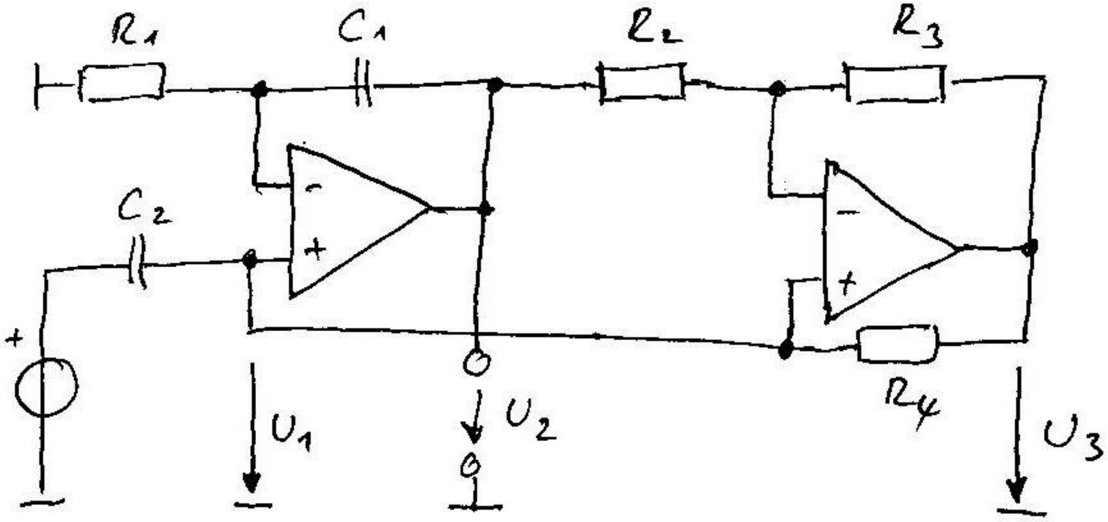
Univ. Prof. Dr.–Ing. H. Wupper
Baumhofstr. 30
44799 Bochum

KLAUSUR HERBST 2005
Aufgabe 1

1.1



$$sC_2(U_1 - E) + G_4(U_1 - U_3) = 0$$

$$G_1 U_1 + sC_1(U_1 - U_2) = 0$$

$$G_2(U_1 - U_2) + G_3(U_1 - U_3) = 0$$

Univ. Prof. Dr.–Ing. H. Wupper
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$$\begin{pmatrix} G_4 + sC_2 & 0 & -G_4 \\ sC_1 + G_1 & -sC_1 & 0 \\ G_2 + G_3 & -G_2 & -G_3 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} sC_2 \\ 0 \\ 0 \end{pmatrix} E$$

$$U_2 = \frac{\begin{vmatrix} G_4 + sC_2 & sC_2 & -G_4 \\ sC_1 + G_1 & 0 & 0 \\ G_2 + G_3 & 0 & -G_3 \end{vmatrix}}{\begin{vmatrix} G_4 + sC_2 & 0 & -G_4 \\ G_1 + sC_1 & -sC_1 & 0 \\ G_2 + G_3 & -G_2 & -G_3 \end{vmatrix}} \cdot E = \frac{sC_2 G_3 (G_1 + sC_1)\, E}{\begin{vmatrix} sC_2 & 0 & -G_4 \\ G_1 \cancel{+ sC_1} & -sC_1 & 0 \\ 0 & -G_2 & -G_3 \end{vmatrix}}$$

$$H(s) = \frac{U_2}{E} = \frac{sC_2 G_3 (G_1 + sC_1)}{s^2 C_1 C_2 G_3 + G_1 G_2 G_4}$$

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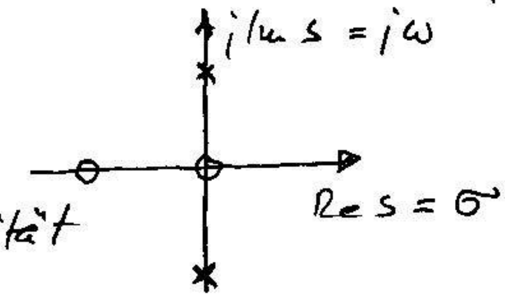
1.2 Nullstellen: $sC_2G_3(G_1+sC_1)=0 \Rightarrow s_{01}=0 \qquad s_{02}=-\frac{1}{C_1R_1}$

Pole: $s_\infty^2 = -\frac{R_3}{C_1C_2R_1R_2R_4}$

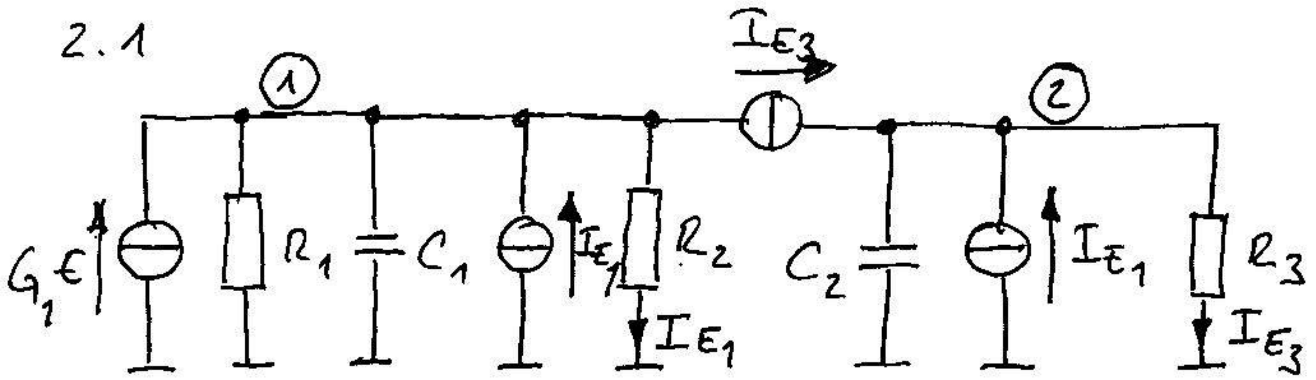


1.3 Pole auf der $j\omega$-Achse: Bedingte Stabilität

1.4 Sinusförmige Spannung im stat. Zustand.

Aufgabe 2

2.1



$I_{E1} = G_2 U_1$

$I_{E3} = G_3 U_2$

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2.2

$$\begin{pmatrix} G_1+G_2+sC_1 & 0 \\ 0 & G_3+sC_2 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} G_1E + I_{E1} - I_{E3} \\ I_{E1} + I_{E3} \end{pmatrix}$$

$$\begin{pmatrix} G_1+G_2+sC_1 & 0 \\ 0 & G_3+sC_2 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} G_1E + G_2U_1 - G_3U_2 \\ G_2U_1 + G_3U_2 \end{pmatrix}$$

$$\begin{pmatrix} G_1+sC_1 & G_3 \\ -G_2 & sC_2 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} G_1E \\ 0 \end{pmatrix}$$

$$\frac{U_1}{E} = \frac{sC_2G_1}{s^2C_1C_2 + sC_2G_1 + G_2G_3}$$

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2.3

$$\frac{U_1}{E} = \frac{\frac{1}{C_1R_1}\cdot s}{s^2 + \frac{1}{C_1R_1}\cdot s + \frac{1}{C_1C_2R_2R_3}}$$

$$a_2 = a_0 = 0$$

$$a_1 = \frac{1}{C_1R_1}$$

$$b_1 = a_1 \qquad b_0 = \frac{1}{C_1C_2R_2R_3}$$

2.4

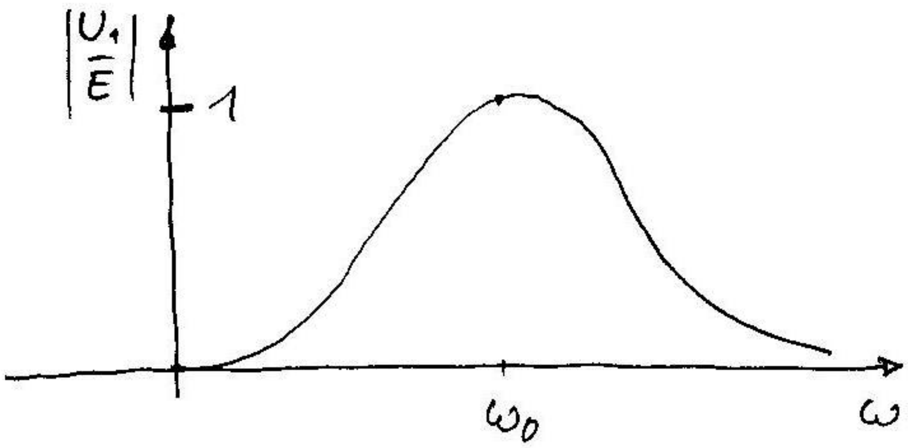
$$\left.\frac{U_1}{E}\right|_{s=j\omega} = \frac{j\omega a_1}{-\omega^2 + j\omega a_1 + b_0}$$

$$\left|\frac{U_1}{E}\right|^2_{s=j\omega} = \frac{a_1^2\omega^2}{(b_0-\omega^2)^2 + a_1^2\omega^2}$$

Max.: $b_0 - \omega^2 = 0$

$$\omega_0 = \sqrt{b_0}$$

2.5



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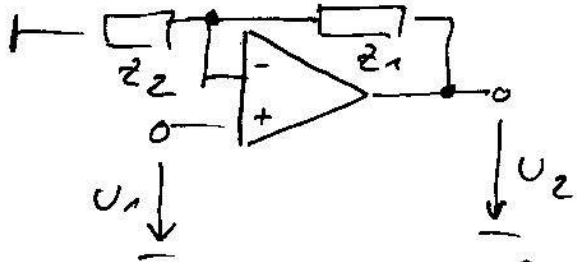
Aufgabe 3

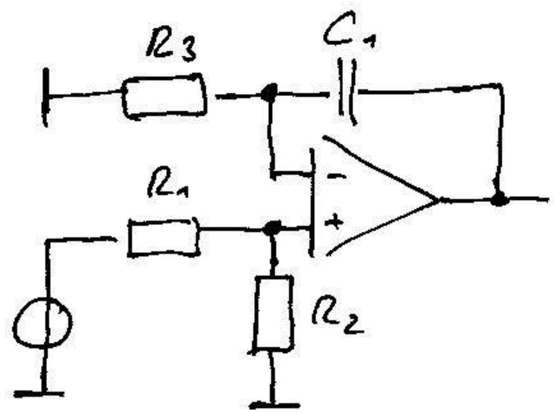
3.1 $$S_{u,0}(f) = \sum_{i=1}^{3} S_{u,R_i}(f)\,|H_i(j\omega)|^2$$

$S_{u,R_1} = 4kTR_1$ $\quad S_{u,R_2} = 4kTR_2$ $\quad S_{u,R_3} = 4kTR_3$



$$U_2 = \left(1 + \frac{Z_1}{Z_2}\right) U_1$$

$\underline{H_1(j\omega)}$



$$H_1 = \frac{R_2}{R_1 + R_2}\left(1 + \frac{1}{j\omega C_1 R_3}\right)$$

$$|H_1(j\omega)|^2 = \frac{R_2^2}{(R_1+R_2)^2}\left(1 + \frac{1}{\omega^2 C_1^2 R_3^2}\right)$$

$$|H_2(j\omega)|^2 = \frac{R_1^2}{(R_1+R_2)^2}\left(1 + \frac{1}{\omega^2 C_1^2 R_3^2}\right)$$

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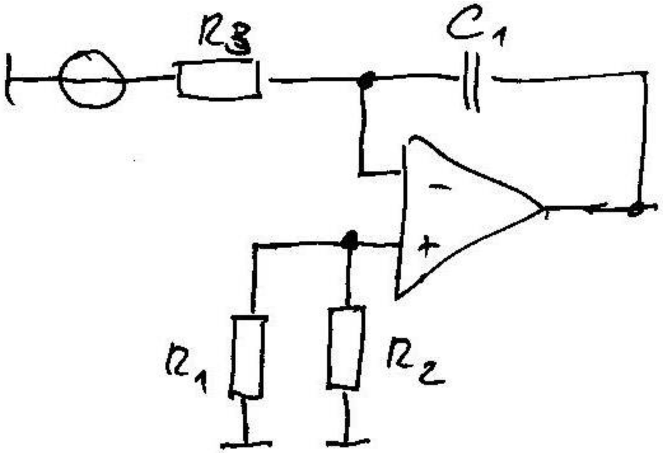

$$H_3(j\omega) = -\frac{1}{j\omega C_1 R_3}$$

$$|H_3(j\omega)|^2 = \frac{1}{\omega^2 C_1^2 R_3^2}$$

3.2

$$U_{2,eff}^2 = \int_0^{f_1} S_{u,0}(f) \cdot df$$

$$= 4kT \int_0^{f_1} \left[\left(1 + \frac{1}{(2\pi f C_1 R_1)^2}\right) \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3}{(2\pi f C_1 R_3)^2} \right] df$$

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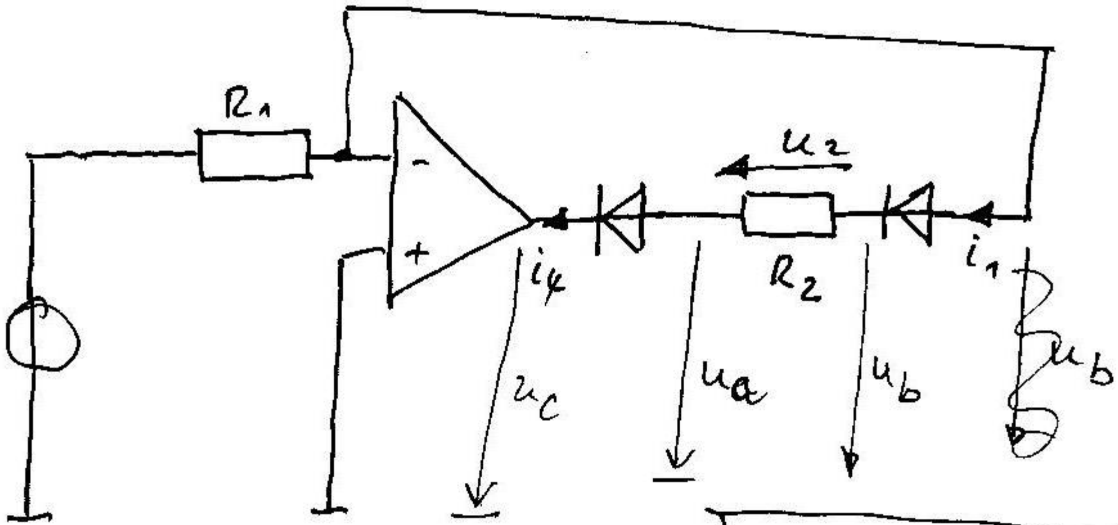
Aufgabe 4

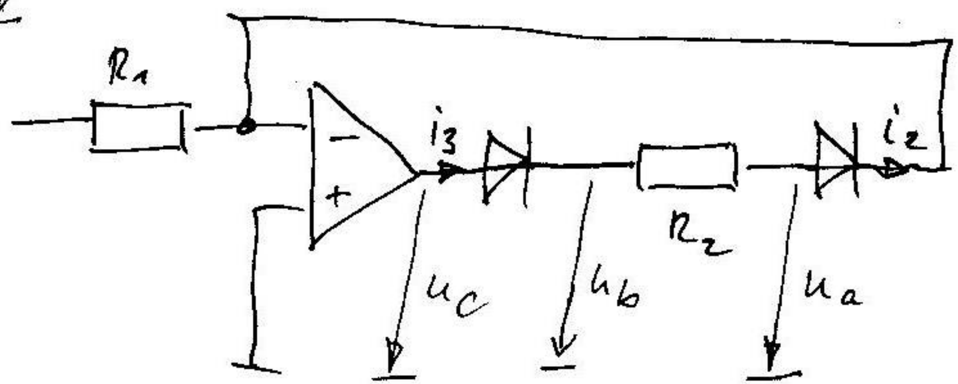
4. 1

Positive Halbwelle der Eingangsspannung

$R_2 = R_1 = 1k$



~~Positive~~ Neg. HW.



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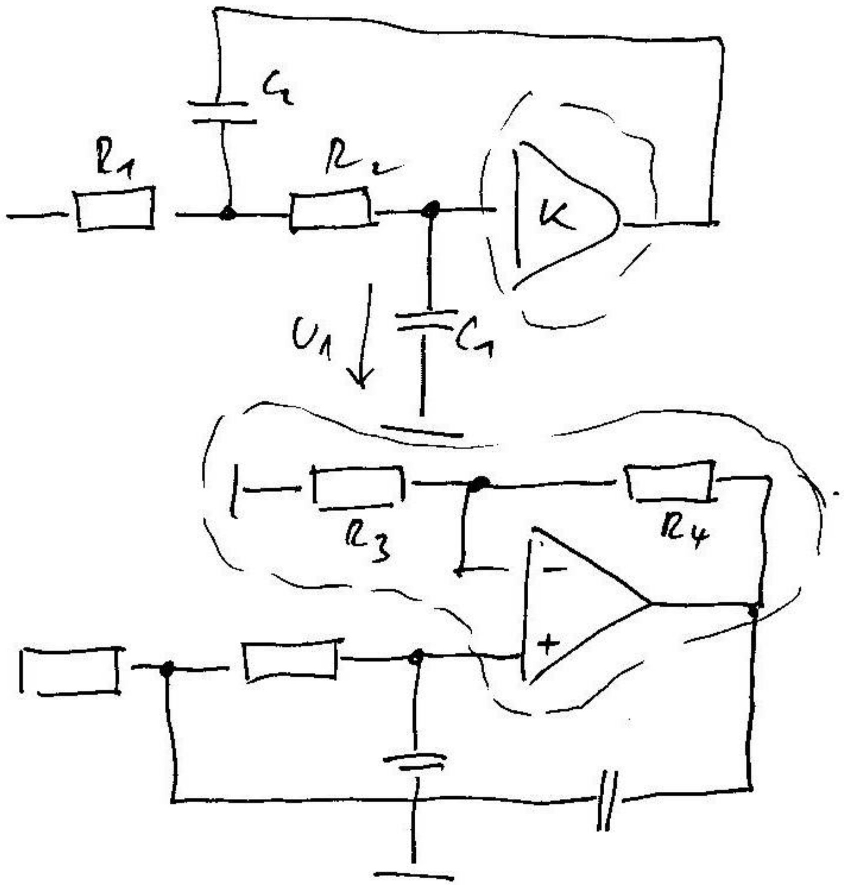
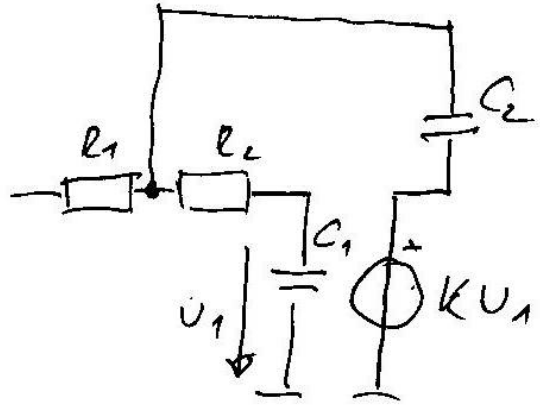
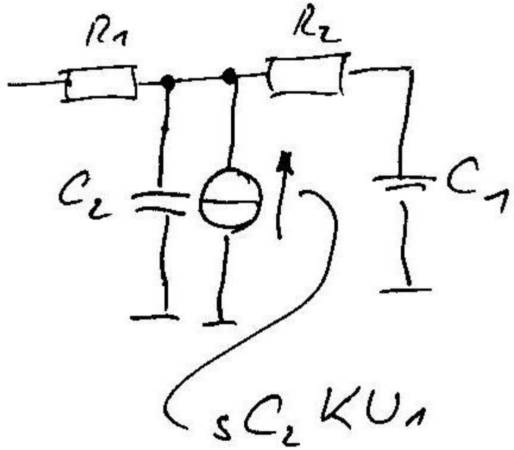
Allgemein







$$K = 1 + \frac{R_4}{R_3}$$