorphisms preserve moreover important structural properties such as S- and T-components, siphons and traps and the free-choice property.

Other concepts for refinement and abstraction of Petri nets and of morphisms [26,17] have been proposed in the literature. However, all these approaches are concerned with marked Petri nets and aim results involving the behavior given by the token game. In contrast, we are concerned with the preliminary task of structuring software requirements down to a working system and aim at structure preservation. Generally, abstraction in our sense is more general than behavior preserving abstraction. However, structure influences behavior. The transition refinement considered in [12] turns out to induce a vicinity respecting homomorphism from the refined net to the coarser net.

As mentioned in the introduction and further in the paper, Petri net quotients can be graphically represented by the less abstract net together with equivalence classes of those elements that are identified by the abstracting morphism. Vicinity respecting quotients play a particular role because they preserve important structural properties. Concepts like aggregation and generalization can be adapted to process modelling and represented as particular vicinity respecting quotients, thus leading to a single diagram including such abstraction elements. Future work will concentrate on the identification of suitable abstraction notions for process models and their representation within single nets.

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