Toward the concept of backtracking computation*

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FernUniversität Hagen, Germany

Abstract

S_1:PP is a new mathematical definition of the Byrd's box model for pure Prolog, with:

- Forward and backward steps
- Modularity

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*paper presented at the SOS'04 Workshop, 30. August 2004, London

Motivation: How to specify (and even prove) properties of computation?

What does it mean that a run-time test is

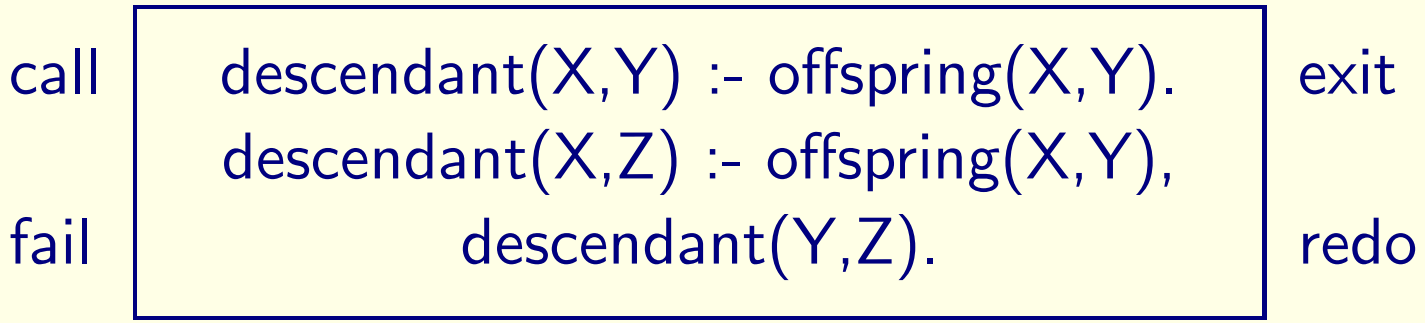
"not affecting the computation" ?

- preserving the success set (what about order etc.?)
- "The transformed program Π' computes wrt the transformed query Q' the same answers and in the same sequence as the original program wrt the original query." (how to specify this?)
- relevant parts of the computation remain the same (...)

Some problems with describing (even pure) Prolog execution

- complex data flow, alternating in forward and backward direction
- backward steps are much more global than forward steps (the most recent choice point may be any distance away)
- lacking in means of simplification
  e. g. abstraction, compositionality, modularity
- representing "state of computation" by data structures that give us too many special cases for theorem proving
  e. g. stack of stacks of frames (ancestor+environment)

Byrd's "box" metaphor of Prolog execution



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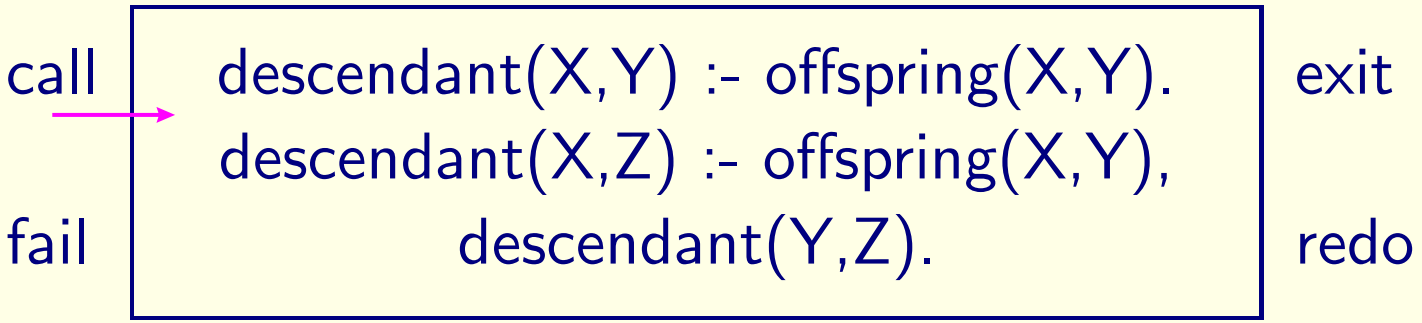

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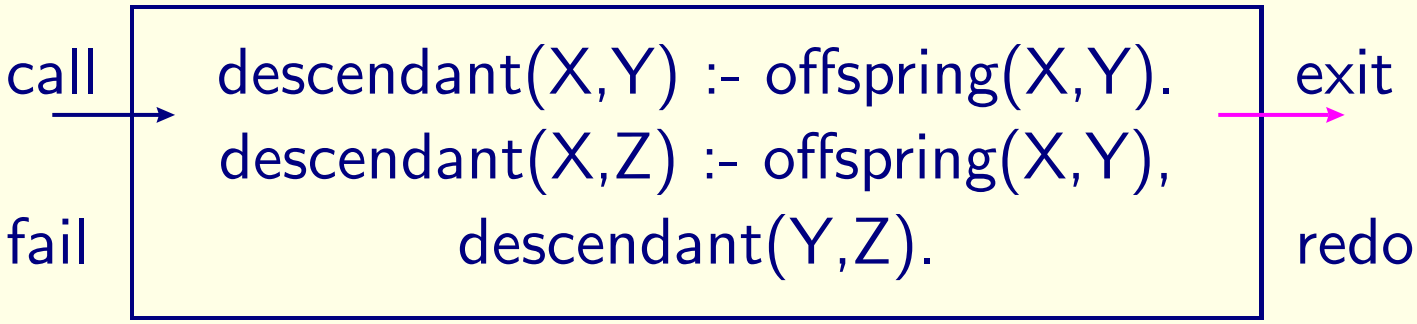

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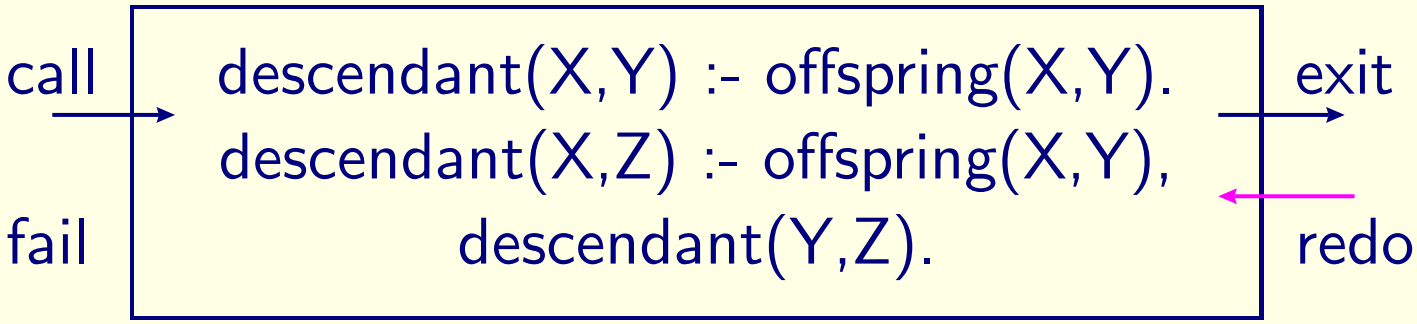

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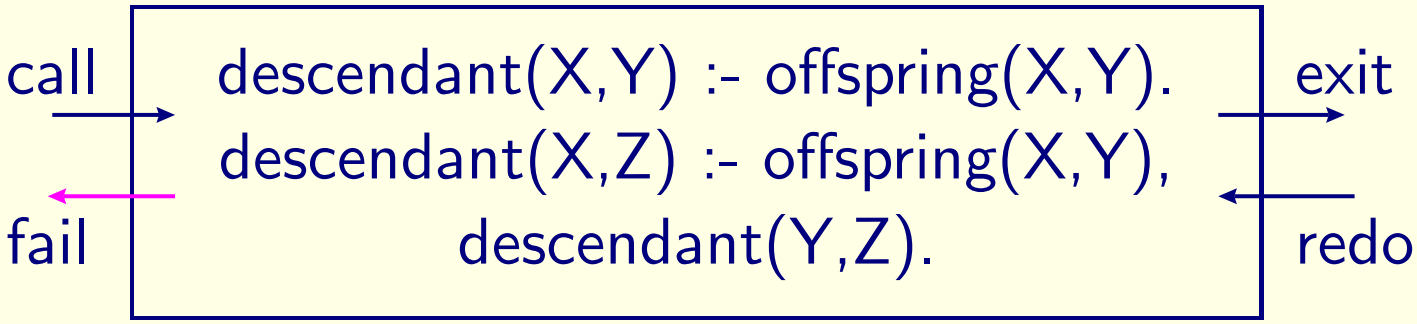
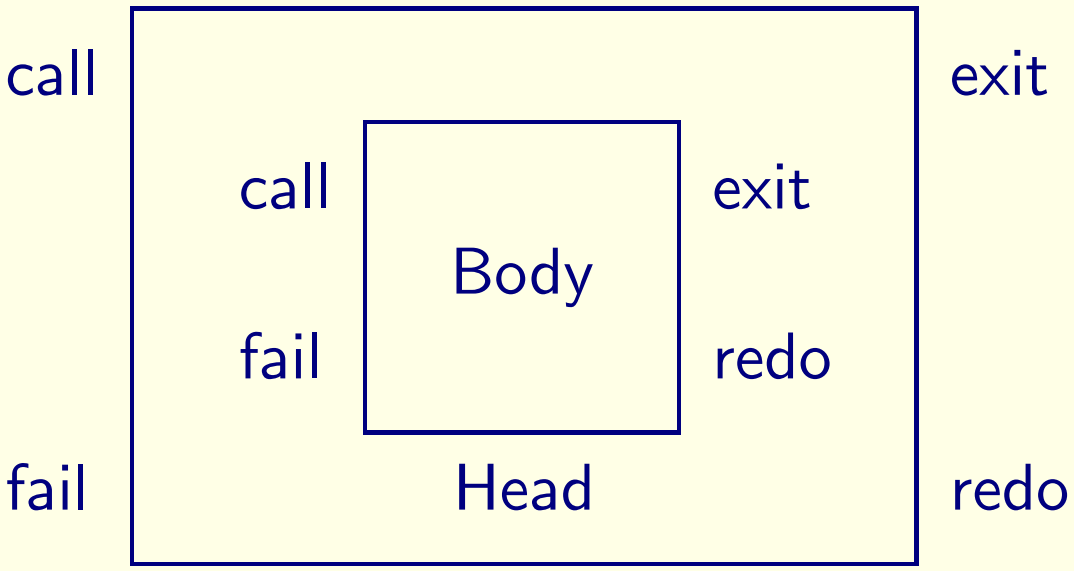


PS. What does the box stand for? Does it have to be the "selected atom"?

... or can it be a general goal?

- focus on general goals is a main idea behind the S_1:PP model
- canonical form of predicates reduces choice to disjunction



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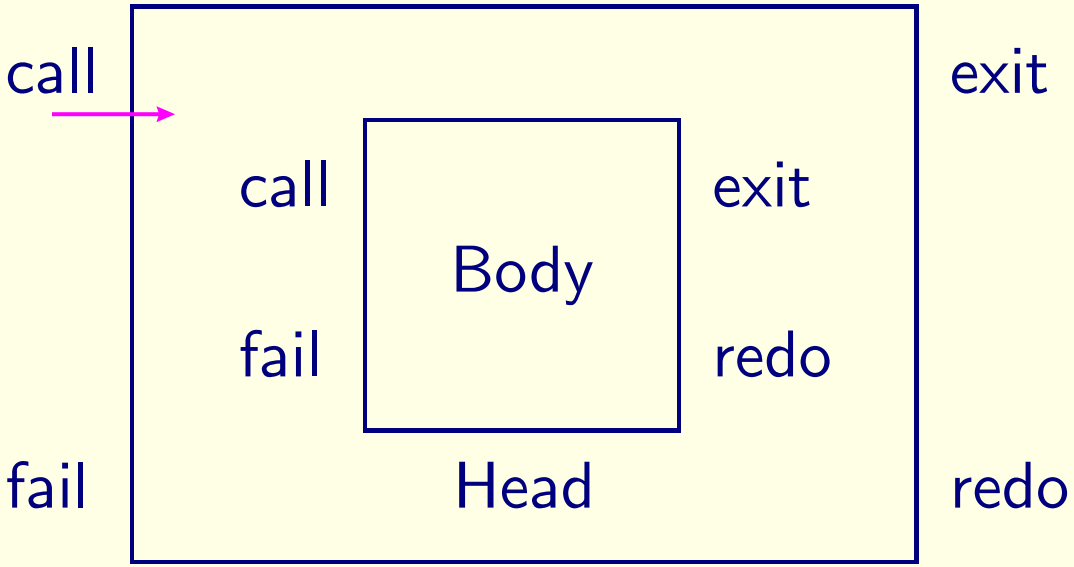

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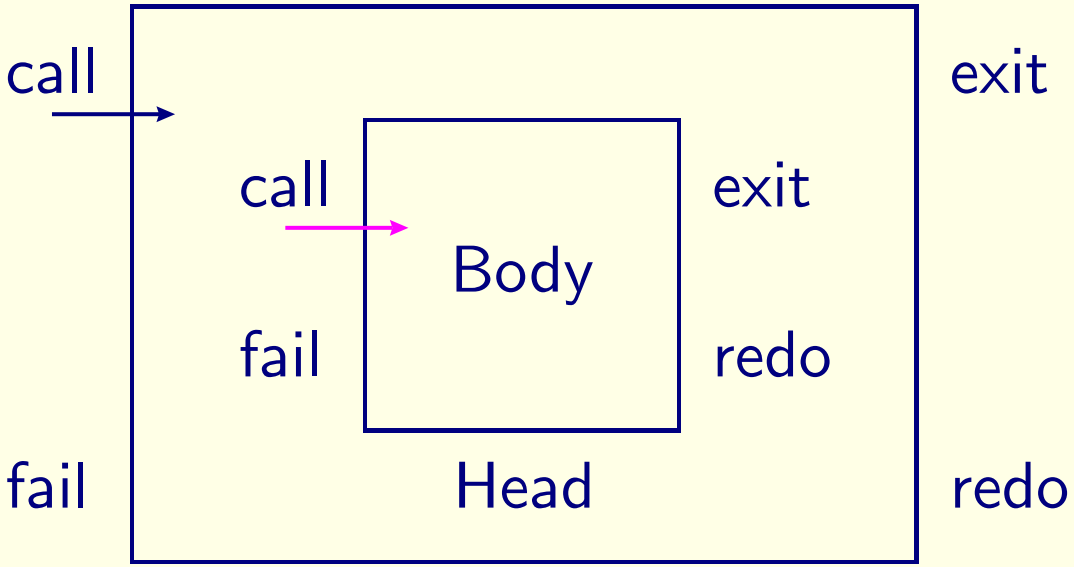

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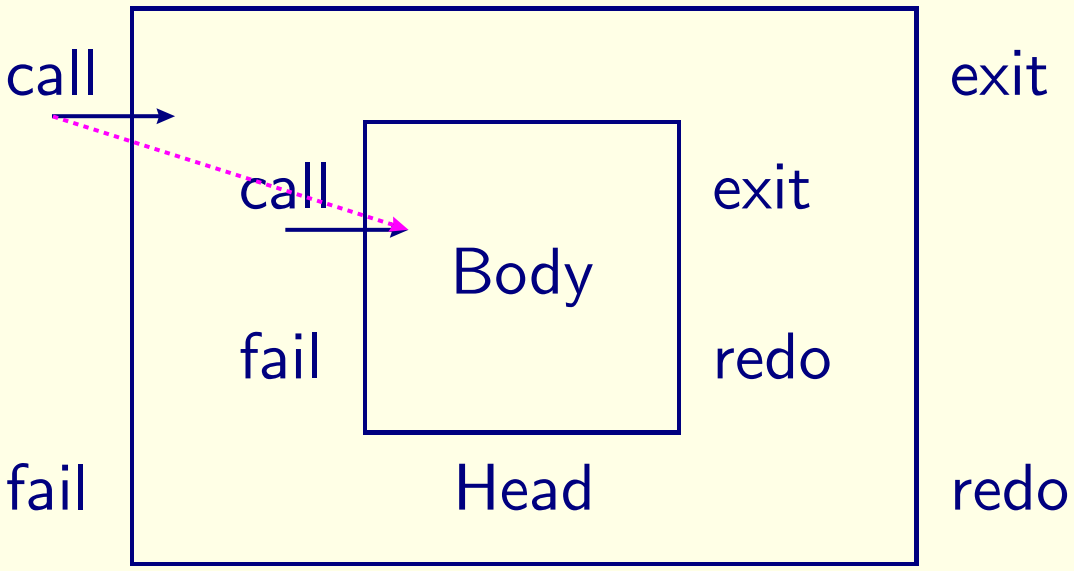

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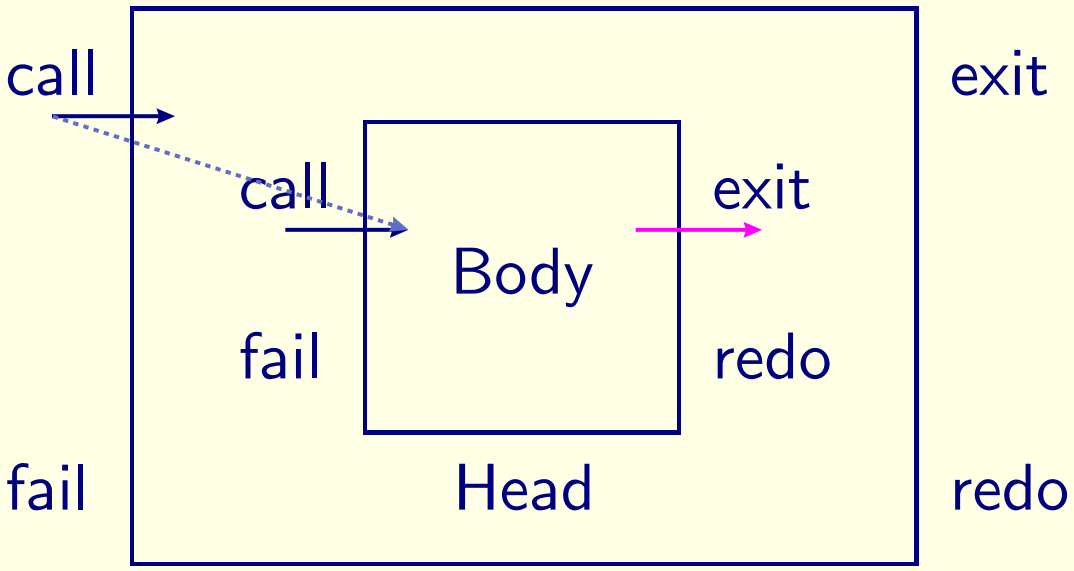

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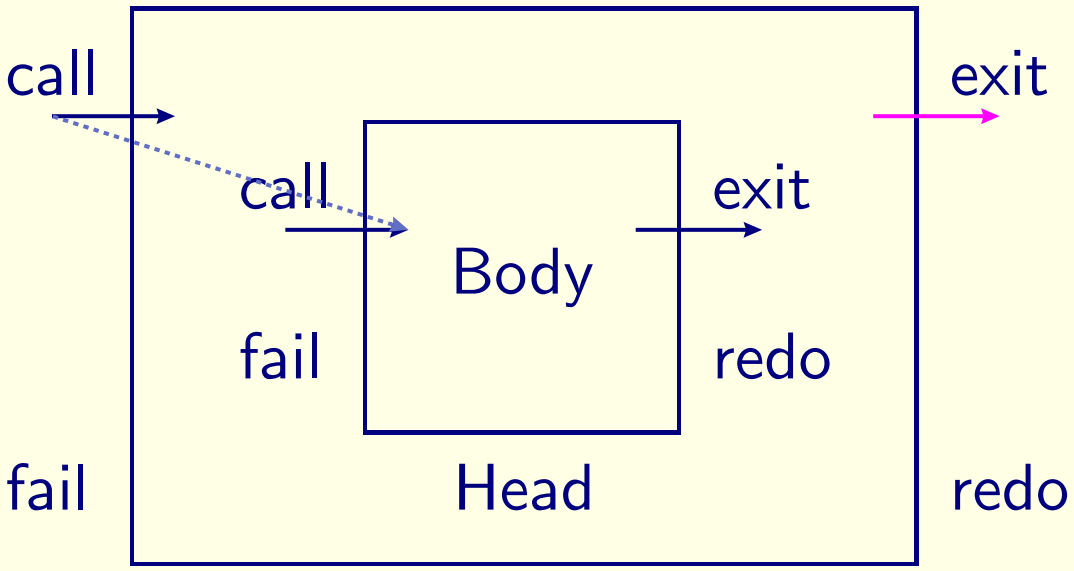

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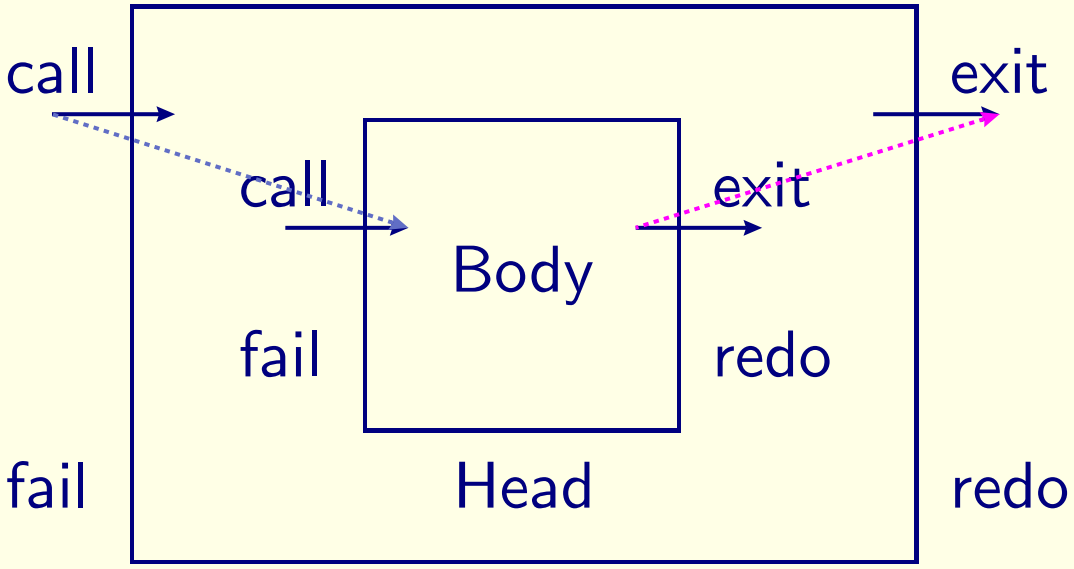

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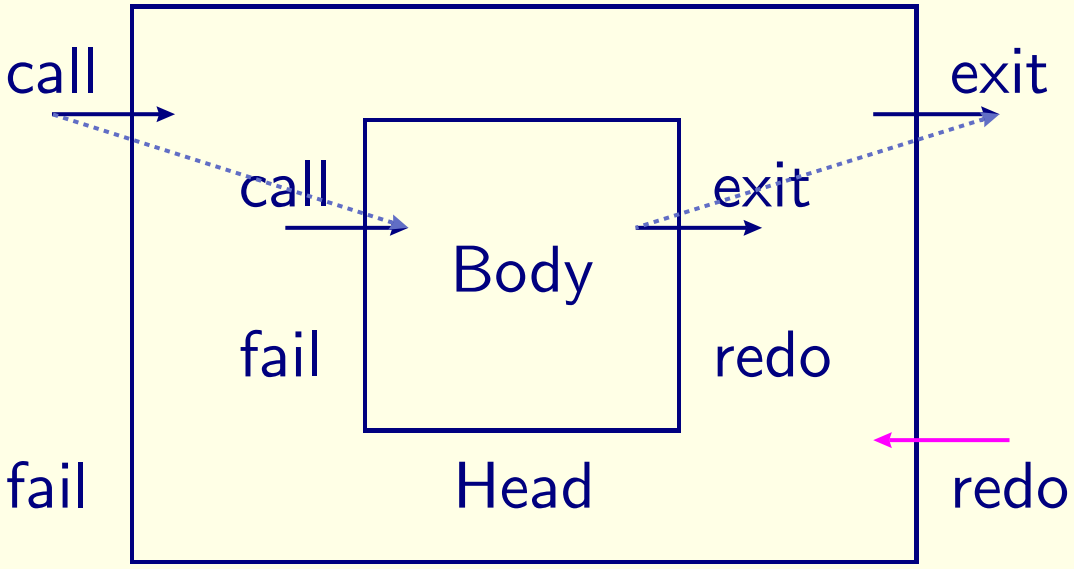

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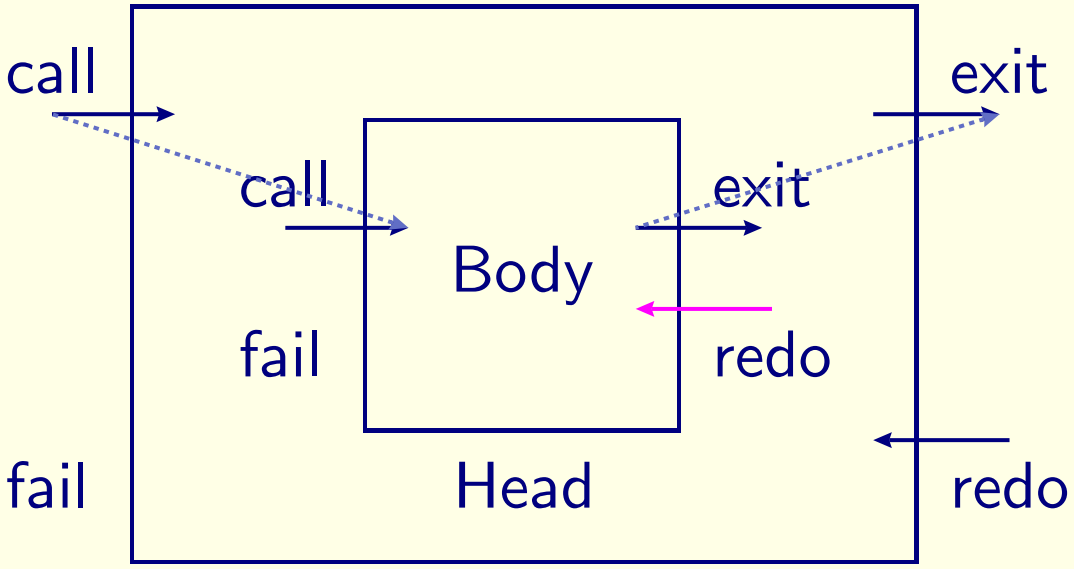

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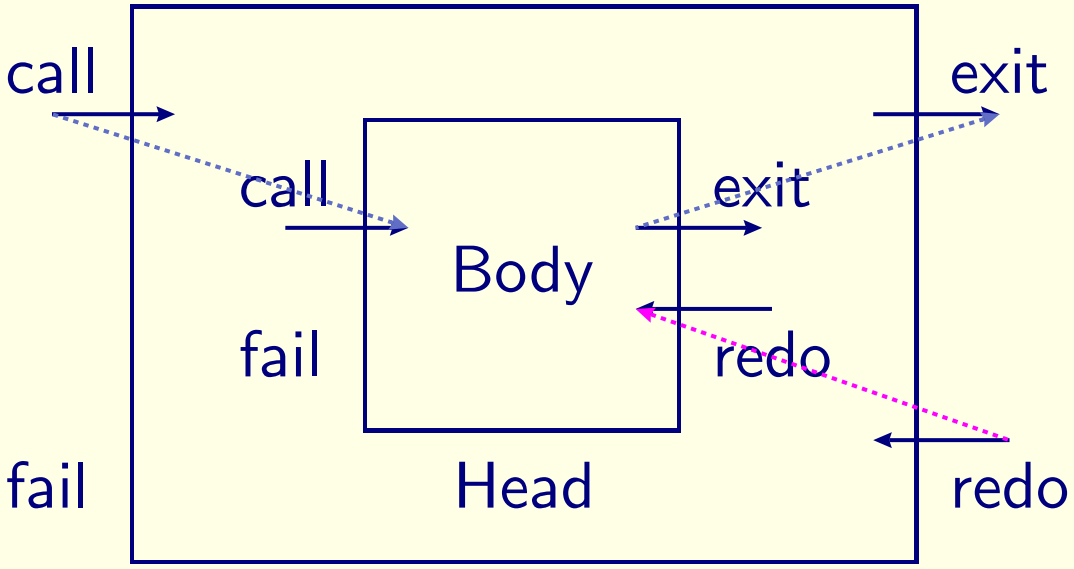

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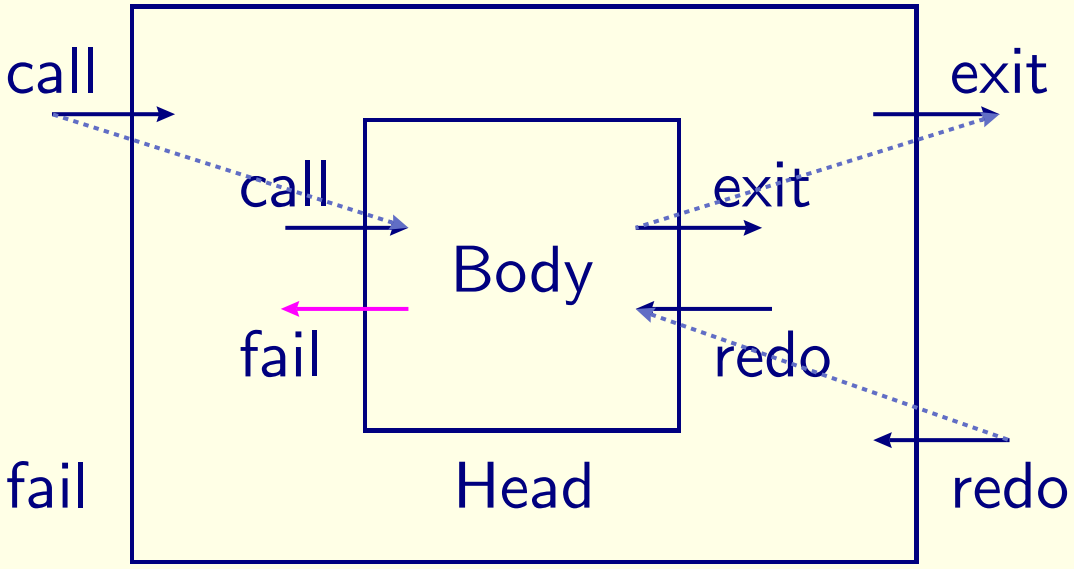

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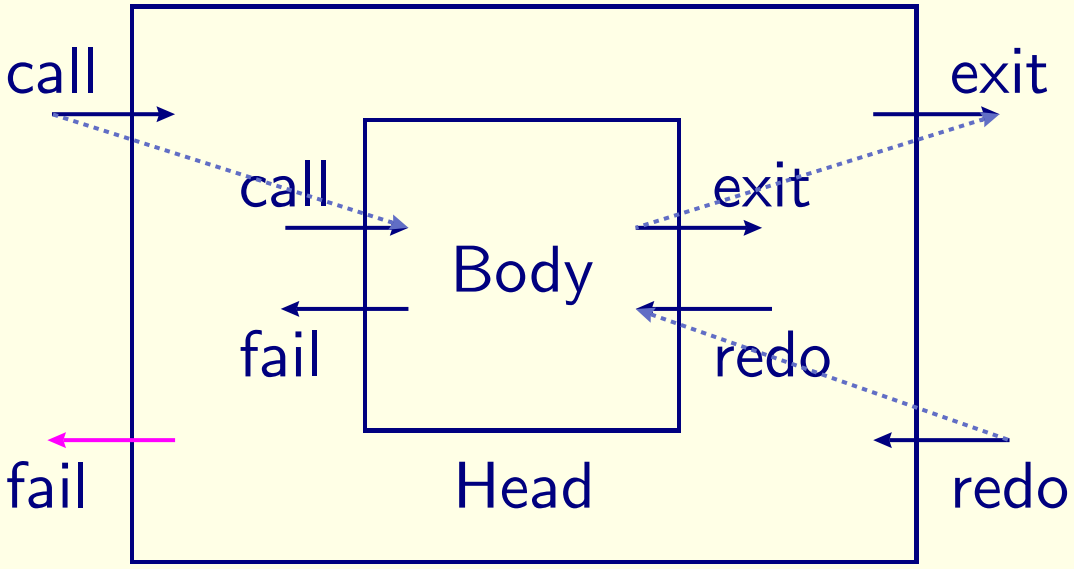

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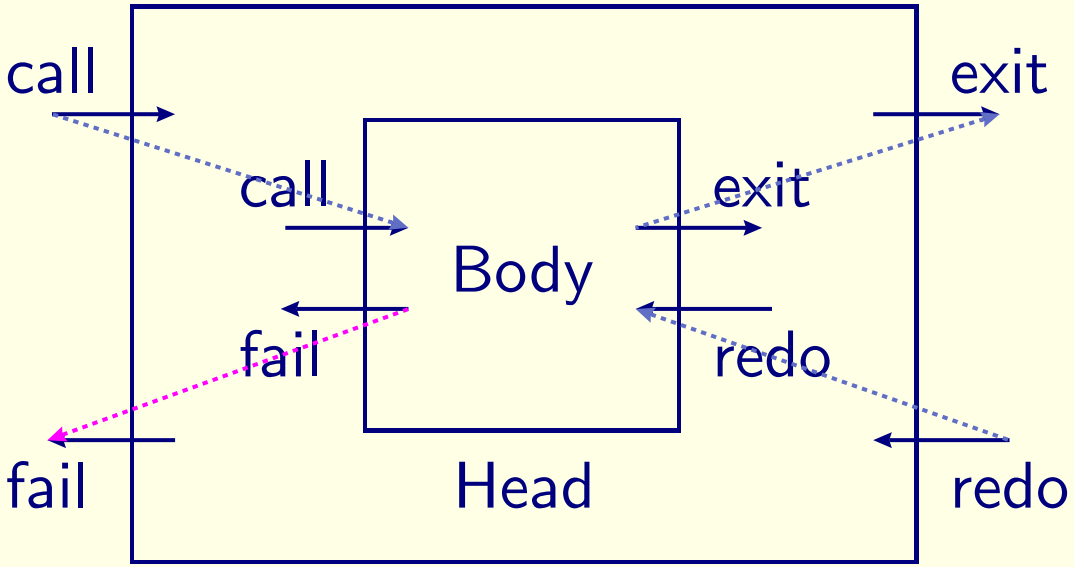

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redo Head ⇾ redo Body
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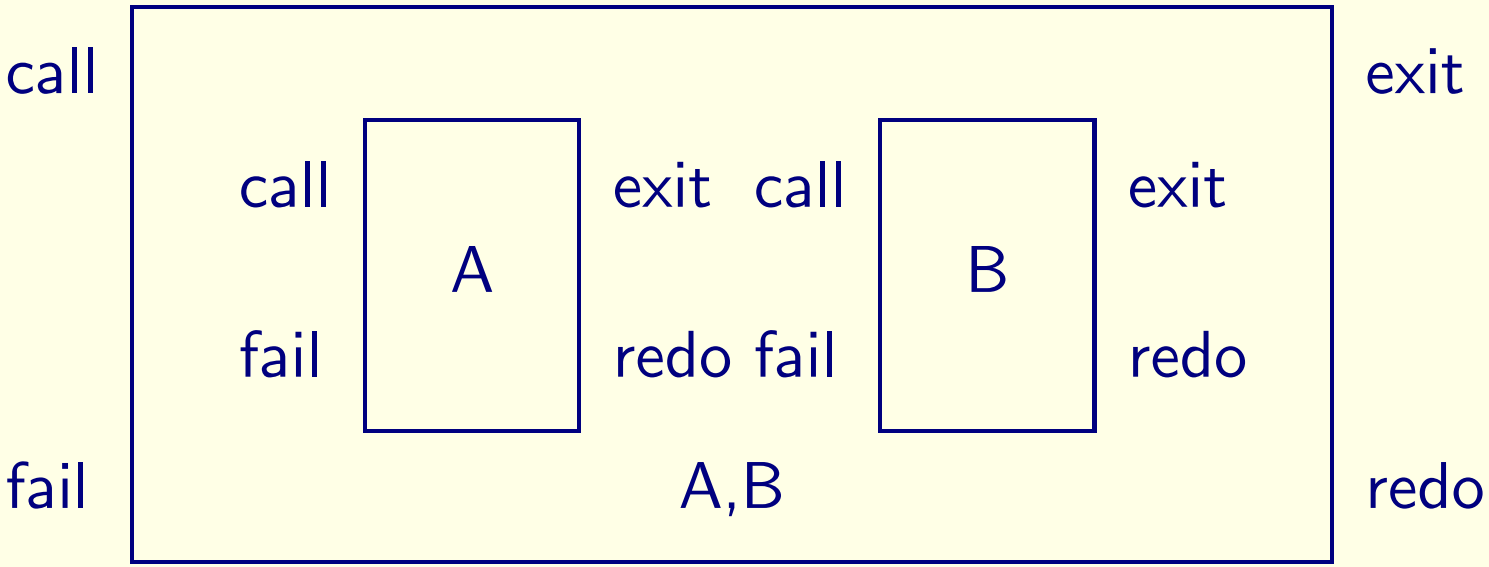
Main idea (cont'd), Conjunction



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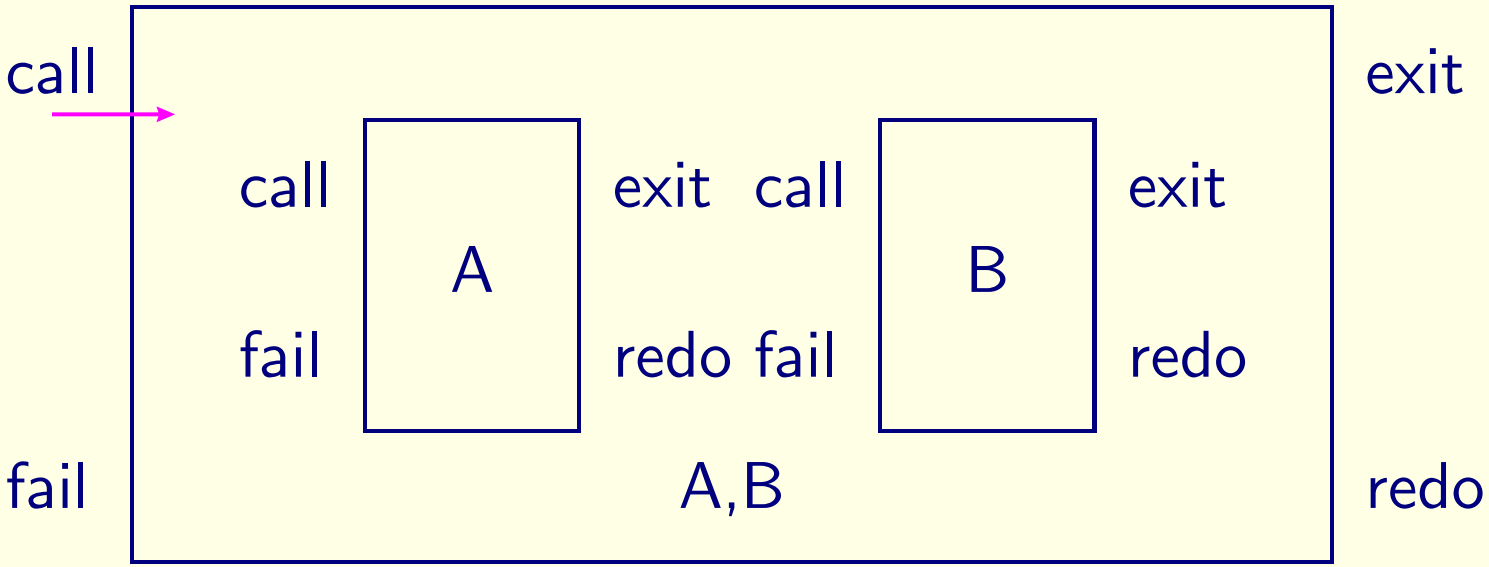

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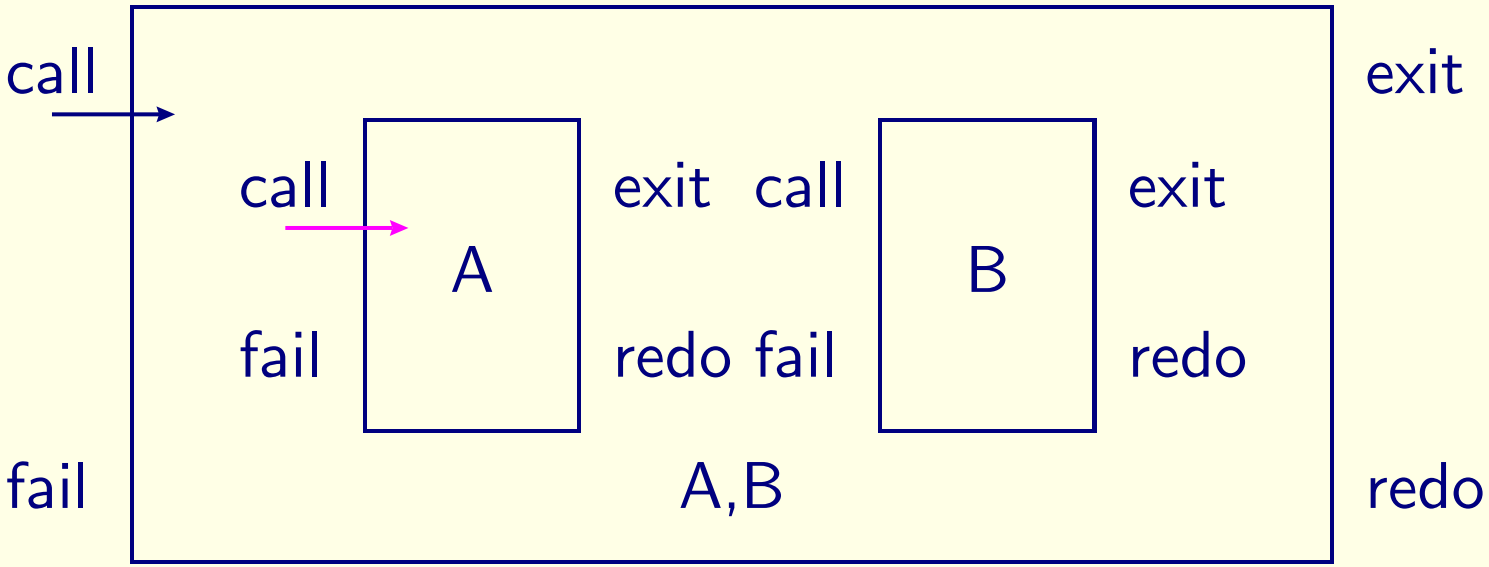

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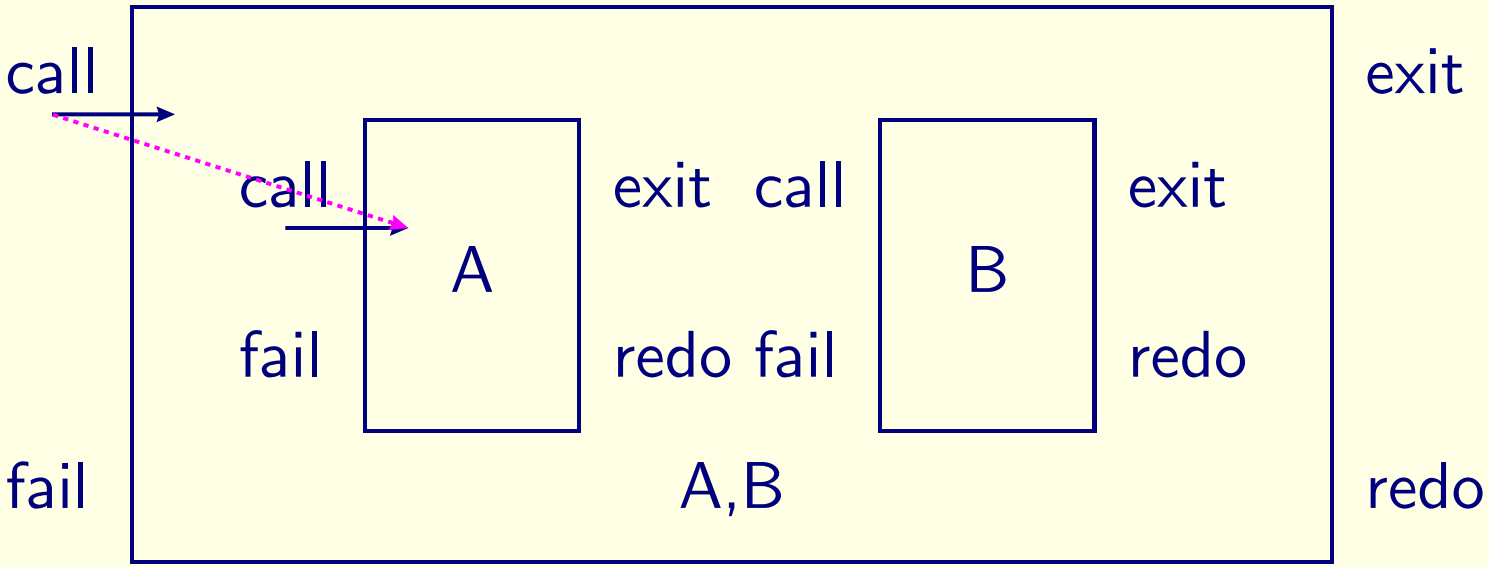

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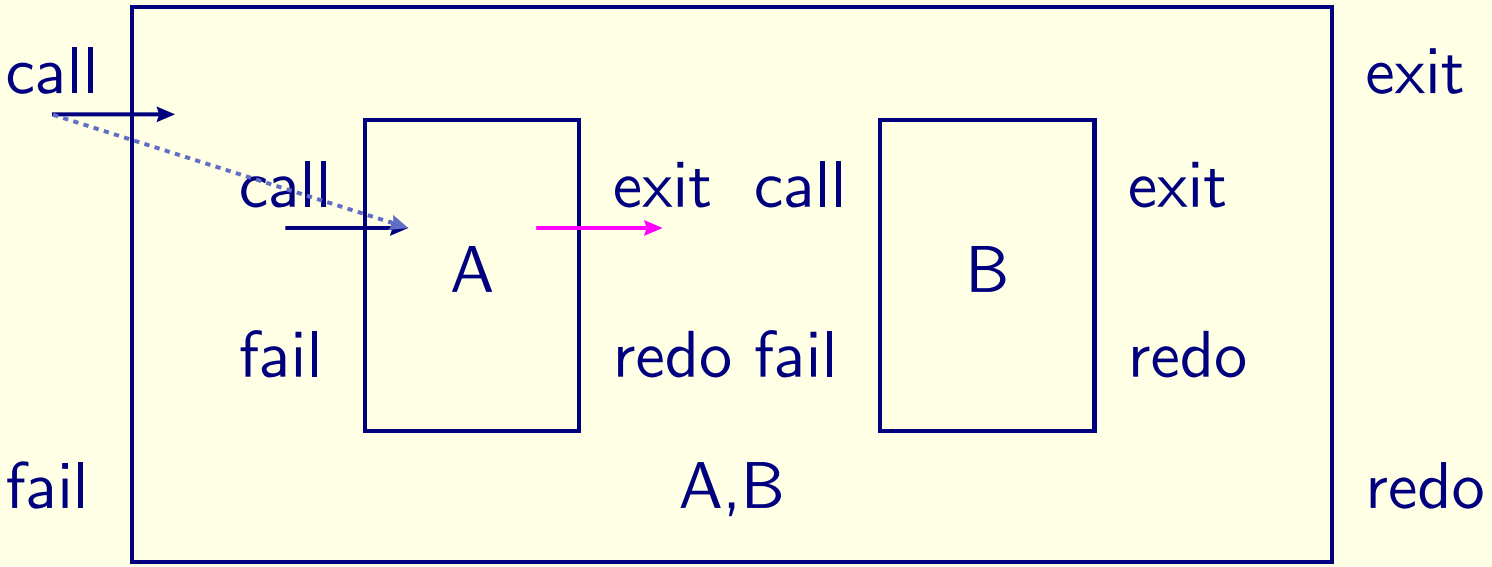

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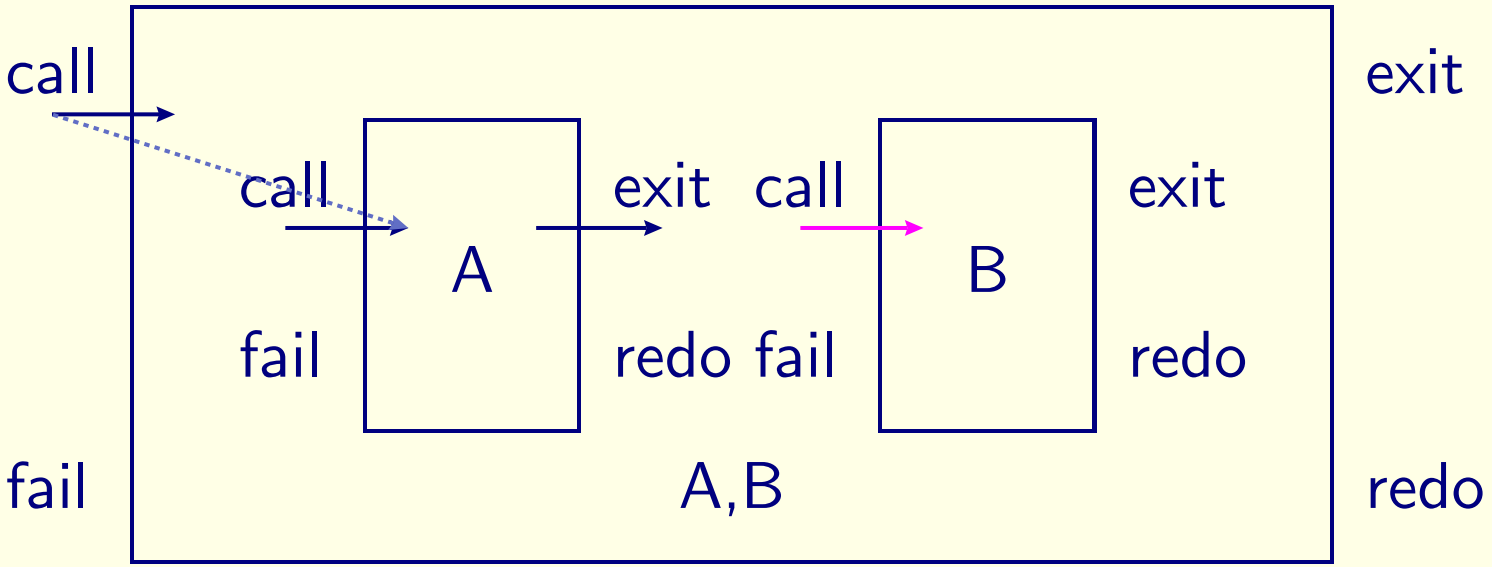

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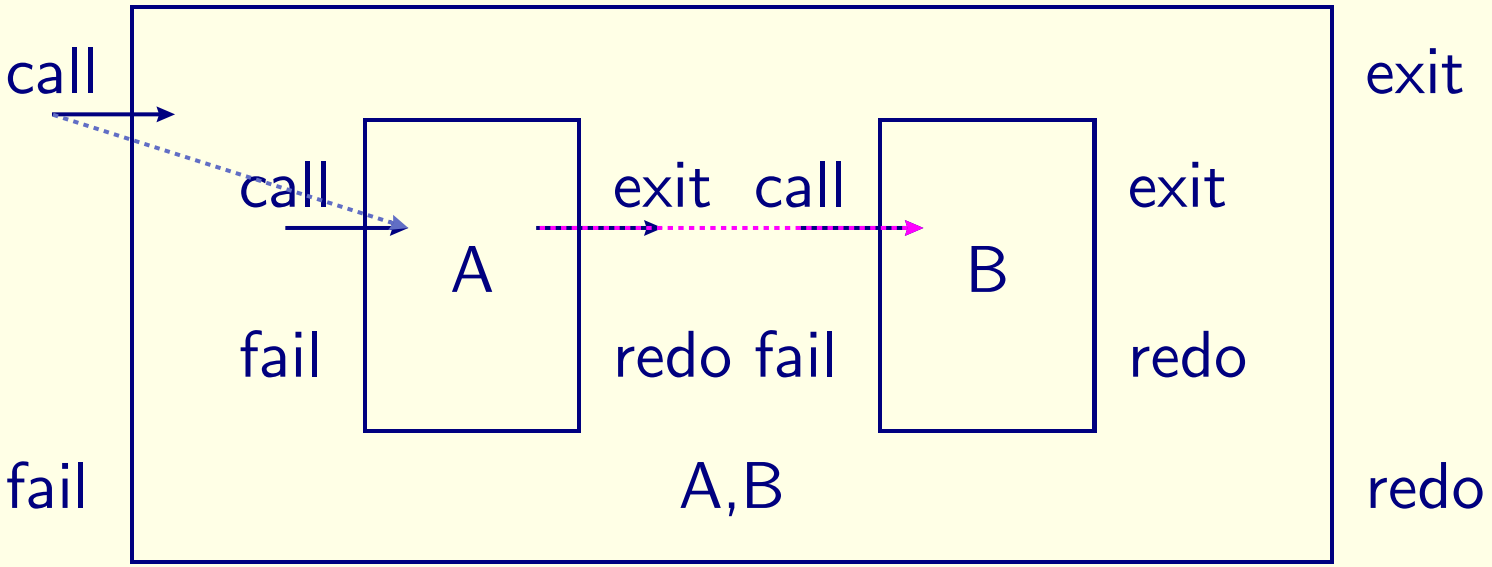

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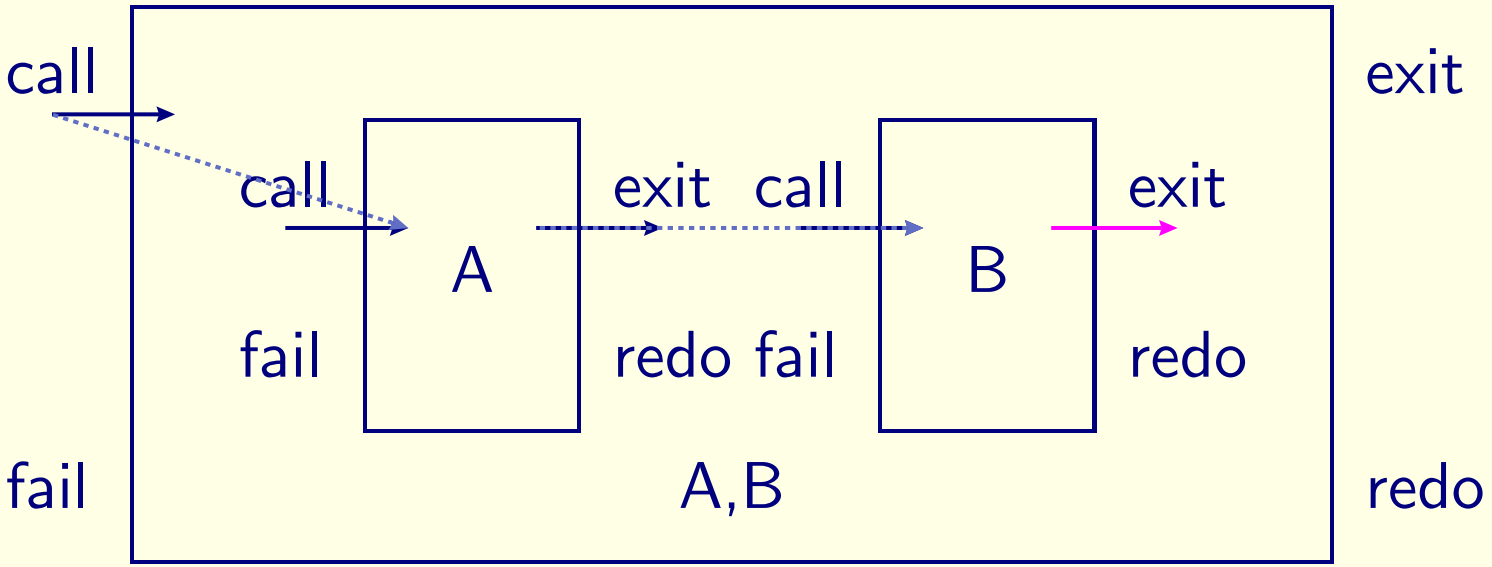

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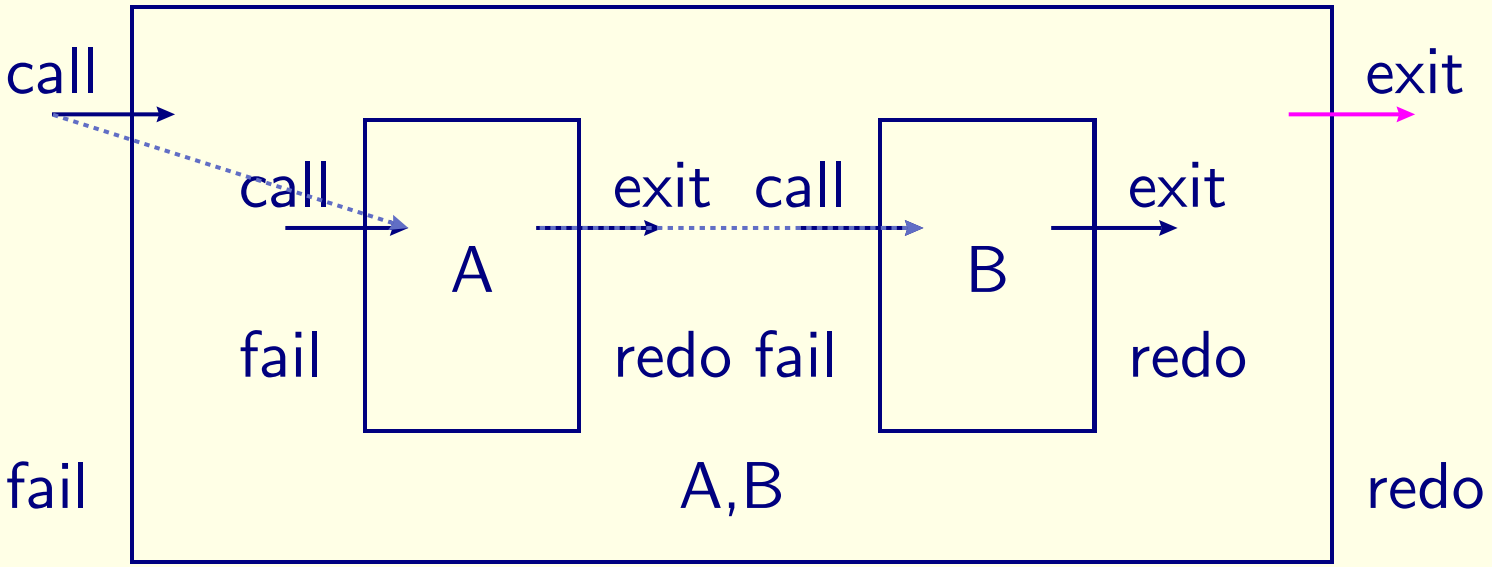

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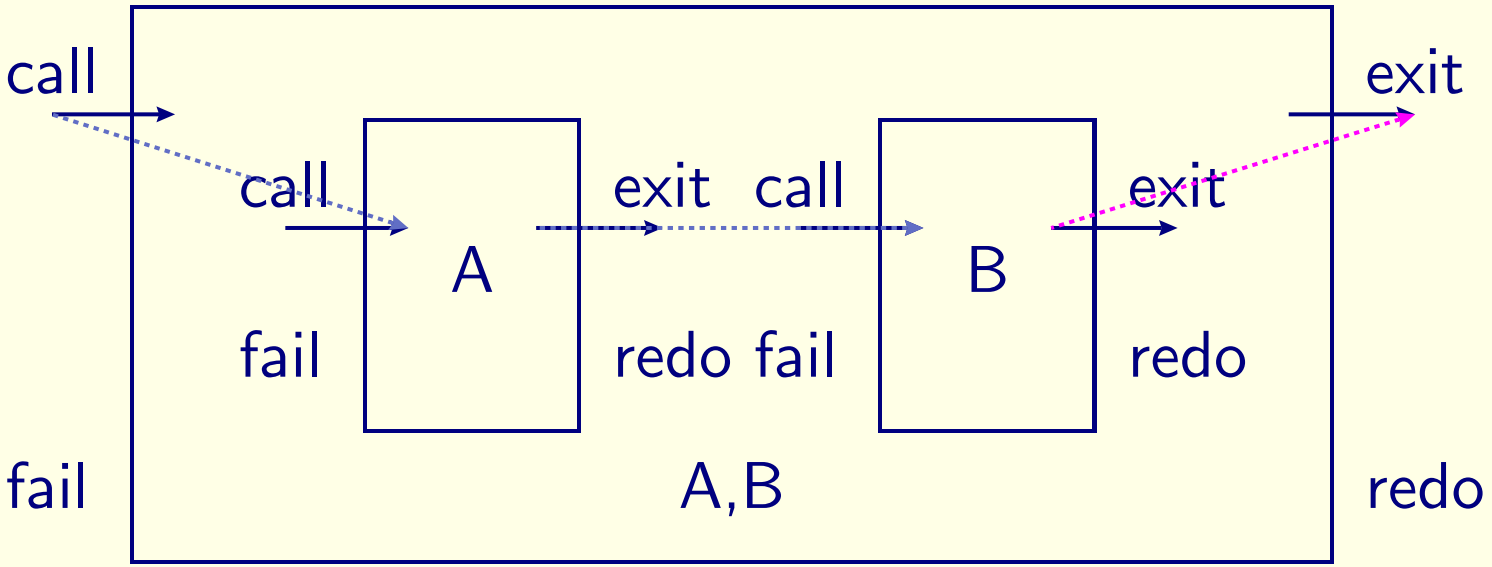

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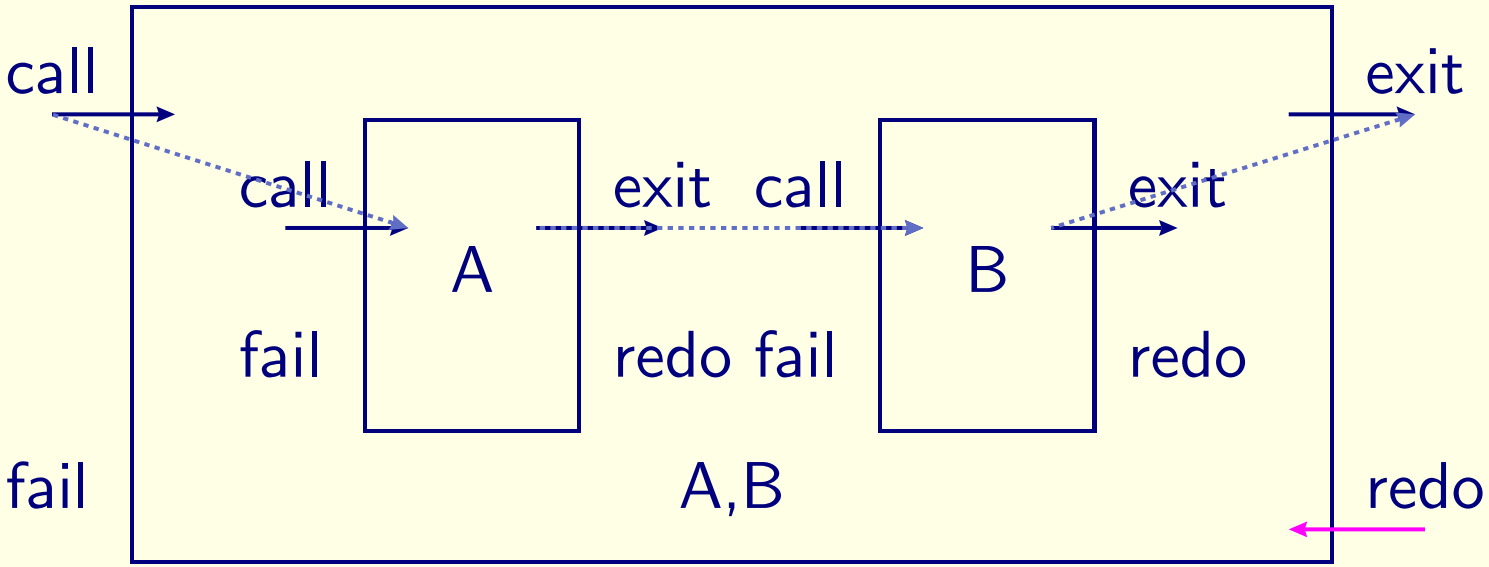

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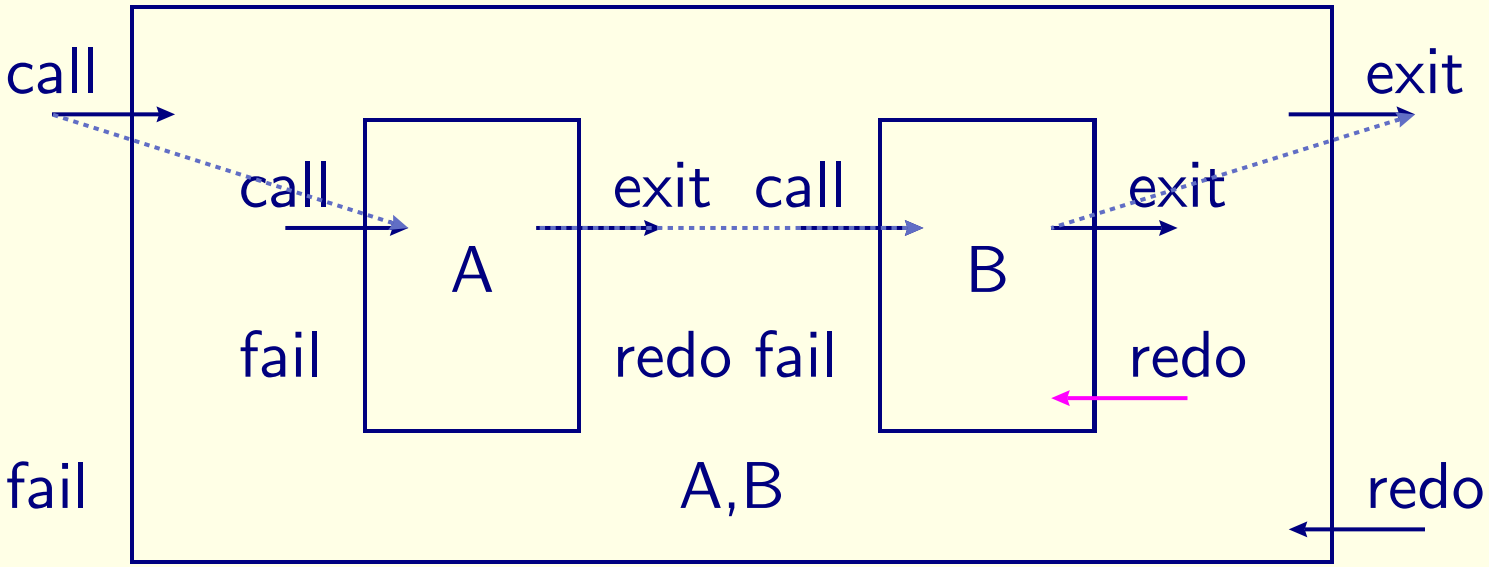

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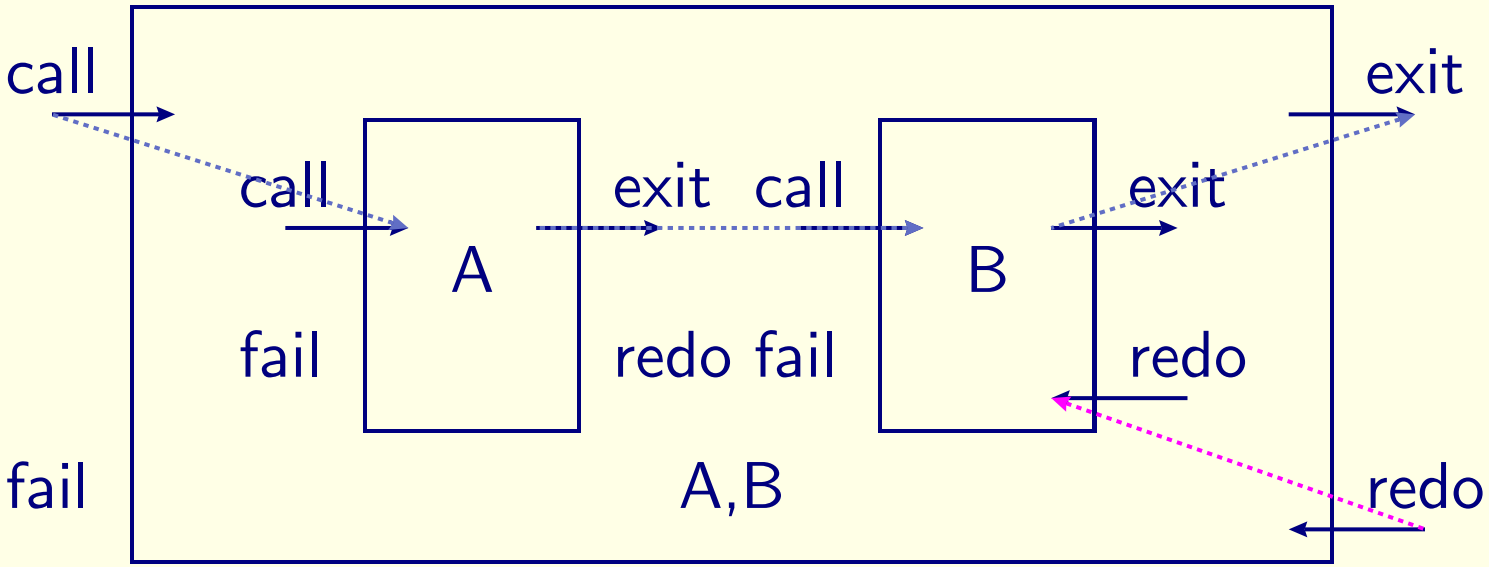

call A,B → call A

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exit B → exit A,B

redo A,B → redo B

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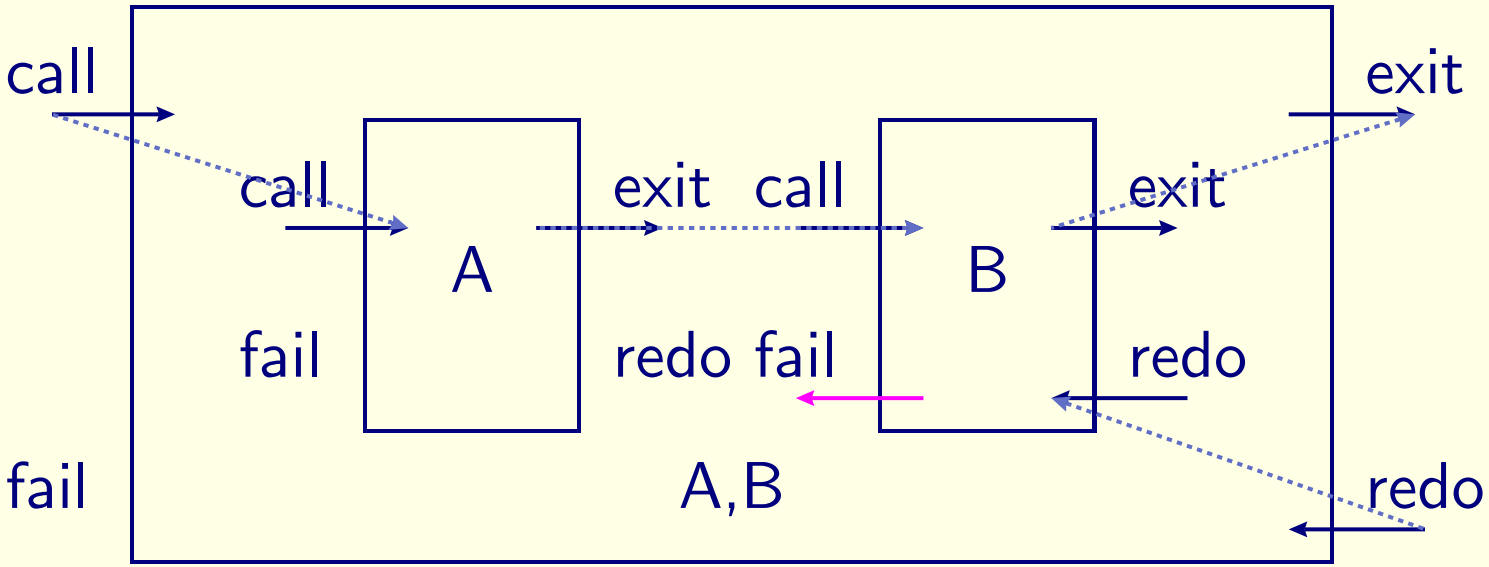

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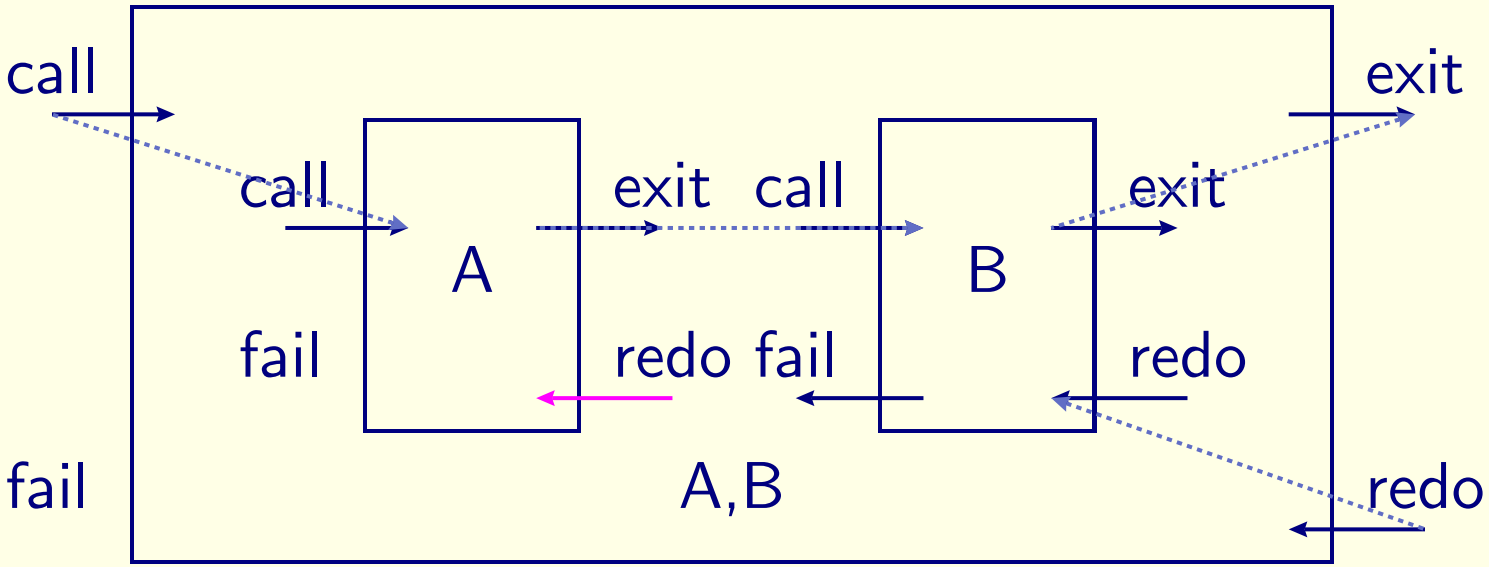

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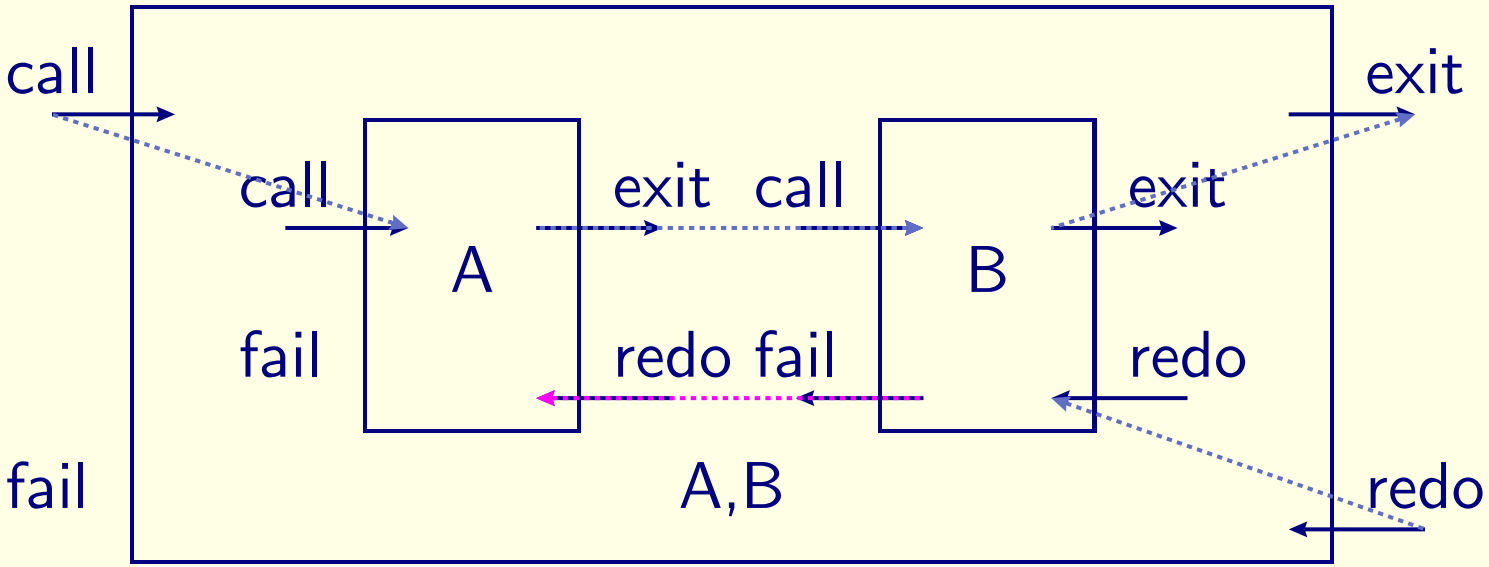

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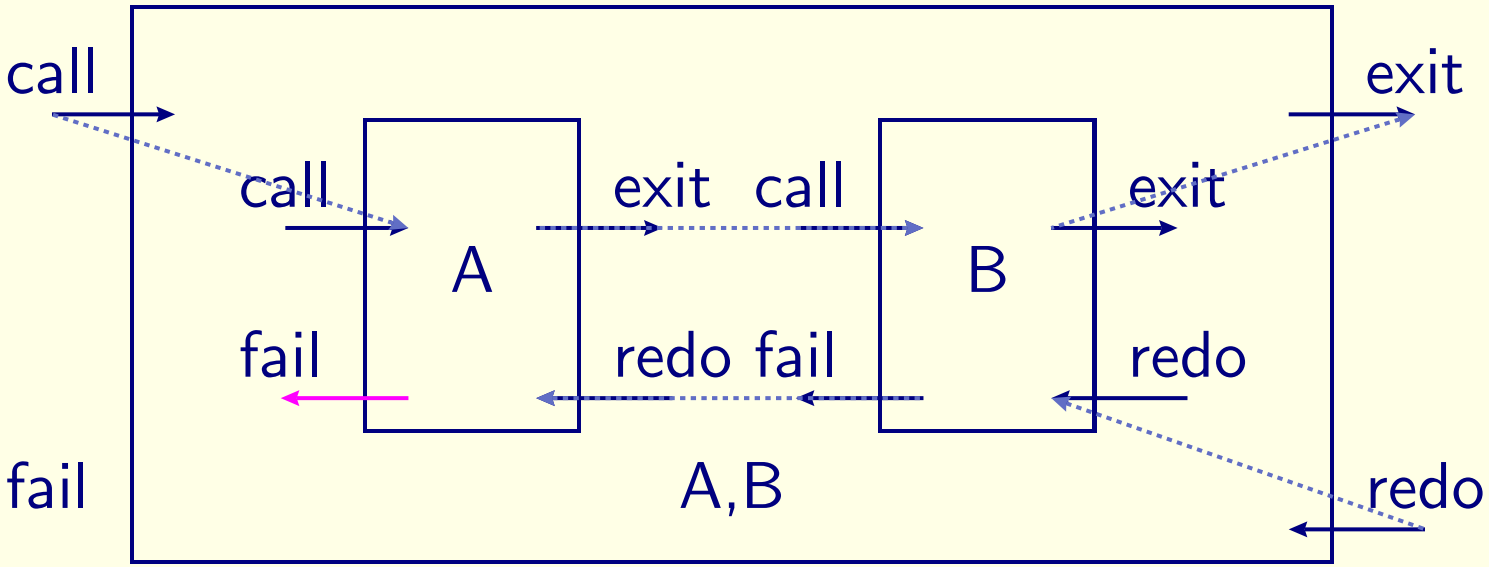

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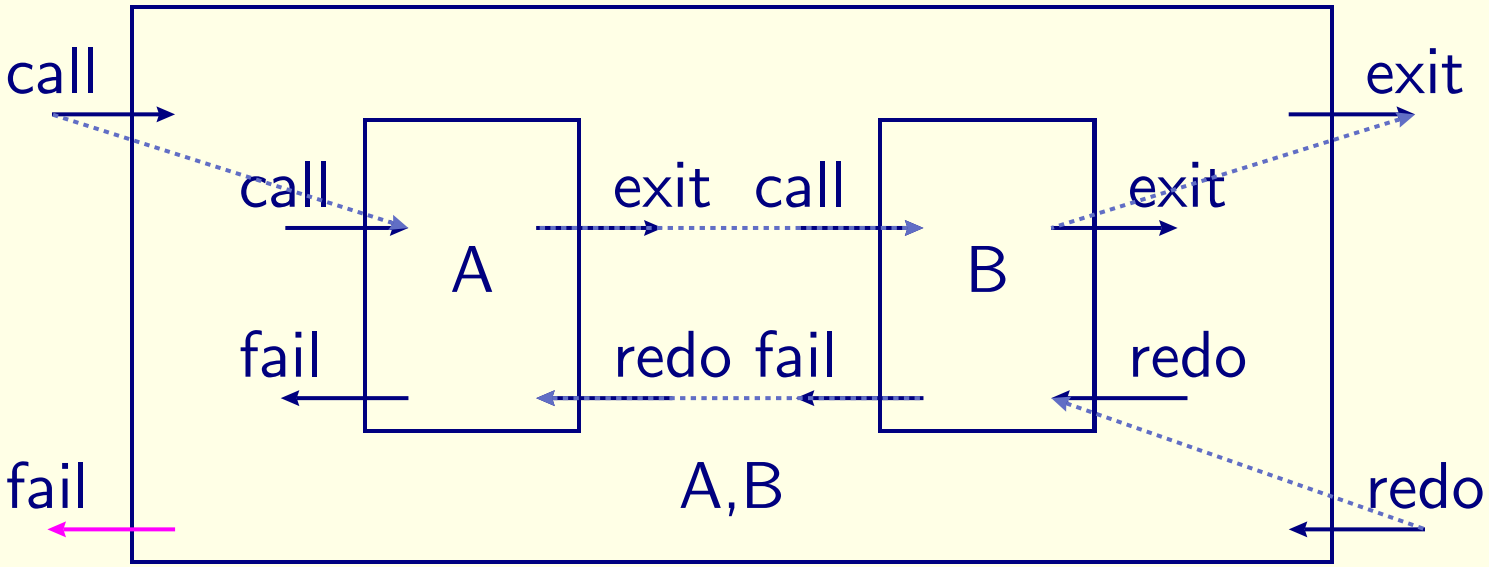

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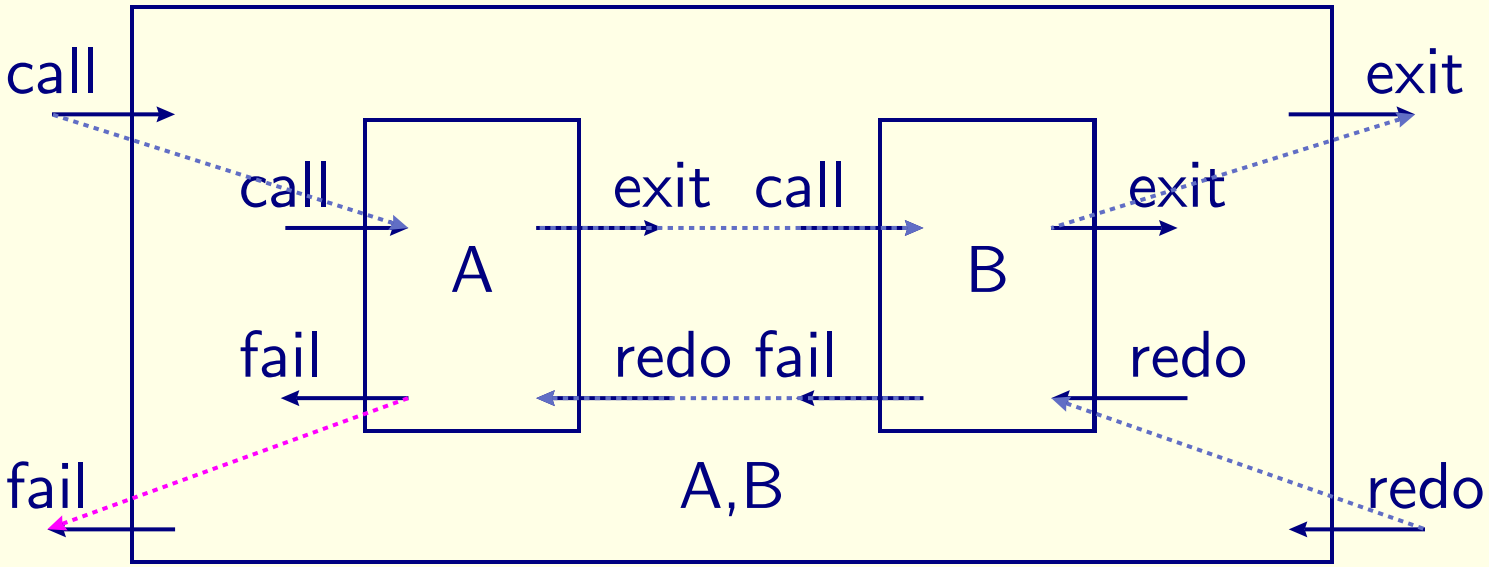

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fail A	→	fail A,B

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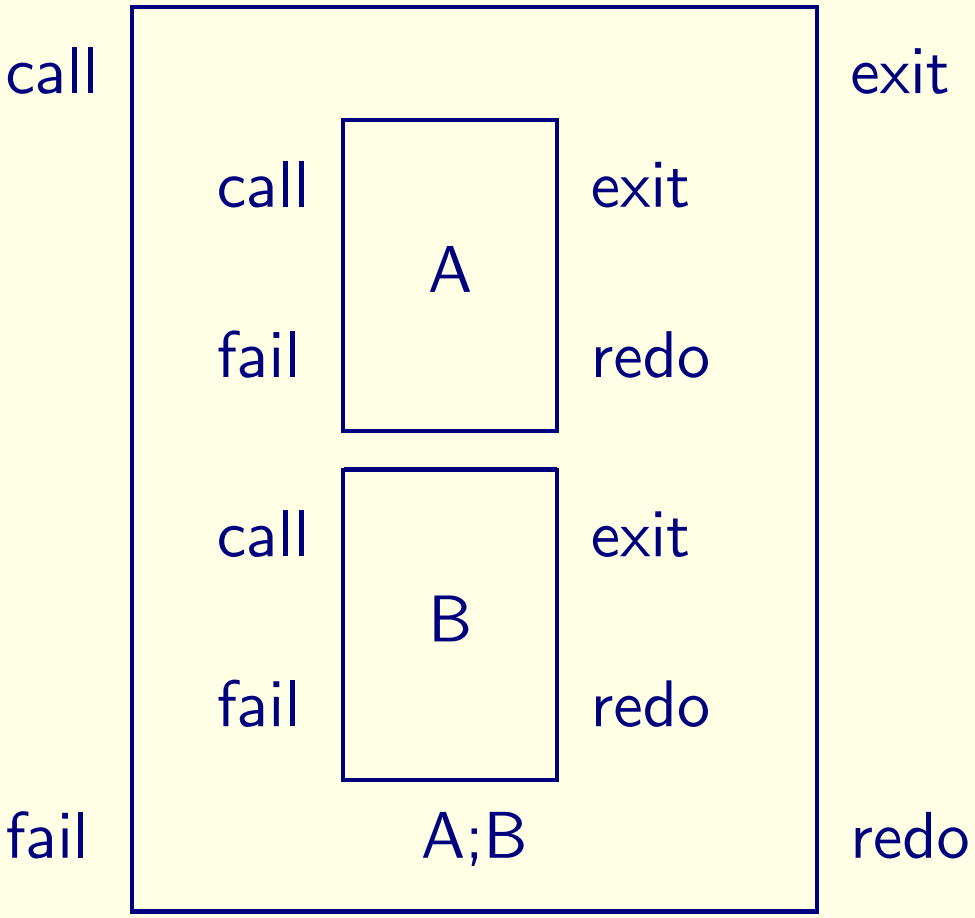

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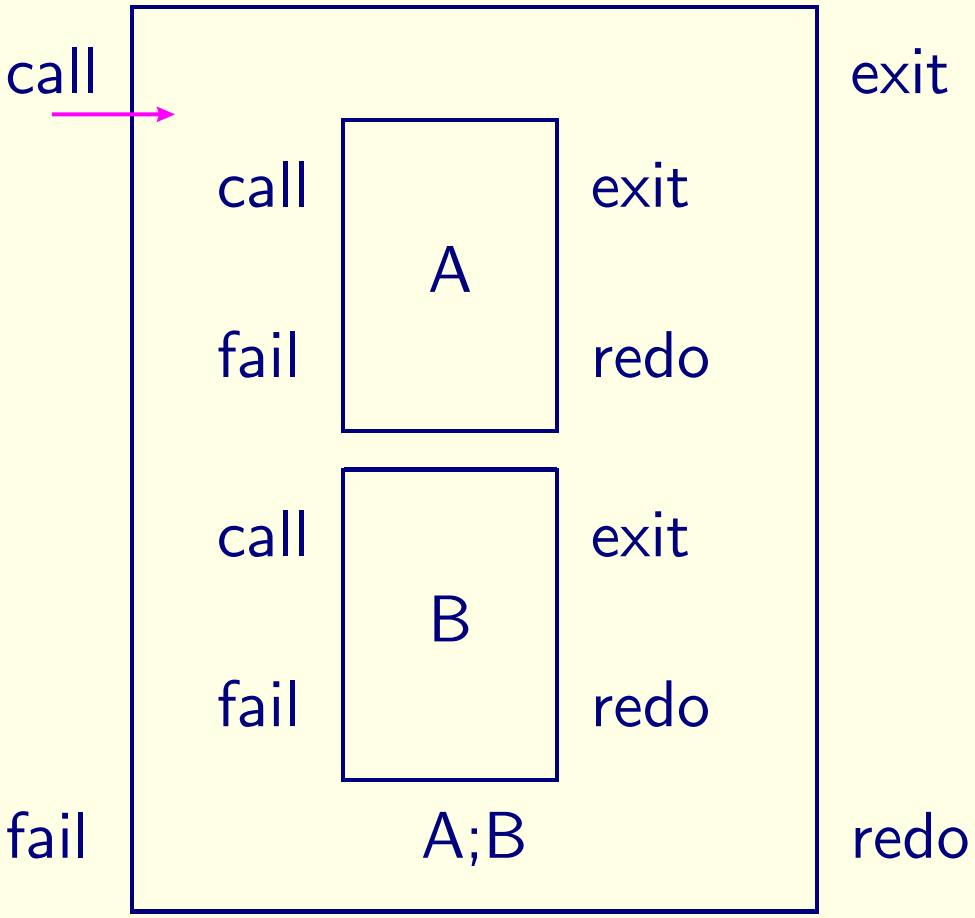

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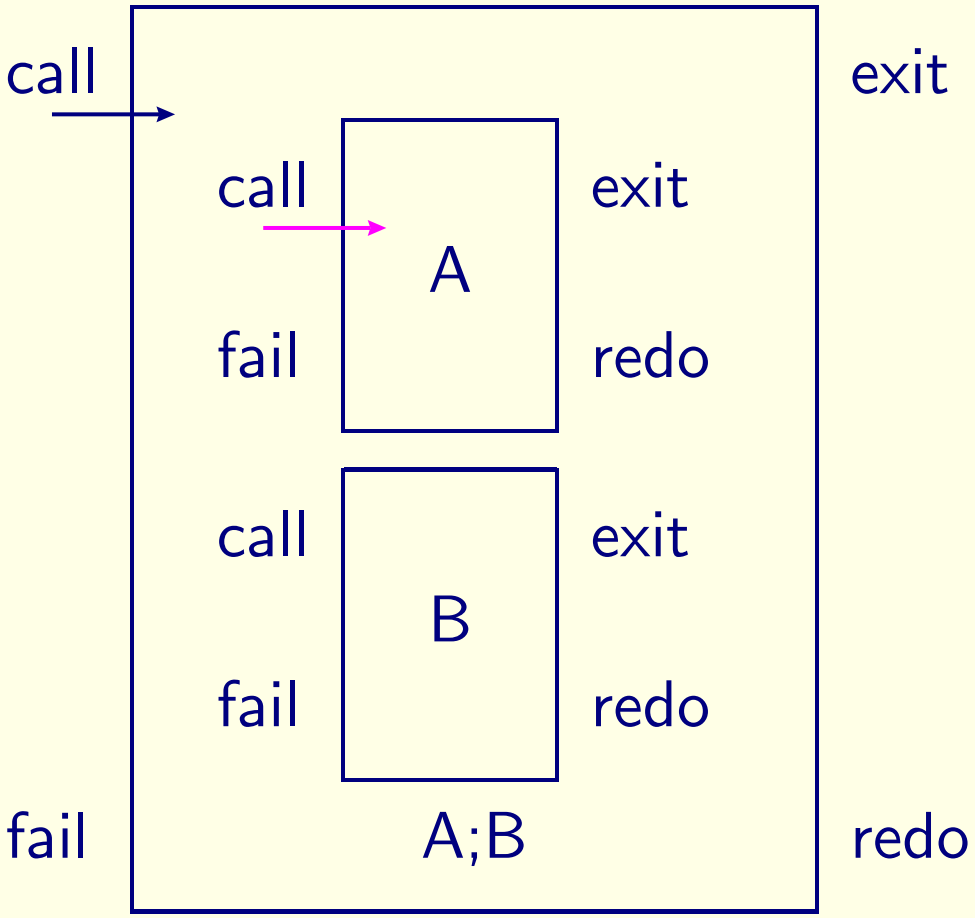

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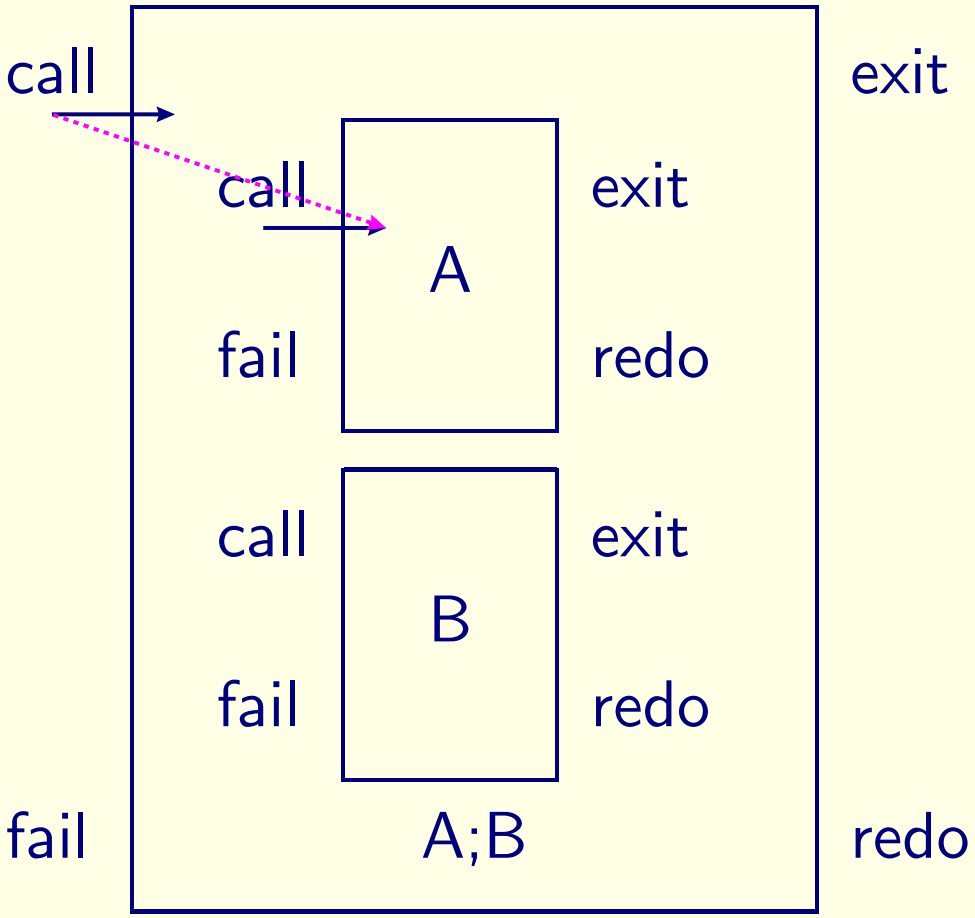

call A;B → call A

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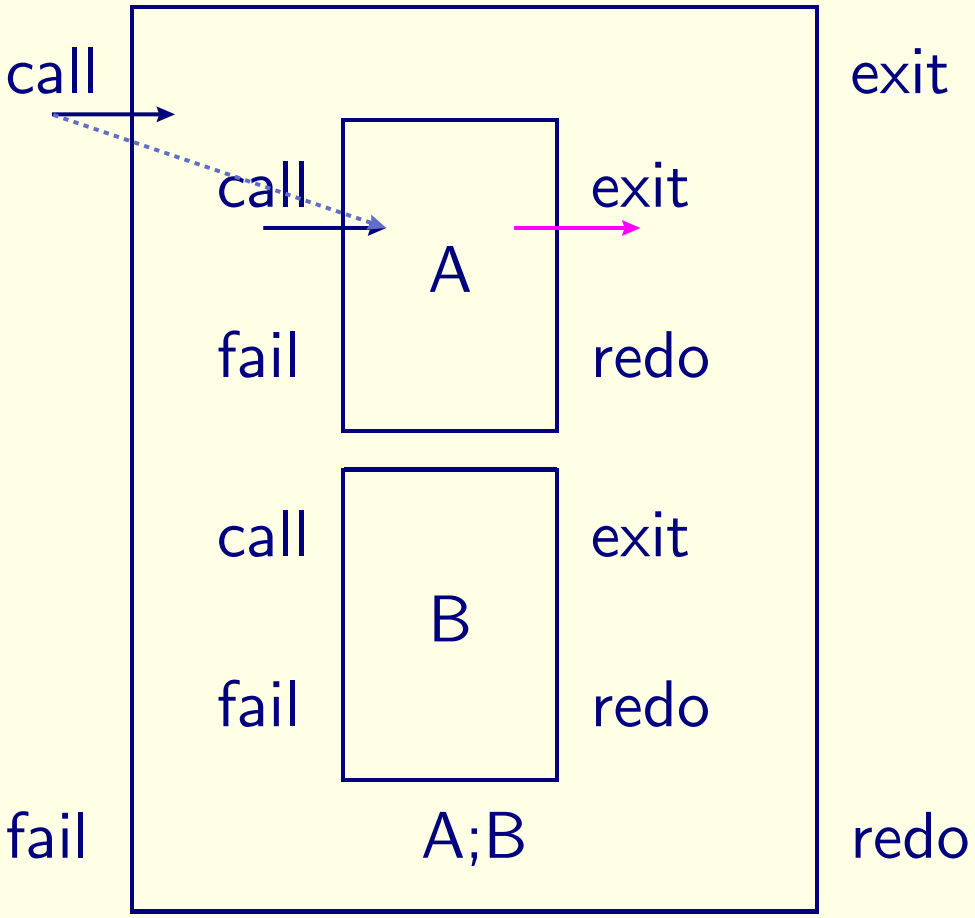

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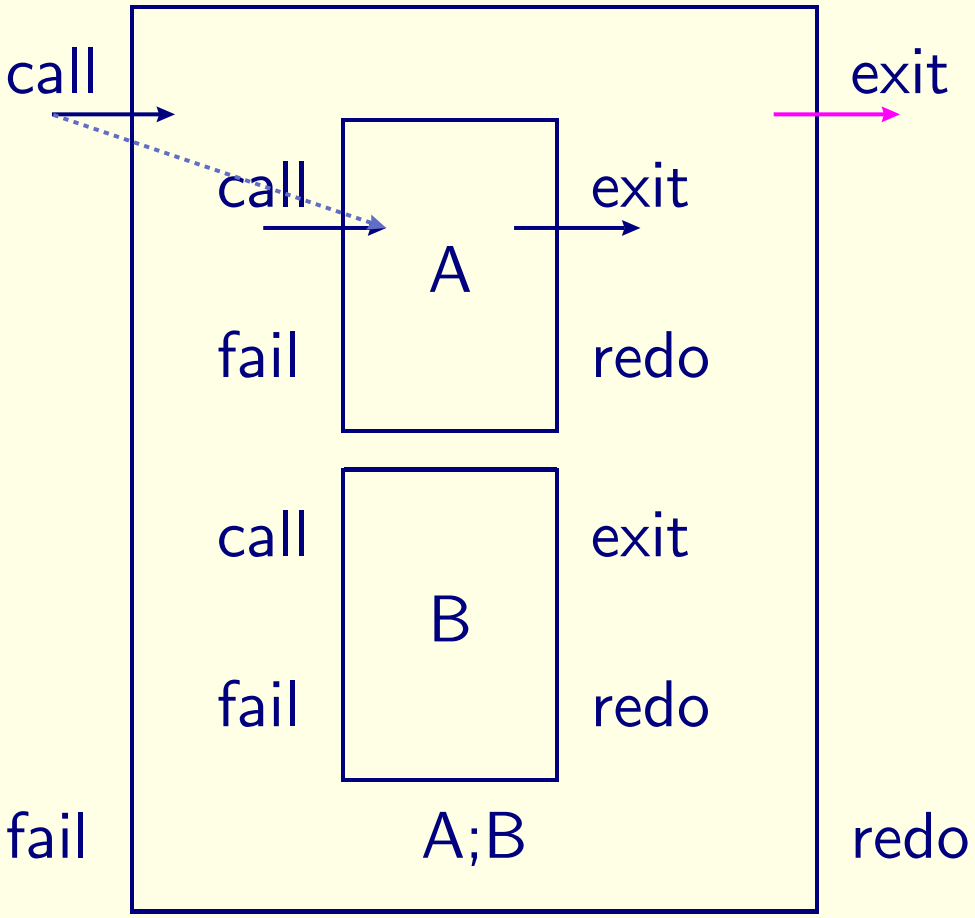

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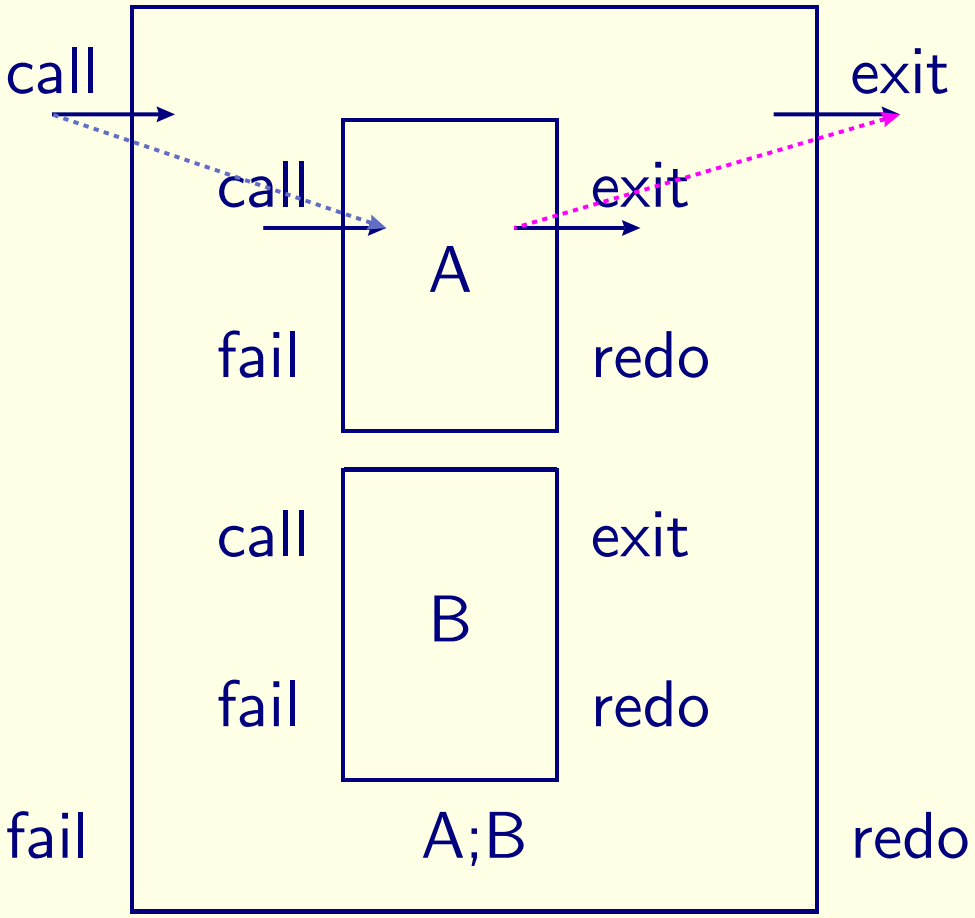

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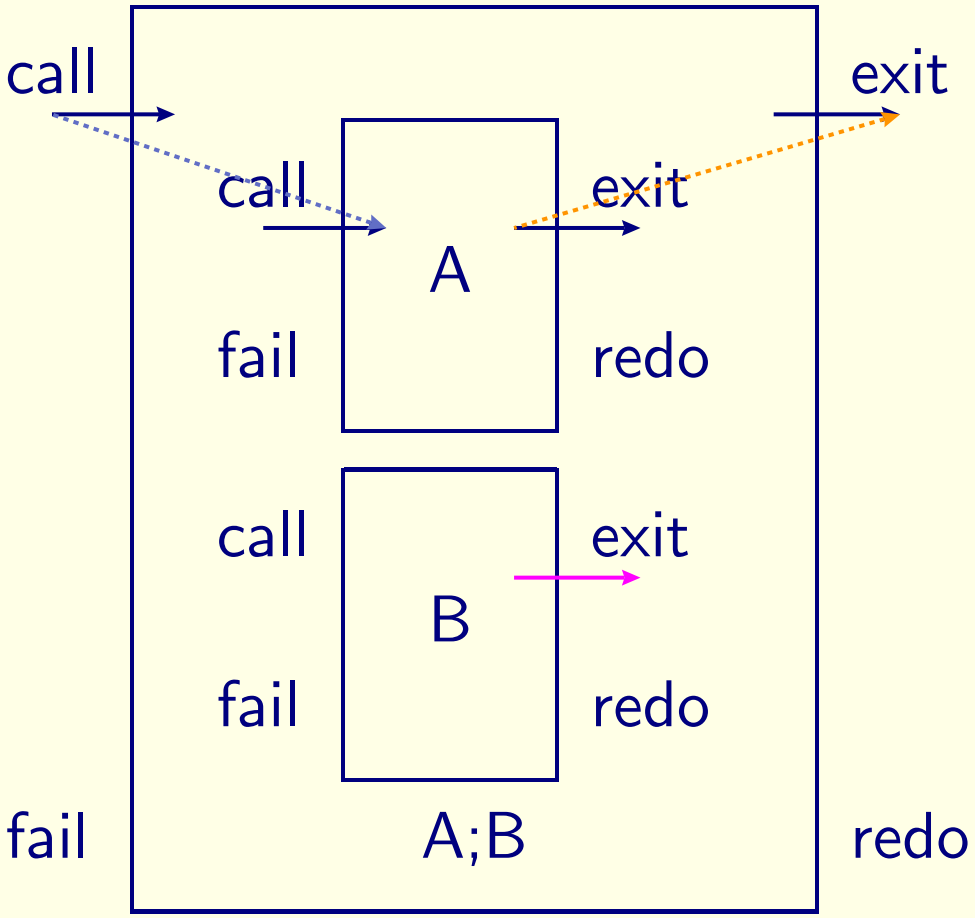

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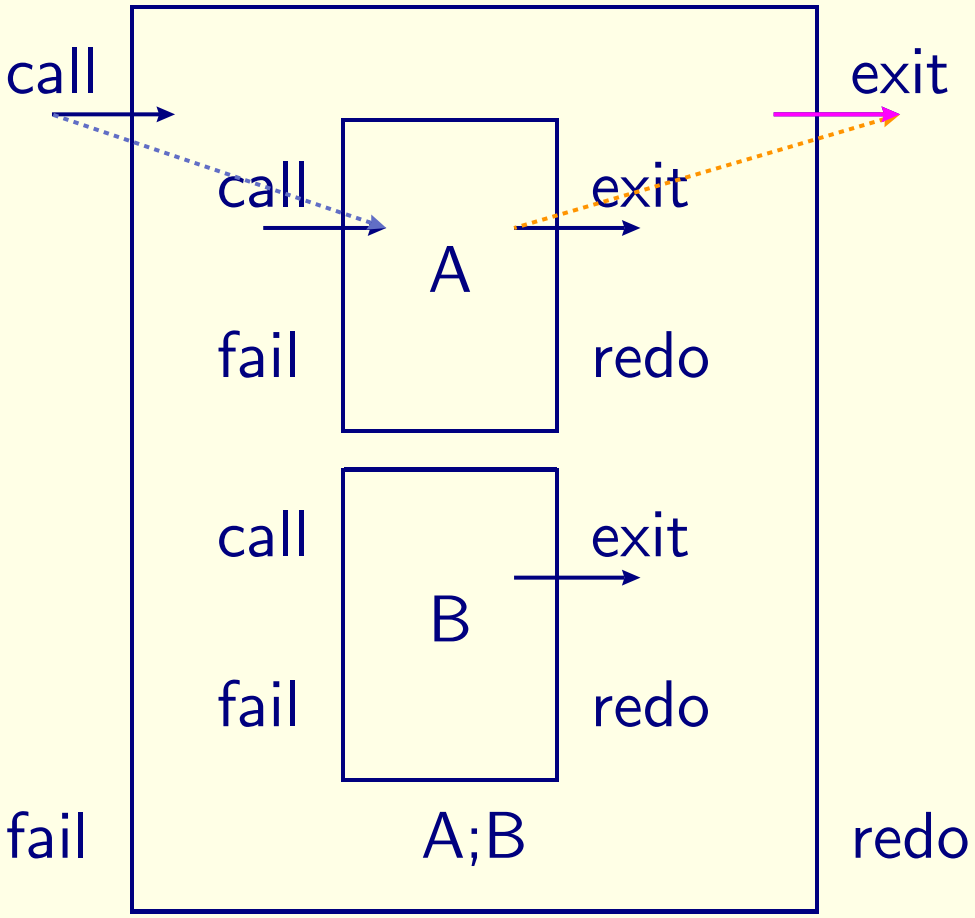

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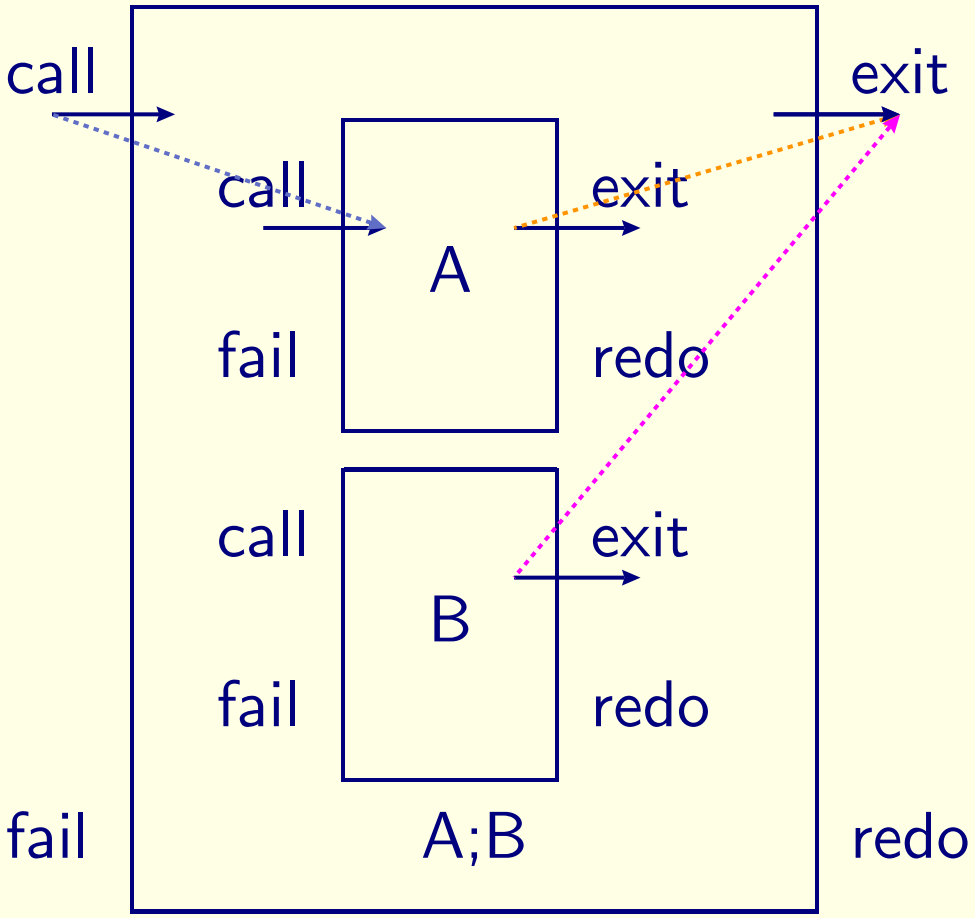
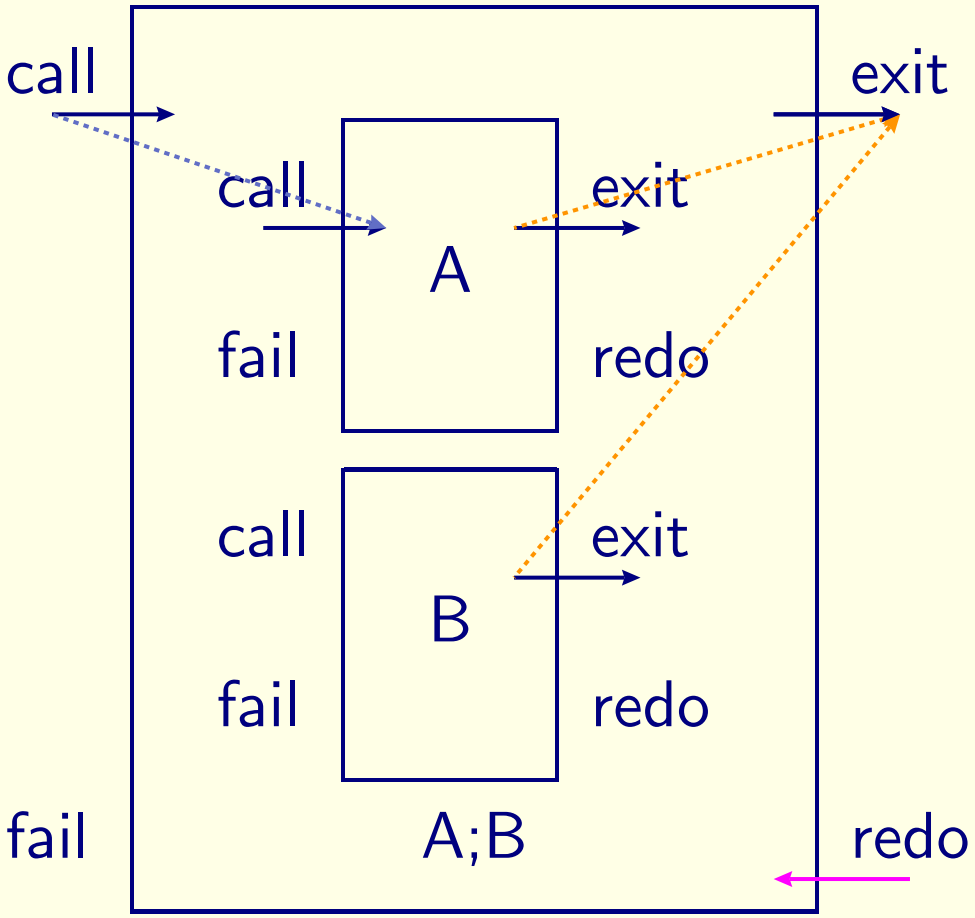


call A;B $\rightarrow$ call A

exit A $\rightarrow^{\alpha}$ exit A;B

exit B $\rightarrow^{\beta}$ exit A;B

Main idea (cont'd), Disjunction

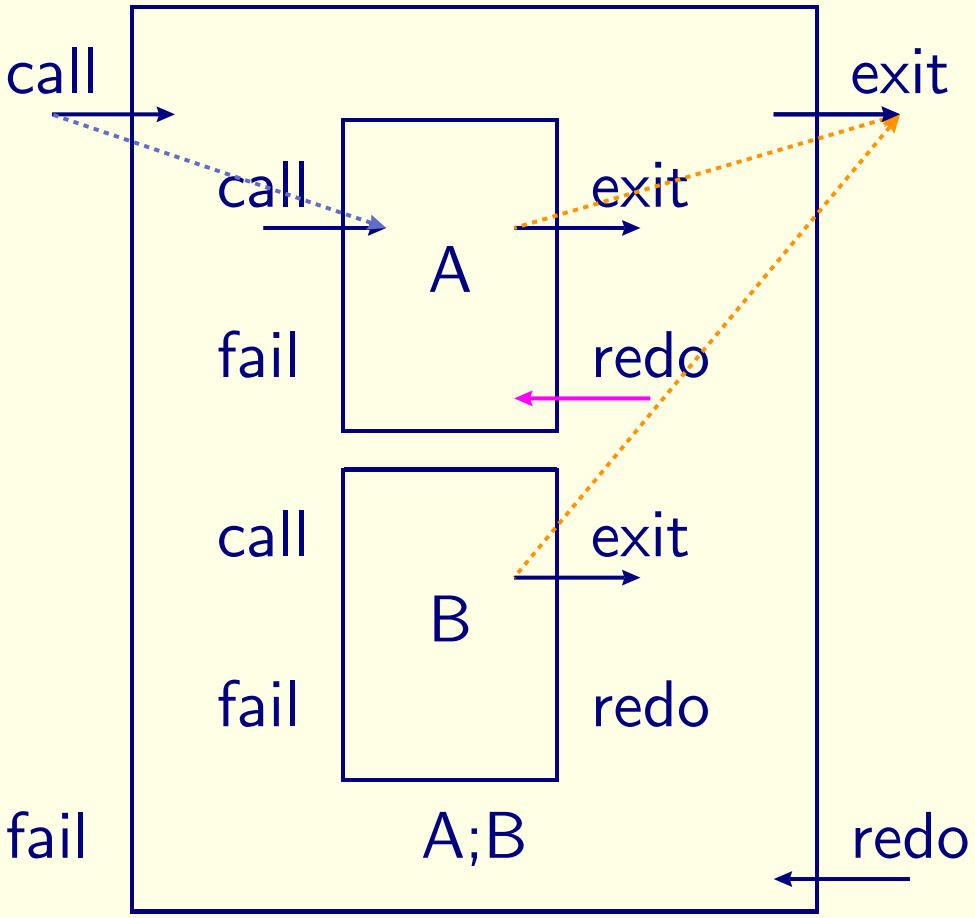


call A;B $\rightarrow$ call A

exit A $\rightarrow^{\alpha}$ exit A;B

exit B $\rightarrow^{\beta}$ exit A;B

Main idea (cont'd), Disjunction

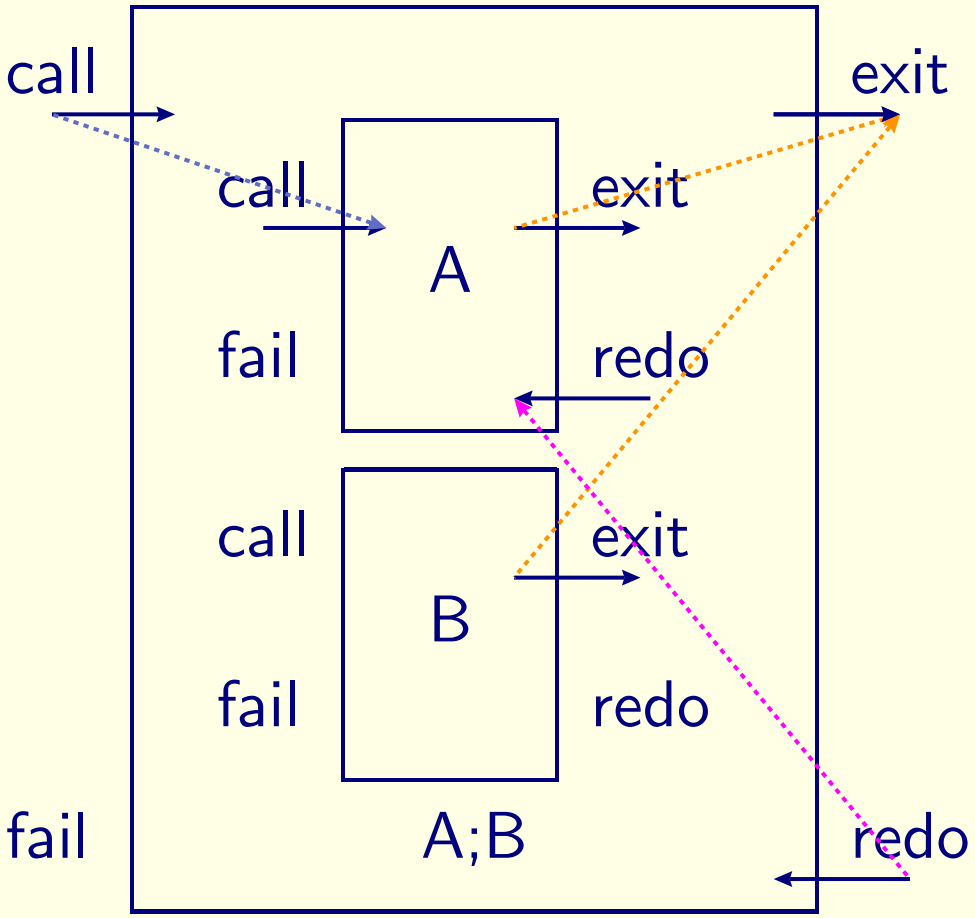


call A;B $\rightarrow$ call A

exit A $\rightarrow^{\alpha}$ exit A;B

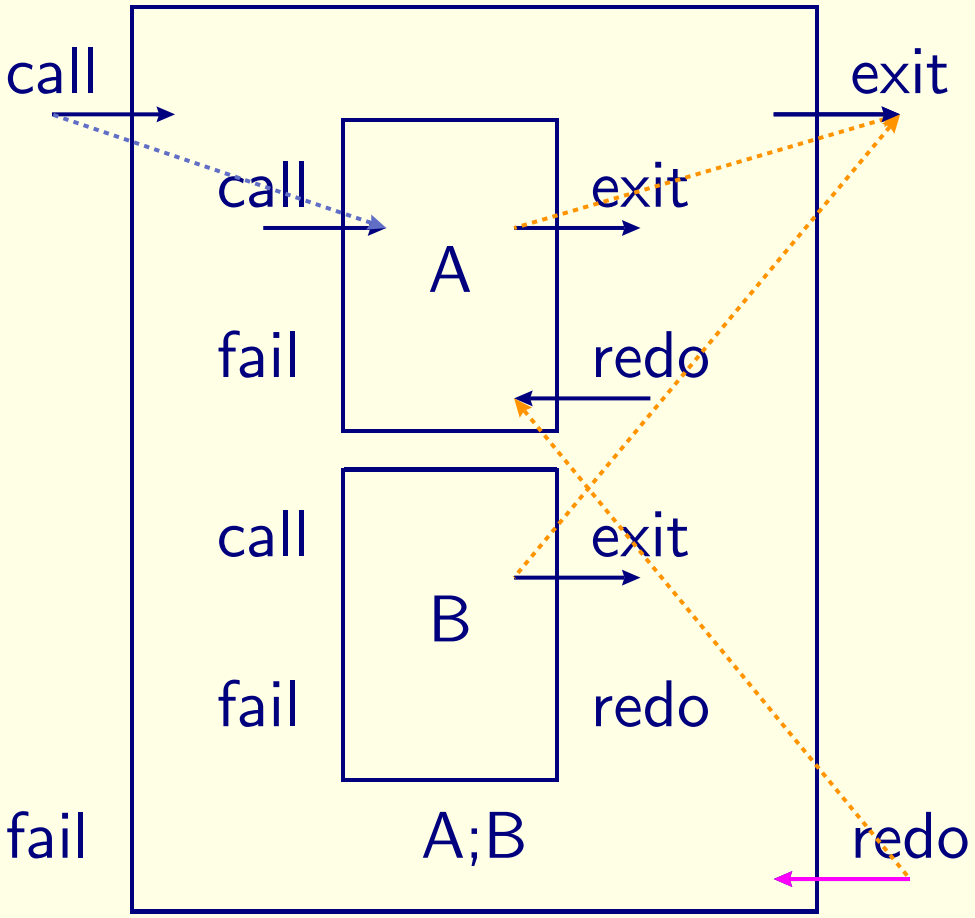
exit B $\rightarrow^{\beta}$ exit A;B

Main idea (cont'd), Disjunction



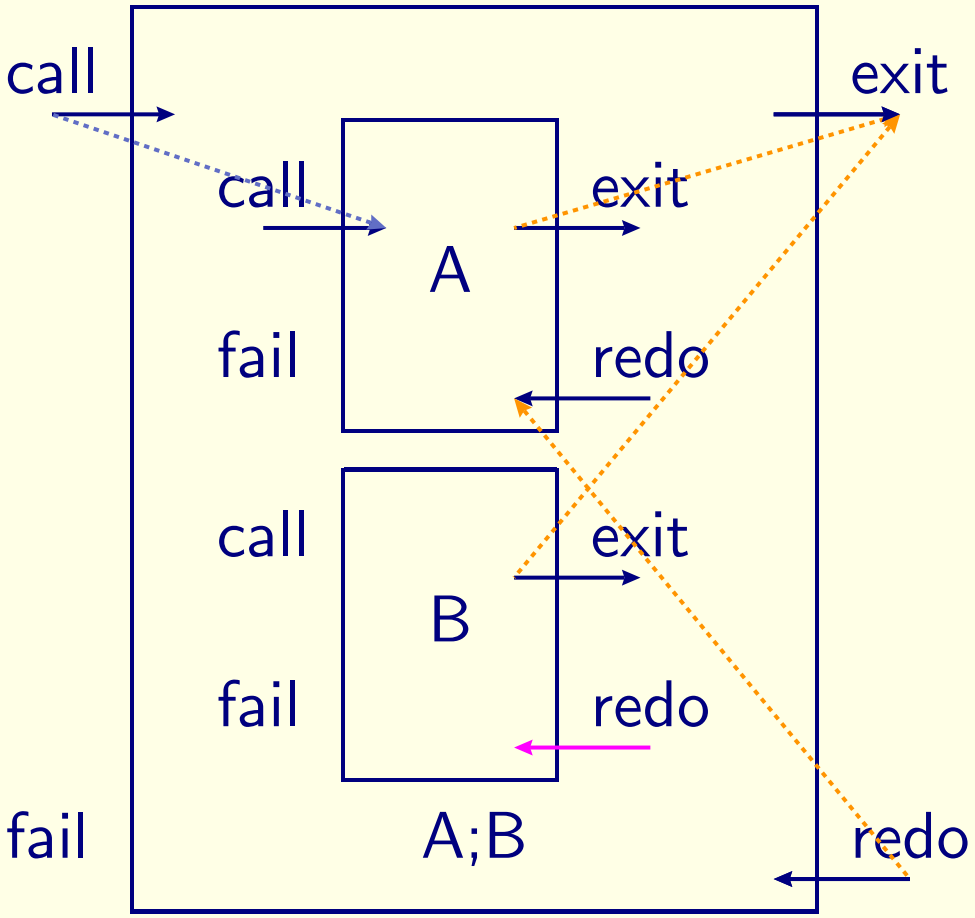
call A;B	$\rightarrowtriangle$	call A
exit A	$\rightarrowtriangle^{\alpha}$	exit A;B
exit B	$\rightarrowtriangle^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrowtriangle$	redo A

Main idea (cont'd), Disjunction



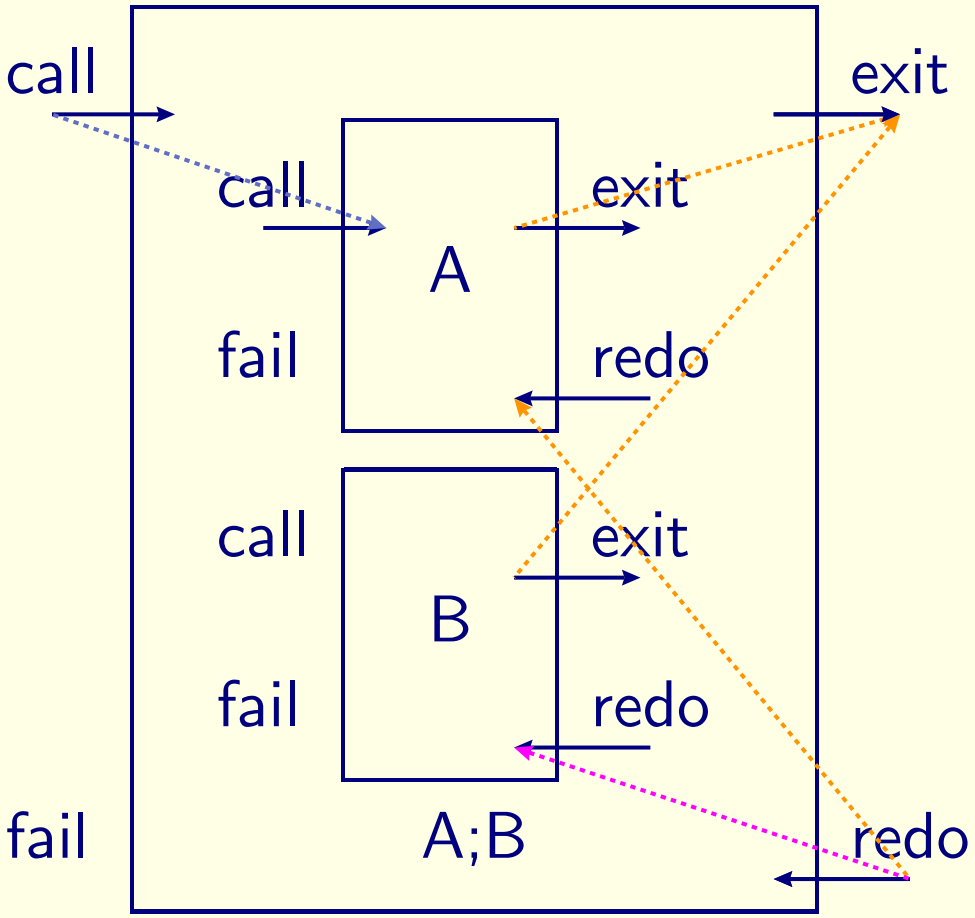
call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A

Main idea (cont'd), Disjunction



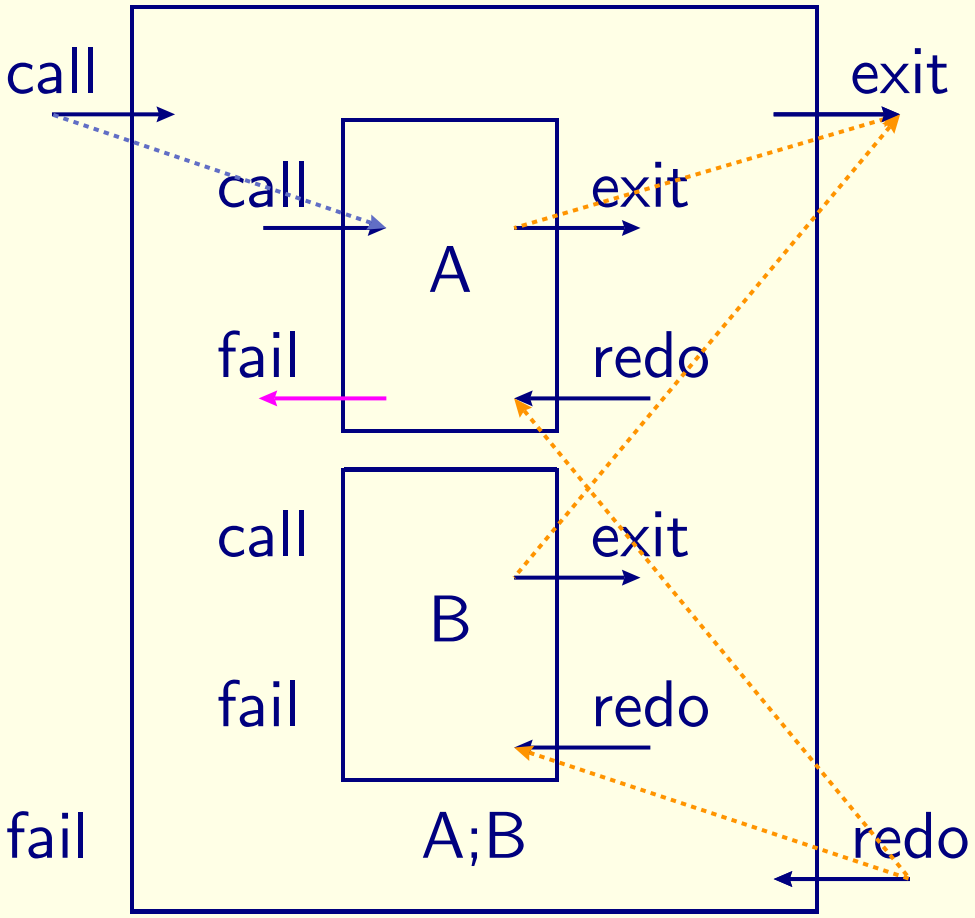
call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A

Main idea (cont'd), Disjunction



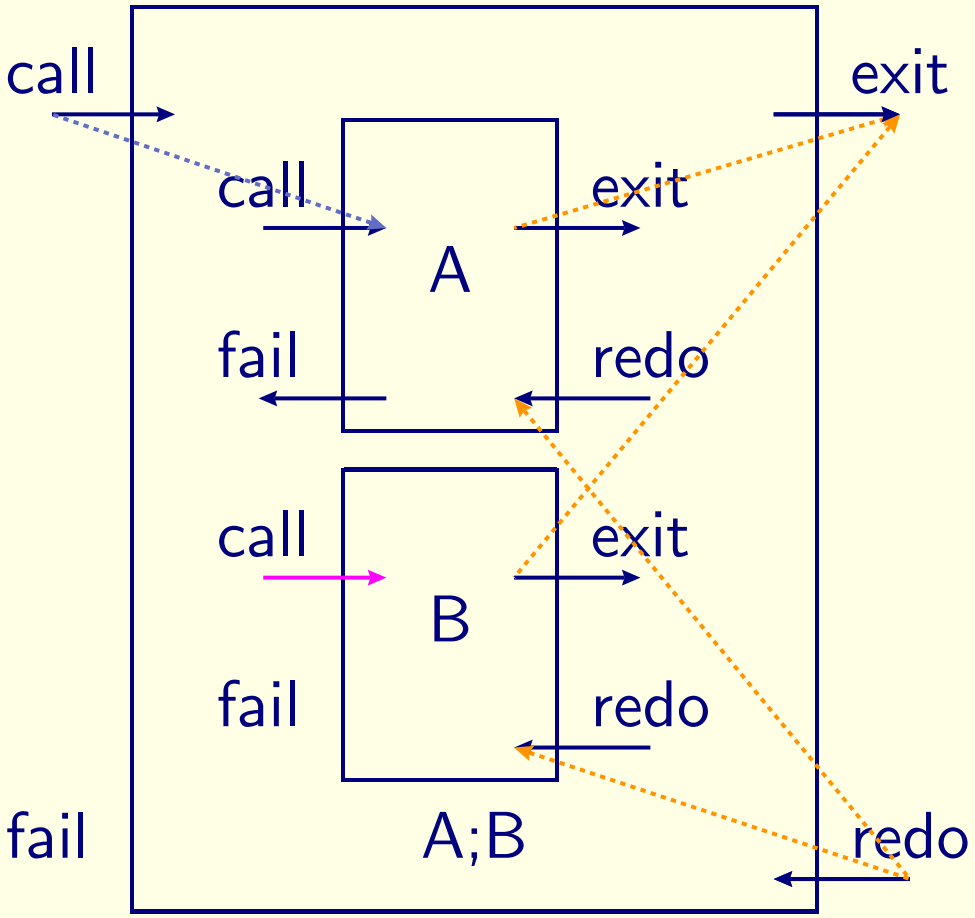
call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A
$^{\beta}$redo A;B	$\rightarrow$	redo B

Main idea (cont'd), Disjunction



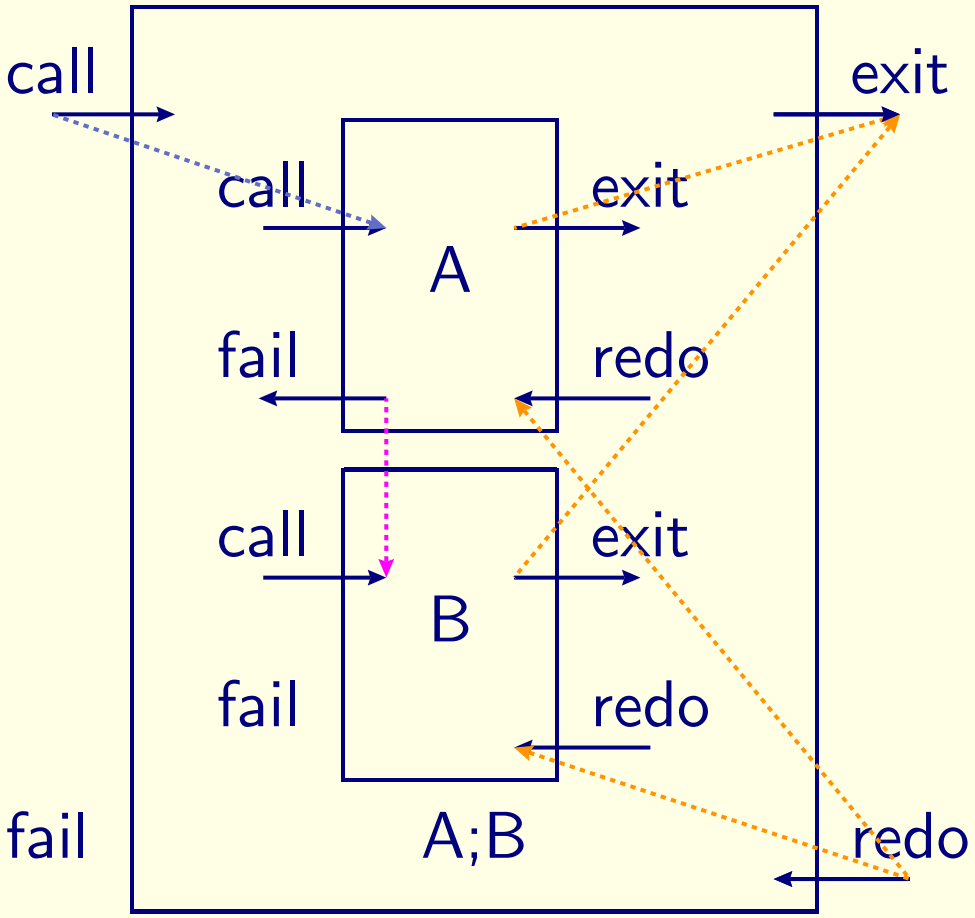
call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A
$^{\beta}$redo A;B	$\rightarrow$	redo B

Main idea (cont'd), Disjunction



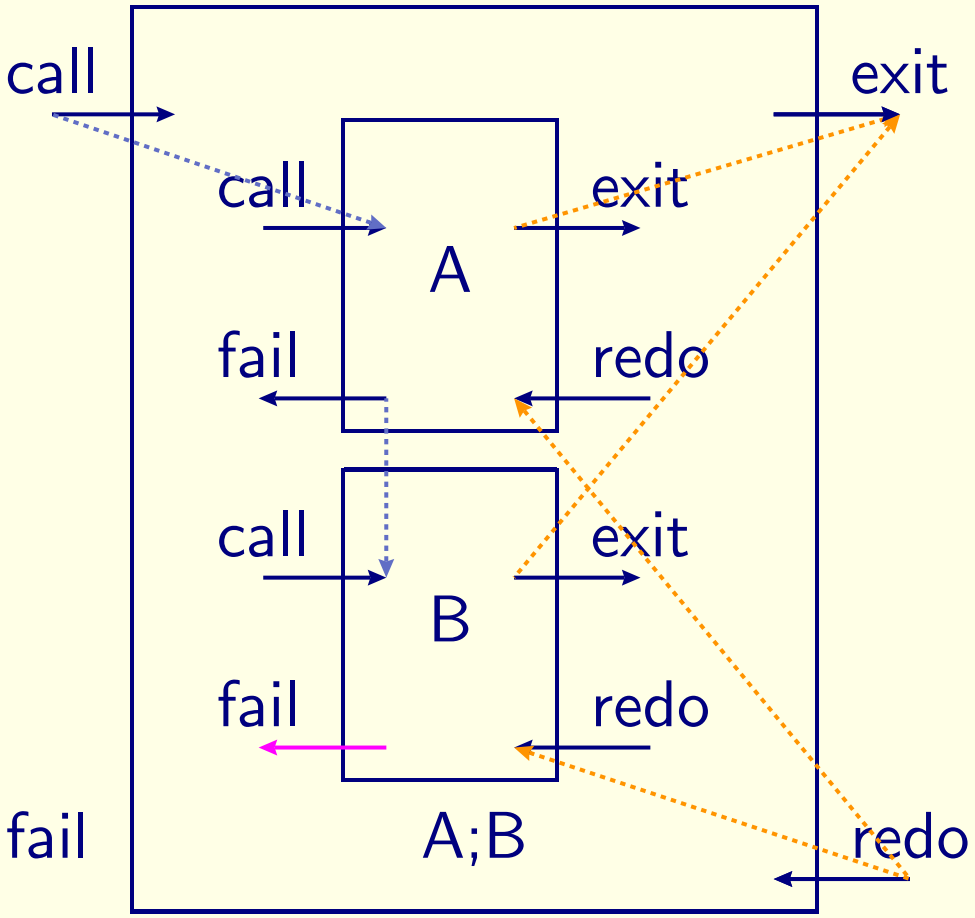
call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A
$^{\beta}$redo A;B	$\rightarrow$	redo B

Main idea (cont'd), Disjunction



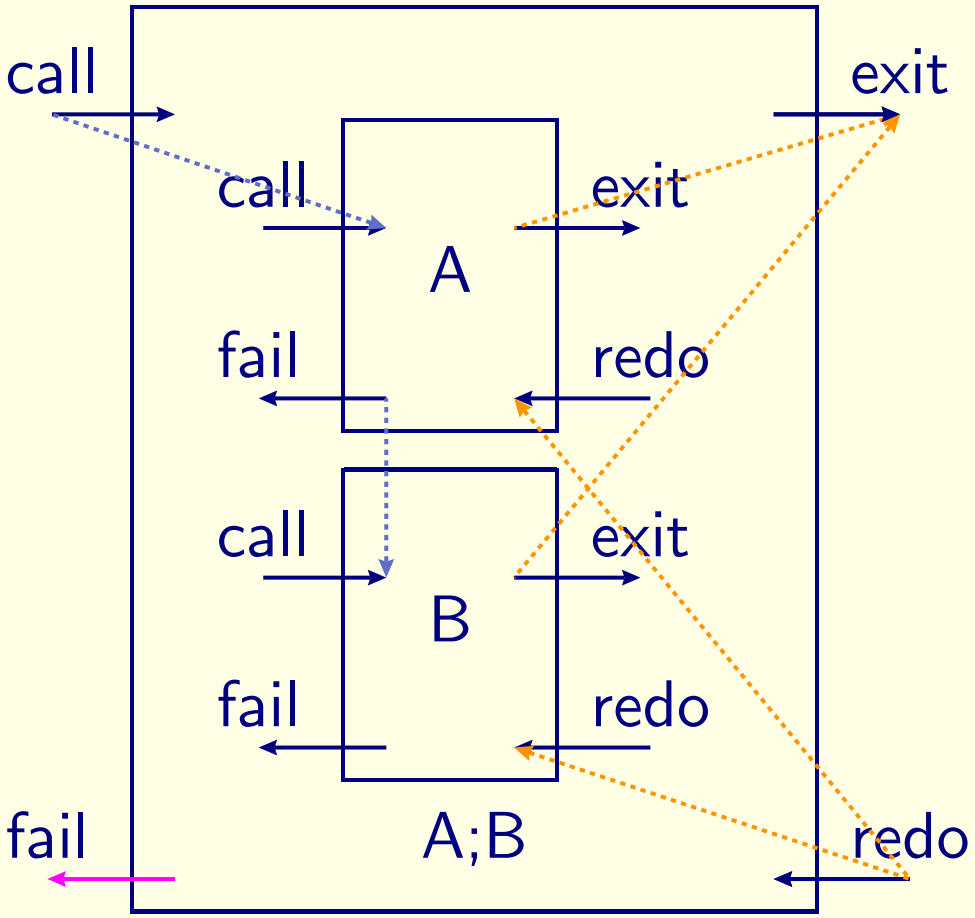
call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A
$^{\beta}$redo A;B	$\rightarrow$	redo B
fail A	$\rightarrow$	call B

Main idea (cont'd), Disjunction



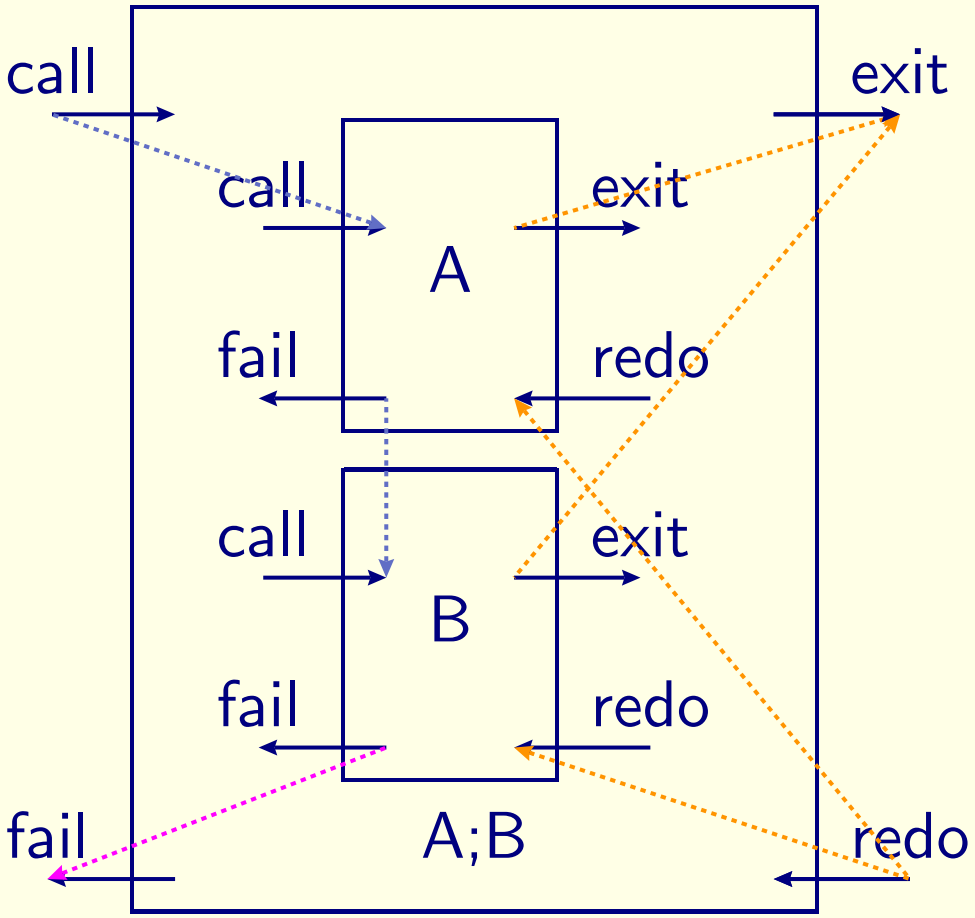
call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A
$^{\beta}$redo A;B	$\rightarrow$	redo B
fail A	$\rightarrow$	call B

Main idea (cont'd), Disjunction



call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A
$^{\beta}$redo A;B	$\rightarrow$	redo B
fail A	$\rightarrow$	call B

Main idea (cont'd), Disjunction

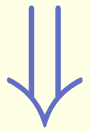


call A;B	$\rightarrow$	call A
exit A	$\rightarrow^{\alpha}$	exit A;B
exit B	$\rightarrow^{\beta}$	exit A;B
$^{\alpha}$redo A;B	$\rightarrow$	redo A
$^{\beta}$redo A;B	$\rightarrow$	redo B
fail A	$\rightarrow$	call B
fail B	$\rightarrow$	fail A;B

A canonical form of Prolog predicates

original program

```
q(a,b).
q(Z,c) :- r(Z).
r(c).
```



canonical representation

```
q(X,Y) :- X=a, Y=b, true; X=Z, Y=c, r(Z).
r(X) :- X=c, true.
```

Filling-in the context

```
main :- good, bad.
good.
```

Filling-in the context

```
main :- good, bad.
good :- true.
```

Filling-in the context

```
main :- good, bad.
good :- true.
```

call main →
  call (good, bad) →
    call good →
      call true →
      exit true →
    exit good →
    call bad →
    fail bad →
    redo good →
      redo true →
      fail true →
    fail good →
  fail (good, bad) →
fail main

Filling-in the context

```
main :- good, bad.
good :- true.
```

call main →
  call (good, bad) →
    call good →
      call true →
      exit true →
    exit good →
    call bad →
    fail bad →
    redo good →
      redo true →
      fail true →
    fail good →
  fail (good, bad) →
fail main

- context information needed: A-stack, stack of ancestors

Filling-in the context

```
main :- good, bad.
good :- true.
```

call main →
  call (good, bad) →
    call good →
      call true →
      exit true →
    exit good →
    call bad →
    fail bad →
    redo good →
      redo true →
      fail true →
    fail good →
  fail (good, bad) →
fail main

- context information needed: A-stack, stack of ancestors
- rest of information: B-stack (bindings for the variables etc.)

```
main :- good, bad.
good :- true.
```

$$call\ \mathsf{main}\,,\ \{nil\},\ \{nil\}$$

```
main :- good, bad.
good :- true.
```

$$call\ \mathsf{main}\ ,\ \{nil\},\ \{nil\}$$

$$\rightarrowtriangle\ \ call\ (\mathsf{good}, \mathsf{bad})\ ,\ \{\mathsf{main} \bullet nil\},\ \{nil\}$$

```
main :- good, bad.
good :- true.
```

$$call\ \mathsf{main}\ ,\ \{nil\},\ \{nil\}$$

$$\rightarrowtriangle\ call\ (\mathsf{good}, \mathsf{bad})\ ,\ \{\mathsf{main} \bullet nil\},\ \{nil\}$$

$$\rightarrowtriangle\ call\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\}$$

main :- good, bad.
good :- true.

$$
\begin{aligned}
& call\ \mathsf{main}\ ,\ \{nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ (\mathsf{good}, \mathsf{bad})\ ,\ \{\mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\}
\end{aligned}
$$

```
main :- good, bad.
good :- true.
```

$$
\begin{aligned}
& call\ \mathsf{main}\ ,\ \{nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ (\mathsf{good}, \mathsf{bad})\ ,\ \{\mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\}
\end{aligned}
$$

main :- good, bad.
good :- true.

$$
\begin{aligned}
& call\ \mathsf{main}\ , \{nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ (\mathsf{good}, \mathsf{bad})\ , \{\mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\}
\end{aligned}
$$

```
main :- good, bad.
good :- true.
```

$$
\begin{aligned}
& call\ \mathsf{main}\ ,\ \{nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ (\mathsf{good}, \mathsf{bad})\ ,\ \{\mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & call\ \mathsf{bad}\ ,\ \{2/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{BY(\mathsf{true}) \bullet nil\}
\end{aligned}
$$

main :- good, bad.
good :- true.

$$
\begin{aligned}
& \mathit{call}\ \mathsf{main}\ , \{\mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ (\mathsf{good}, \mathsf{bad})\ , \{\mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{good}\ , \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{true}\ , \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{exit}\ \mathsf{true}\ , \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{exit}\ \mathsf{good}\ , \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{BY}(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{bad}\ , \{\mathit{2}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{BY}(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{fail}\ \mathsf{bad}\ , \{\mathit{2}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{BY}(\mathsf{true}) \bullet \mathit{nil}\}
\end{aligned}
$$

```
main :- good, bad.
good :- true.
```

$$
\begin{aligned}
& call\ \mathsf{main}\ ,\ \{nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ (\mathsf{good}, \mathsf{bad})\ ,\ \{\mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & call\ \mathsf{bad}\ ,\ \{2/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{bad}\ ,\ \{2/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & redo\ \mathsf{good}\ ,\ \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\},\ \{BY(\mathsf{true}) \bullet nil\}
\end{aligned}
$$

main :- good, bad.
good :- true.

$$
\begin{aligned}
& \mathit{call}\ \mathsf{main}\ , \{\mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ (\mathsf{good}, \mathsf{bad})\ , \{\mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{good}\ , \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{true}\ , \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{exit}\ \mathsf{true}\ , \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{exit}\ \mathsf{good}\ , \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{BY}(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{bad}\ , \{\mathit{2}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{BY}(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{fail}\ \mathsf{bad}\ , \{\mathit{2}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{BY}(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{redo}\ \mathsf{good}\ , \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{BY}(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{redo}\ \mathsf{true}\ , \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\}, \{\mathit{nil}\}
\end{aligned}
$$

```
main :- good, bad.
good :- true.
```

$$
\begin{aligned}
& \mathit{call}\ \mathsf{main}\ ,\ \{\mathit{nil}\},\ \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ (\mathsf{good}, \mathsf{bad})\ ,\ \{\mathsf{main} \bullet \mathit{nil}\},\ \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{exit}\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{exit}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{BY(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{call}\ \mathsf{bad}\ ,\ \{\mathit{2}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{BY(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{fail}\ \mathsf{bad}\ ,\ \{\mathit{2}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{BY(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{redo}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{BY(\mathsf{true}) \bullet \mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{redo}\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{\mathit{nil}\} \\
\rightarrowtriangle\ & \mathit{fail}\ \mathsf{true}\ ,\ \{\mathsf{good} \bullet \mathit{1}/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet \mathit{nil}\},\ \{\mathit{nil}\}
\end{aligned}
$$

```
main :- good, bad.
good :- true.
```

$$
\begin{aligned}
& call\ \mathsf{main}\ , \{nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ (\mathsf{good}, \mathsf{bad})\ , \{\mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & call\ \mathsf{bad}\ , \{2/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{bad}\ , \{2/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & redo\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & redo\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\}
\end{aligned}
$$

main :- good, bad.
good :- true.

$call$ main , $\{nil\}$, $\{nil\}$

$\rightarrowtriangle$ $call$ (good, bad) , $\{\text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $call$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $call$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $exit$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $exit$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $call$ bad , $\{2/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $fail$ bad , $\{2/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $redo$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $redo$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $fail$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $fail$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $fail$ (good, bad) , $\{\text{main} \bullet nil\}$, $\{nil\}$

main :- good, bad.
good :- true.

$$
\begin{aligned}
& call\ \mathsf{main}\ , \{nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ (\mathsf{good}, \mathsf{bad})\ , \{\mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & call\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & exit\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & call\ \mathsf{bad}\ , \{2/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{bad}\ , \{2/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & redo\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{BY(\mathsf{true}) \bullet nil\} \\
\rightarrowtriangle\ & redo\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{true}\ , \{\mathsf{good} \bullet 1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{good}\ , \{1/\mathsf{good}, \mathsf{bad} \bullet \mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & fail\ (\mathsf{good}, \mathsf{bad})\ , \{\mathsf{main} \bullet nil\}, \{nil\} \\
\rightarrowtriangle\ & fail\ \mathsf{main}\ , \{nil\}, \{nil\}
\end{aligned}
$$

S_1:PP Language of events

Example $\mathit{fail}\ \mathsf{bad}\ \langle \frac{BY(\mathsf{true}) \bullet \mathit{nil}}{2/\mathsf{good,bad} \bullet \mathsf{main} \bullet \mathit{nil}} \rangle$

Grammar

event	::=	port goal $\langle \frac{\text{B-stack}}{\text{A-stack}} \rangle$
port	::=	$\mathit{call} \mid \mathit{exit} \mid \mathit{fail} \mid \mathit{redo}$
goal	::=	true | fail | atom | term=term | goal;goal | goal,goal
A-stack	::=	nil | ancestor • A-stack
ancestor	::=	atom | tag/goal;goal | tag/goal,goal
tag	::=	$1 \mid 2$
B-stack	::=	nil | bet • B-stack
bet	::=	mgu | memo
memo	::=	$BY(\text{goal}) \mid OR(\text{tag})$

S_1:PP Language of events

Example $fail$ bad $\langle \frac{BY(\text{true}) \bullet nil}{2/\text{good,bad} \bullet \text{main} \bullet nil} \rangle$

Grammar

event	::=	port goal $\langle \frac{\text{B-stack}}{\text{A-stack}} \rangle$
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S_1:PP Language of events

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S_1:PP Language of events

Example $fail$ bad $\langle \frac{BY(\text{true}) \bullet nil}{2/\text{good,bad} \bullet \text{main} \bullet nil} \rangle$

Grammar

event	::=	port goal $\langle \frac{\text{B-stack}}{\text{A-stack}} \rangle$
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A-stack	::=	nil | ancestor • A-stack
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S_1:PP Language of events

Example $fail$ bad $\langle \dfrac{BY(\text{true}) \bullet nil}{2/\text{good,bad} \bullet \text{main} \bullet nil} \rangle$

Grammar

event	::=	port goal $\langle \frac{\text{B-stack}}{\text{A-stack}} \rangle$
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S_1:PP Language of events

Example $fail$ bad $\langle \frac{BY(\text{true}) \bullet nil}{2/\text{good,bad} \bullet \text{main} \bullet nil} \rangle$

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S_1:PP Language of events

Example $fail$ bad $\langle \frac{BY(\text{true}) \bullet nil}{2/\text{good,bad} \bullet \text{main} \bullet nil} \rangle$

Grammar

event	::=	port goal $\langle \frac{\text{B-stack}}{\text{A-stack}} \rangle$
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B-stack	::=	nil | bet • B-stack
bet	::=	mgu | memo
memo	::=	BY(goal) | OR(tag)

S_1:PP Calculus in 20 rules

First of all, how to specify the built-in predicates true and fail?

True

Fail

S_1:PP Calculus in 20 rules

First of all, how to specify the built-in predicates true and fail?

True

$$call\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \rightarrowtriangle exit\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:true:1})$$

Fail

S_1:PP Calculus in 20 rules

First of all, how to specify the built-in predicates true and fail?

True

$$call\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ exit\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:true:1})$$

$$redo\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ fail\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:true:2})$$

Fail

S_1:PP Calculus in 20 rules

First of all, how to specify the built-in predicates true and fail?

True

$$call\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ exit\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:true:1})$$

$$redo\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ fail\ \text{true}\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:true:2})$$

Fail

$$call\ \text{fail}\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ fail\ \text{fail}\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:fail})$$

S_1:PP Calculus: **Explicit unification**

(S_1:unif:1)

(S_1:unif:2)

S_1:PP Calculus: **Explicit unification**

$$call\ T_1 = T_2\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrow \begin{cases} exit\ T_1 = T_2\ \langle \frac{\sigma \bullet \boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle, & \text{if } mgu\ldots^{\dagger} \\ fail\ T_1 = T_2\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle, & \text{otherwise} \end{cases} \qquad (S_1\text{:unif:1})$$

$(S_1\text{:unif:2})$

---

† if $mgu(\boldsymbol{\Sigma}(T_1), \boldsymbol{\Sigma}(T_2)) = \sigma$.

S_1:PP Calculus: **Explicit unification**

$$call\ T_1 = T_2\ \langle \frac{\Sigma}{U} \rangle \ \rightarrow\ \begin{cases} exit\ T_1 = T_2\ \langle \frac{\sigma \bullet \Sigma}{U} \rangle, & \text{if } mgu\ldots^{\dagger} \\ fail\ T_1 = T_2\ \langle \frac{\Sigma}{U} \rangle, & \text{otherwise} \end{cases} \qquad (S_1\text{:unif:1})$$

$$redo\ T_1 = T_2\ \langle \frac{\sigma \bullet \Sigma}{U} \rangle \ \rightarrow\ fail\ T_1 = T_2\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:unif:2})$$

---

† if $mgu(\Sigma(T_1), \Sigma(T_2)) = \sigma$.

S_1:PP Calculus: User-defined atom

(S_1:atom:1)

(S_1:atom:2)

(S_1:atom:3)

(S_1:atom:4)

S_1:PP Calculus: User-defined atom

$$call\ G_A\ \langle \frac{\Sigma}{U} \rangle \rightarrowtriangle \begin{cases} call\ \sigma(B)\ \langle \frac{\Sigma}{G_A \bullet U} \rangle, & \text{if } H{:-}B\ \ldots^{\ddagger} \\ fail\ G_A\ \langle \frac{\Sigma}{U} \rangle, & \text{otherwise} \end{cases} \qquad (S_1\text{:atom:1})$$

(S_1:atom:2)

(S_1:atom:3)

(S_1:atom:4)

[‡] if $H{:-}B$ is a fresh renaming of a clause in Π, and $\sigma = mgu(G'_A, H)$ with $G'_A := \Sigma(G_A)$ and $\sigma(G'_A) = G'_A$.

S_1:PP Calculus: User-defined atom

$$call\ G_A\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle \begin{cases} call\ \sigma(B)\ \langle \frac{\Sigma}{G_A \bullet U} \rangle, & \text{if } H{:-}B\ \ldots^{\ddagger} \\ fail\ G_A\ \langle \frac{\Sigma}{U} \rangle, & \text{otherwise} \end{cases} \qquad (S_1\text{:atom:1})$$

$$exit\ B\ \langle \frac{\Sigma}{G_A \bullet U} \rangle \ \rightarrowtriangle \ exit\ G_A\ \langle \frac{BY(B) \bullet \Sigma}{U} \rangle \qquad (S_1\text{:atom:2})$$

(S_1:atom:3)

(S_1:atom:4)

[‡] if $H{:-}B$ is a fresh renaming of a clause in Π, and $\sigma = mgu(G'_A, H)$ with $G'_A := \Sigma(G_A)$ and $\sigma(G'_A) = G'_A$.

S_1:PP Calculus: User-defined atom

$$call\ G_A\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle \begin{cases} call\ \sigma(B)\ \langle \frac{\Sigma}{G_A \bullet U} \rangle, & \text{if } H{:-}B\ \ldots^{\ddagger} \\ fail\ G_A\ \langle \frac{\Sigma}{U} \rangle, & \text{otherwise} \end{cases} \qquad (S_1\text{:atom:1})$$

$$exit\ B\ \langle \frac{\Sigma}{G_A \bullet U} \rangle \ \rightarrowtriangle \ exit\ G_A\ \langle \frac{BY(B) \bullet \Sigma}{U} \rangle \qquad (S_1\text{:atom:2})$$

$$fail\ B\ \langle \frac{\Sigma}{G_A \bullet U} \rangle \ \rightarrowtriangle \ fail\ G_A\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:atom:3})$$

$(S_1\text{:atom:4})$

[‡] if $H{:-}B$ is a fresh renaming of a clause in Π, and $\sigma = mgu(G'_A, H)$ with $G'_A := \Sigma(G_A)$ and $\sigma(G'_A) = G'_A$.

S_1:PP Calculus: User-defined atom

$$call\ G_A\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle \begin{cases} call\ \sigma(B)\ \langle \frac{\Sigma}{G_A \bullet U} \rangle, & \text{if } H{:-}B\ \ldots^{\ddagger} \\ fail\ G_A\ \langle \frac{\Sigma}{U} \rangle, & \text{otherwise} \end{cases} \qquad (S_1\text{:atom:1})$$

$$exit\ B\ \langle \frac{\Sigma}{G_A \bullet U} \rangle \ \rightarrowtriangle \ exit\ G_A\ \langle \frac{BY(B) \bullet \Sigma}{U} \rangle \qquad (S_1\text{:atom:2})$$

$$fail\ B\ \langle \frac{\Sigma}{G_A \bullet U} \rangle \ \rightarrowtriangle \ fail\ G_A\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:atom:3})$$

$$redo\ G_A\ \langle \frac{BY(B) \bullet \Sigma}{U} \rangle \ \rightarrowtriangle \ redo\ B\ \langle \frac{\Sigma}{G_A \bullet U} \rangle \qquad (S_1\text{:atom:4})$$

---

‡ if $H{:-}B$ is a fresh renaming of a clause in Π, and $\sigma = mgu(G'_A, H)$ with $G'_A := \Sigma(G_A)$ and $\sigma(G'_A) = G'_A$.

S_1:PP Calculus: **Conjunction**

(S_1:conj:1)

(S_1:conj:2)

(S_1:conj:3)

(S_1:conj:4)

(S_1:conj:5)

(S_1:conj:6)

S_1:PP Calculus: **Conjunction**

$$call\ A, B\ \langle \frac{\Sigma}{U} \rangle \rightarrowtriangle call\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \qquad (S_1\text{:conj:1})$$

(S_1:conj:2)

(S_1:conj:3)

(S_1:conj:4)

(S_1:conj:5)

(S_1:conj:6)

S_1:PP Calculus: **Conjunction**

$$call\ A, B\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ call\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \qquad (S_1\text{:conj:1})$$

$$exit\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \ \rightarrowtriangle\ call\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \qquad (S_1\text{:conj:2})$$

(S_1:conj:3)

(S_1:conj:4)

(S_1:conj:5)

(S_1:conj:6)

S_1:PP Calculus: **Conjunction**

$$call\ A, B\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ call\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \qquad (S_1\text{:conj:1})$$

$$exit\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \ \rightarrowtriangle\ call\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \qquad (S_1\text{:conj:2})$$

$$fail\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \ \rightarrowtriangle\ fail\ A, B\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:conj:3})$$

(S$_1$:conj:4)

(S$_1$:conj:5)

(S$_1$:conj:6)

S_1:PP Calculus: **Conjunction**

$$call\ A, B\ \langle \frac{\Sigma}{U} \rangle \rightarrowtriangle call\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \quad (S_1\text{:conj:1})$$

$$exit\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \rightarrowtriangle call\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \quad (S_1\text{:conj:2})$$

$$fail\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \rightarrowtriangle fail\ A, B\ \langle \frac{\Sigma}{U} \rangle \quad (S_1\text{:conj:3})$$

$$exit\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \rightarrowtriangle exit\ A, B\ \langle \frac{\Sigma}{U} \rangle \quad (S_1\text{:conj:4})$$

(S_1:conj:5)

(S_1:conj:6)

S_1:PP Calculus: **Conjunction**

$$call\ A, B\ \langle \frac{\Sigma}{U} \rangle \rightarrow call\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \quad (S_1\text{:conj:1})$$

$$exit\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \rightarrow call\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \quad (S_1\text{:conj:2})$$

$$fail\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \rightarrow fail\ A, B\ \langle \frac{\Sigma}{U} \rangle \quad (S_1\text{:conj:3})$$

$$exit\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \rightarrow exit\ A, B\ \langle \frac{\Sigma}{U} \rangle \quad (S_1\text{:conj:4})$$

$$fail\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \rightarrow redo\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \quad (S_1\text{:conj:5})$$

(S$_1$:conj:6)

S_1:PP Calculus: **Conjunction**

$$call\ A, B\ \langle \frac{\Sigma}{U} \rangle \rightarrow call\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \quad (S_1\text{:conj:1})$$

$$exit\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \rightarrow call\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \quad (S_1\text{:conj:2})$$

$$fail\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \rightarrow fail\ A, B\ \langle \frac{\Sigma}{U} \rangle \quad (S_1\text{:conj:3})$$

$$exit\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \rightarrow exit\ A, B\ \langle \frac{\Sigma}{U} \rangle \quad (S_1\text{:conj:4})$$

$$fail\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \rightarrow redo\ A\ \langle \frac{\Sigma}{1/A,B \bullet U} \rangle \quad (S_1\text{:conj:5})$$

$$redo\ A, B\ \langle \frac{\Sigma}{U} \rangle \rightarrow redo\ B\ \langle \frac{\Sigma}{2/A,B \bullet U} \rangle \quad (S_1\text{:conj:6})$$

S_1:PP Calculus: **Disjunction**

(S_1:disj:1)

(S_1:disj:2)

(S_1:disj:3)

(S_1:disj:4)

(S_1:disj:5)

S_1:PP Calculus: **Disjunction**

$$call\ A; B\ \langle \frac{\Sigma}{U} \rangle \rightarrowtriangle call\ A\ \langle \frac{\Sigma}{1/A;B \bullet U} \rangle \qquad (S_1\text{:disj:1})$$

(S_1:disj:2)

(S_1:disj:3)

(S_1:disj:4)

(S_1:disj:5)

S_1:PP Calculus: **Disjunction**

$$call\ A; B\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ call\ A\ \langle \frac{\Sigma}{1/A;B \bullet U} \rangle \qquad (S_1\text{:disj:1})$$

$$fail\ A\ \langle \frac{\Sigma}{1/A;B \bullet U} \rangle \ \rightarrowtriangle\ call\ B\ \langle \frac{\Sigma}{2/A;B \bullet U} \rangle \qquad (S_1\text{:disj:2})$$

(S_1:disj:3)

(S_1:disj:4)

(S_1:disj:5)

S_1:PP Calculus: **Disjunction**

$$call\ A; B\ \langle \frac{\Sigma}{U} \rangle \ \rightarrowtriangle\ call\ A\ \langle \frac{\Sigma}{1/A;B \bullet U} \rangle \qquad (S_1\text{:disj:1})$$

$$fail\ A\ \langle \frac{\Sigma}{1/A;B \bullet U} \rangle \ \rightarrowtriangle\ call\ B\ \langle \frac{\Sigma}{2/A;B \bullet U} \rangle \qquad (S_1\text{:disj:2})$$

$$fail\ B\ \langle \frac{\Sigma}{2/A;B \bullet U} \rangle \ \rightarrowtriangle\ fail\ A; B\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:disj:3})$$

(S_1:disj:4)

(S_1:disj:5)

S_1:PP Calculus: **Disjunction**

$$call\ A; B\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrowtriangle call\ A\ \langle \frac{\boldsymbol{\Sigma}}{1/A;B \bullet \boldsymbol{U}} \rangle \quad (S_1\text{:disj:1})$$

$$fail\ A\ \langle \frac{\boldsymbol{\Sigma}}{1/A;B \bullet \boldsymbol{U}} \rangle \rightarrowtriangle call\ B\ \langle \frac{\boldsymbol{\Sigma}}{2/A;B \bullet \boldsymbol{U}} \rangle \quad (S_1\text{:disj:2})$$

$$fail\ B\ \langle \frac{\boldsymbol{\Sigma}}{2/A;B \bullet \boldsymbol{U}} \rangle \rightarrowtriangle fail\ A; B\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \quad (S_1\text{:disj:3})$$

$$exit\ C\ \langle \frac{\boldsymbol{\Sigma}}{N/A;B \bullet \boldsymbol{U}} \rangle \rightarrowtriangle exit\ A; B\ \langle \frac{OR(N) \bullet \boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle, \text{ with } C = \lfloor N/A; B \rfloor \quad (S_1\text{:disj:4})$$

$(S_1\text{:disj:5})$

S_1:PP Calculus: **Disjunction**

$$call\ A; B\ \langle \frac{\Sigma}{U} \rangle \rightarrowtriangle call\ A\ \langle \frac{\Sigma}{1/A;B \bullet U} \rangle \qquad (S_1\text{:disj:1})$$

$$fail\ A\ \langle \frac{\Sigma}{1/A;B \bullet U} \rangle \rightarrowtriangle call\ B\ \langle \frac{\Sigma}{2/A;B \bullet U} \rangle \qquad (S_1\text{:disj:2})$$

$$fail\ B\ \langle \frac{\Sigma}{2/A;B \bullet U} \rangle \rightarrowtriangle fail\ A; B\ \langle \frac{\Sigma}{U} \rangle \qquad (S_1\text{:disj:3})$$

$$exit\ C\ \langle \frac{\Sigma}{N/A;B \bullet U} \rangle \rightarrowtriangle exit\ A; B\ \langle \frac{OR(N) \bullet \Sigma}{U} \rangle, \text{ with } C = \lfloor N/A; B \rfloor \qquad (S_1\text{:disj:4})$$

$$redo\ A; B\ \langle \frac{OR(N) \bullet \Sigma}{U} \rangle \rightarrowtriangle redo\ C\ \langle \frac{\Sigma}{N/A;B \bullet U} \rangle, \text{ with } C = \lfloor N/A; B \rfloor \qquad (S_1\text{:disj:5})$$

Legal event and derivation

- Initial event: $call\ Q\ \langle \frac{nil}{nil} \rangle$, where Q is any goal formula.

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- Legal event: reachable from an initial event.

Legal event and derivation

- Initial event: $call\ Q\ \langle \frac{nil}{nil} \rangle$, where Q is any goal formula.
- Legal event: reachable from an initial event.
- Final event: a legal event that does not lead to another event.

$$call\ Q\ \langle \tfrac{nil}{nil} \rangle \quad \rightarrow\!\!\triangleright^* \quad E_0 \ \rightarrow\!\!\triangleright^* \ E$$

Legal event and derivation

- Initial event: $call\ Q\ \langle \frac{nil}{nil} \rangle$, where Q is any goal formula.
- Legal event: reachable from an initial event.
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Legal event and derivation

- Initial event: $call\ Q\ \langle \frac{nil}{nil} \rangle$, where Q is any goal formula.
- Legal event: reachable from an initial event.
- Final event: a legal event that does not lead to another event.

$$call\ Q\ \langle \tfrac{nil}{nil} \rangle \quad \rightarrow^* \quad \underbrace{E_0 \rightarrow^* E}_{\text{legal derivation}} \qquad E: \text{legal event}$$

In S_1:PP, a legal event can have only one legal predecessor, and only one successor (Theorem 4.2 in [Kul04]).

$$
\begin{aligned}
&\mathit{call}\ \mathsf{main}\ ,\ \{\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{call}\ (\mathsf{good},\mathsf{bad})\ ,\ \{\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{call}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{call}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{exit}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{exit}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{call}\ \mathsf{bad}\ ,\ \{\mathit{2}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{fail}\ \mathsf{bad}\ ,\ \{\mathit{2}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{redo}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{redo}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{fail}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{fail}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{fail}\ (\mathsf{good},\mathsf{bad})\ ,\ \{\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\quad\twoheadrightarrow\ \mathit{fail}\ \mathsf{main}\ ,\ \{\mathit{nil}\},\ \{\mathit{nil}\}
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{call}\ \mathsf{main}\ ,\ \{\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{call}\ (\mathsf{good},\mathsf{bad})\ ,\ \{\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{call}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{call}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{exit}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{exit}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{call}\ \mathsf{bad}\ ,\ \{\mathit{2}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{fail}\ \mathsf{bad}\ ,\ \{\mathit{2}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{redo}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{BY}(\mathsf{true})\bullet\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{redo}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{fail}\ \mathsf{true}\ ,\ \{\mathsf{good}\bullet\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{fail}\ \mathsf{good}\ ,\ \{\mathit{1}/\mathsf{good},\mathsf{bad}\bullet\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{fail}\ (\mathsf{good},\mathsf{bad})\ ,\ \{\mathsf{main}\bullet\mathit{nil}\},\ \{\mathit{nil}\}\\
&\rightarrowtriangle\ \mathit{fail}\ \mathsf{main}\ ,\ \{\mathit{nil}\},\ \{\mathit{nil}\}
\end{aligned}
$$

$call$ main , $\{nil\}$, $\{nil\}$

$\rightarrowtriangle$ $call$ (good, bad) , $\{\text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $call$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $call$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $exit$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $exit$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $call$ bad , $\{2/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $fail$ bad , $\{2/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $redo$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{BY(\text{true}) \bullet nil\}$

$\rightarrowtriangle$ $redo$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $fail$ true , $\{\text{good} \bullet 1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $fail$ good , $\{1/\text{good}, \text{bad} \bullet \text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $fail$ (good, bad) , $\{\text{main} \bullet nil\}$, $\{nil\}$

$\rightarrowtriangle$ $fail$ main , $\{nil\}$, $\{nil\}$

Aggregations: Simple pass

- forward derivation relative to U: $Push\ G\ \langle \frac{\Sigma}{U} \rangle \underbrace{\rightarrow \rightarrow \ldots \rightarrow}_{\text{no } Pop\_\langle \frac{\bar{\Sigma}}{U} \rangle} E$

Aggregations: Simple pass

- forward derivation relative to $\boldsymbol{U}$: $Push\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \overset{\triangleright}{\longrightarrow} E$

Aggregations: Simple pass

- forward derivation relative to $\boldsymbol{U}$: $Push\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \overset{\triangleright}{\longrightarrow} E$

- backward derivation relative to $\boldsymbol{U}$: $Pop\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \underbrace{\longrightarrow \longrightarrow \ldots \longrightarrow}_{\text{no } Push\ \_\ \langle \frac{=}{\boldsymbol{U}} \rangle} E$

Aggregations: Simple pass

- forward derivation relative to $\boldsymbol{U}$: $Push\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \xrightarrow{\triangleright} E$
- backward derivation relative to $\boldsymbol{U}$: $Pop\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \xrightarrow{\triangleleft} E$

Aggregations: Simple pass

- forward derivation relative to $\boldsymbol{U}$: $Push\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \xrightarrow{\triangleright} E$
- backward derivation relative to $\boldsymbol{U}$: $Pop\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \xrightarrow{\triangleleft} E$
- simple pass relative to G,$\boldsymbol{U}$: $Push\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle \xrightarrow{\triangleright} Pop\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle$
  $Pop\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle \xrightarrow{\triangleleft} Push\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle$

Aggregations: Simple pass

- forward derivation relative to $\boldsymbol{U}$: $Push\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \overset{\triangleright}{\longrightarrow} E$

- backward derivation relative to $\boldsymbol{U}$: $Pop\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \overset{\triangleleft}{\longrightarrow} E$

- simple pass relative to G,$\boldsymbol{U}$:

$$\begin{array}{lcl} Push\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle & \longrightarrow & Pop\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle \\ Pop\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle & \longrightarrow & Push\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle \\ \_\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle & \longrightarrow & \_\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle \\ \text{stage} & & \text{stage} \end{array}$$

Aggregations: Composed pass

Based on:

- If $Pop\ G\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $Pop\ G\ \langle \frac{\Sigma}{U} \rangle \longleftarrow Push\ G\ \langle \frac{\Sigma^{\circ}}{U} \rangle$. (Lemma 5.6)
- If $fail\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $fail\ H\ \langle \frac{\Sigma}{U} \rangle \Leftarrow^{*} call\ H\ \langle \frac{\Sigma}{U} \rangle$, where $call\ H\ \langle \frac{=}{U} \rangle$ does not appear within the derivation. Furthermore, if $redo\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $redo\ H\ \langle \frac{\Sigma}{U} \rangle \longleftarrow exit\ H\ \langle \frac{\Sigma}{U} \rangle$. (Theorem 6.1)

Aggregations: Composed pass

Based on:

- If $Pop\ G\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $Pop\ G\ \langle \frac{\Sigma}{U} \rangle \longleftarrow Push\ G\ \langle \frac{\Sigma^{\circ}}{U} \rangle$. (Lemma 5.6)
- If $fail\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $fail\ H\ \langle \frac{\Sigma}{U} \rangle \Leftarrow^{*} call\ H\ \langle \frac{\Sigma}{U} \rangle$, where $call\ H\ \langle \frac{=}{U} \rangle$ does not appear within the derivation. Furthermore, if $redo\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $redo\ H\ \langle \frac{\Sigma}{U} \rangle \longleftarrow exit\ H\ \langle \frac{\Sigma}{U} \rangle$. (Theorem 6.1)

$redo\ G\ \langle \frac{=}{U} \rangle$

Aggregations: Composed pass

Based on:

- If $Pop\ G\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $Pop\ G\ \langle \frac{\Sigma}{U} \rangle \longleftarrow Push\ G\ \langle \frac{\Sigma^{\circ}}{U} \rangle$. (Lemma 5.6)
- If $fail\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $fail\ H\ \langle \frac{\Sigma}{U} \rangle \Leftarrow^{*} call\ H\ \langle \frac{\Sigma}{U} \rangle$, where $call\ H\ \langle \frac{=}{U} \rangle$ does not appear within the derivation. Furthermore, if $redo\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $redo\ H\ \langle \frac{\Sigma}{U} \rangle \longleftarrow exit\ H\ \langle \frac{\Sigma}{U} \rangle$. (Theorem 6.1)

$redo\ G\ \langle \frac{=}{U} \rangle \longleftarrow exit\ G\ \langle \frac{=}{U} \rangle$

Aggregations: Composed pass

Based on:

- If $Pop\ G\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $Pop\ G\ \langle \frac{\Sigma}{U} \rangle \longleftarrow Push\ G\ \langle \frac{\Sigma^{\circ}}{U} \rangle$. (Lemma 5.6)
- If $fail\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $fail\ H\ \langle \frac{\Sigma}{U} \rangle \Leftarrow^{*} call\ H\ \langle \frac{\Sigma}{U} \rangle$, where $call\ H\ \langle \frac{=}{U} \rangle$ does not appear within the derivation. Furthermore, if $redo\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $redo\ H\ \langle \frac{\Sigma}{U} \rangle \longleftarrow exit\ H\ \langle \frac{\Sigma}{U} \rangle$. (Theorem 6.1)

$redo\ G\ \langle \frac{=}{U} \rangle \longleftarrow exit\ G\ \langle \frac{=}{U} \rangle \longleftarrow Push\ G\ \langle \frac{=}{U} \rangle$

Aggregations: Composed pass

Based on:

- If $Pop\ G\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $Pop\ G\ \langle \frac{\Sigma}{U} \rangle \longleftarrow Push\ G\ \langle \frac{\Sigma^{\circ}}{U} \rangle$. (Lemma 5.6)
- If $fail\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $fail\ H\ \langle \frac{\Sigma}{U} \rangle \Leftarrow^{*} call\ H\ \langle \frac{\Sigma}{U} \rangle$, where $call\ H\ \langle \frac{=}{U} \rangle$ does not appear within the derivation. Furthermore, if $redo\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $redo\ H\ \langle \frac{\Sigma}{U} \rangle \longleftarrow exit\ H\ \langle \frac{\Sigma}{U} \rangle$. (Theorem 6.1)

$$redo\ G\ \langle \tfrac{=}{U} \rangle \longleftarrow exit\ G\ \langle \tfrac{=}{U} \rangle \longleftarrow Push\ G\ \langle \tfrac{=}{U} \rangle \longleftarrow \ldots \longleftarrow call\ G\ \langle \tfrac{=}{U} \rangle$$

Aggregations: Composed pass

Based on:

- If $Pop\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$ is legal, then $Pop\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \longleftarrow Push\ G\ \langle \frac{\boldsymbol{\Sigma}^\circ}{\boldsymbol{U}} \rangle$. (Lemma 5.6)
- If $fail\ H\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$ is legal, then $fail\ H\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \leftarrow^* call\ H\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$, where $call\ H\ \langle \frac{=}{\boldsymbol{U}} \rangle$ does not appear within the derivation. Furthermore, if $redo\ H\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$ is legal, then $redo\ H\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \longleftarrow exit\ H\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$. (Theorem 6.1)

$$redo\ G\ \langle \tfrac{=}{\boldsymbol{U}} \rangle \longleftarrow exit\ G\ \langle \tfrac{=}{\boldsymbol{U}} \rangle \longleftarrow Push\ G\ \langle \tfrac{=}{\boldsymbol{U}} \rangle \longleftarrow \ldots \longleftarrow call\ G\ \langle \tfrac{=}{\boldsymbol{U}} \rangle$$

Definition of composed pass relative to $G,\boldsymbol{U}$:

$$E_1^{G,\boldsymbol{U}} \quad \underbrace{\longrightarrow\longrightarrow \ldots \longrightarrow}_{\text{no } call\ G\ \langle \frac{=}{\boldsymbol{U}} \rangle} \quad E_2^{G,\boldsymbol{U}}$$

also called composable sequence

Aggregations: Composed pass

Based on:

- If $Pop\ G\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $Pop\ G\ \langle \frac{\Sigma}{U} \rangle \longleftarrow Push\ G\ \langle \frac{\Sigma^{\circ}}{U} \rangle$. (Lemma 5.6)
- If $fail\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $fail\ H\ \langle \frac{\Sigma}{U} \rangle \Leftarrow^{*} call\ H\ \langle \frac{\Sigma}{U} \rangle$, where $call\ H\ \langle \frac{=}{U} \rangle$ does not appear within the derivation. Furthermore, if $redo\ H\ \langle \frac{\Sigma}{U} \rangle$ is legal, then $redo\ H\ \langle \frac{\Sigma}{U} \rangle \longleftarrow exit\ H\ \langle \frac{\Sigma}{U} \rangle$. (Theorem 6.1)

$$redo\ G\ \langle \tfrac{=}{U} \rangle \longleftarrow exit\ G\ \langle \tfrac{=}{U} \rangle \longleftarrow Push\ G\ \langle \tfrac{=}{U} \rangle \longleftarrow \ldots \longleftarrow call\ G\ \langle \tfrac{=}{U} \rangle$$

Definition of composed pass relative to G,U:

$$E_1^{G,U} \Longrightarrow E_2^{G,U}$$

Some properties of S_1:PP

Uniqueness of steps: transition relation, as well as its converse on legal events, are functional (Theorem 4.2)

$\longrightarrow$ forward and backward derivation steps are possible

For application, see the sketches on non-interference and modularity of conjunction.

Independence on the context of derivation: the same goal goes through the same stages (Theorem 7.6)

Theorem 7.6 (independence) *Let the following sequences be composable, where* $m, k \geq 1$, *and let* $\boldsymbol{\Theta}(H) = \boldsymbol{\Sigma}(G)$.

$$call\ G\ \langle \tfrac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \longrightarrow E_1^{G,\boldsymbol{U}} \longrightarrow \ldots \longrightarrow E_m^{G,\boldsymbol{U}}$$
$$call\ H\ \langle \tfrac{\boldsymbol{\Theta}}{\boldsymbol{V}} \rangle \longrightarrow E_1^{H,\boldsymbol{V}} \longrightarrow \ldots \longrightarrow E_k^{H,\boldsymbol{V}}$$

Then for any i *in common* ($i \leq m, k$) *holds:*

$$\textit{If } E_i^{G,\boldsymbol{U}} := \Gamma\ G\ \langle \tfrac{\boldsymbol{\Sigma}'}{\boldsymbol{U}} \rangle \ \textit{ then } \ E_i^{H,\boldsymbol{V}} = \Gamma\ H\ \langle \tfrac{\boldsymbol{\Sigma}' - \boldsymbol{\Sigma} + \boldsymbol{\Theta}}{\boldsymbol{V}} \rangle.$$

A legal derivation is modular, due to compositionality of rules wrt conjunction and disjunction, and stages (Theorem 7.2, Theorem 7.3)

$\longrightarrow$ abstracting parts of a derivation is possible

Theorem 7.2 ($\oplus$) *If* $call\ G\ \langle \frac{nil}{nil} \rangle \rightarrowtriangle E_1 \rightarrowtriangle \ldots \rightarrowtriangle E_n \rightarrowtriangle Pop\ G\ \langle \frac{\boldsymbol{\Delta}}{nil} \rangle$, *then for every* G°, $\boldsymbol{U}$, $\boldsymbol{\Sigma}$ *such that* $call\ G^\circ\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$ *is legal and* $\boldsymbol{\Sigma}(G^\circ) = G$ *holds:*

$$call\ G^\circ\ \langle \tfrac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrowtriangle E_1{}^{\S} \oplus \langle \tfrac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrowtriangle \ldots \rightarrowtriangle E_n{}^{\S} \oplus \langle \tfrac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrowtriangle Pop\ G^\circ\ \langle \tfrac{\boldsymbol{\Delta} + \boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$$

is also a legal derivation.

Theorem 7.3 ($\ominus$) *Let* $call\ G\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrowtriangle E_1 \rightarrowtriangle \ldots \rightarrowtriangle E_n \rightarrowtriangle Pop\ G\ \langle \frac{\boldsymbol{\Omega}}{\boldsymbol{U}} \rangle$ *be legal, and* $E_i \neq Pop\ \_\ \langle \frac{=}{\boldsymbol{U}} \rangle$ *for every* i. *Then for* $G' := \boldsymbol{\Sigma}(G)$ *holds:*

$$call\ G'\ \langle \tfrac{nil}{nil} \rangle \rightarrowtriangle E'_1 \ominus \langle \tfrac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrowtriangle \ldots \rightarrowtriangle E'_n \ominus \langle \tfrac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle \rightarrowtriangle Pop\ G'\ \langle \tfrac{\boldsymbol{\Omega} - \boldsymbol{\Sigma}}{nil} \rangle$$

is also a legal derivation. Here if $E_i = \Gamma\ H\ \langle \frac{\boldsymbol{\Theta}}{\boldsymbol{V}} \rangle$, *then* $E'_i := \Gamma\ H'\ \langle \frac{\boldsymbol{\Theta}}{\boldsymbol{V}'} \rangle$, *where* $\boldsymbol{\Sigma}(H) = H'$ *and* $\boldsymbol{\Sigma}(\boldsymbol{V}) = \boldsymbol{V}'$.

---

§ possibly the fresh variables of E_i have to be renamed

Modeling Prolog execution in S_1:PP

Theorem 8.1 (existential termination, success, failure) *Prolog computation of a goal G relative to Π terminates existentially if there is a simple pass*

$$call\ G\ \langle \tfrac{nil}{nil} \rangle \longrightarrow Pop\ G\ \langle \tfrac{\mathbf{\Delta}}{nil} \rangle$$

In case $Pop = exit$, the computation is successful, otherwise it is failed.

Theorem 8.2 (the first computed answer) *If $call\ G\ \langle \tfrac{nil}{nil} \rangle \longrightarrow exit\ G\ \langle \tfrac{\mathbf{\Delta}}{nil} \rangle$, then $subst(\mathbf{\Delta})\mid_G$ is the first computed answer substitution for G relative to Π in Prolog.*

Theorem 8.3 (all computed answers, universal termination) *In a composable sequence*

$$call\ G\ \langle \tfrac{nil}{1/G,\mathsf{fail} \bullet nil} \rangle \longrightarrow^{2k-1} exit\ G\ \langle \tfrac{\mathbf{\Delta}}{1/G,\mathsf{fail} \bullet nil} \rangle$$

is $subst(\mathbf{\Delta})\mid_G$ the kth computed answer substitution for G relative to Π in Prolog. Furthermore, G is universally terminating relative to Π if

$$call\ G\ \langle \tfrac{nil}{1/G,\mathsf{fail} \bullet nil} \rangle \Longrightarrow fail\ G\ \langle \tfrac{nil}{1/G,\mathsf{fail} \bullet nil} \rangle$$

Vanilla meta-interpreter

Let $exit\ A, B\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$ be a legal event. How could it have been derived? There is only one way:

$exit\ A, B\ \langle \frac{\boldsymbol{\Sigma}}{\boldsymbol{U}} \rangle$

$\Leftarrow$ $exit\ B\ \langle \frac{\boldsymbol{\Sigma}}{2/A,B \bullet \boldsymbol{U}} \rangle$, by ($S_1$:conj:4)

$\Longleftarrow$ $call\ B\ \langle \frac{\boldsymbol{\Sigma}^\circ}{2/A,B \bullet \boldsymbol{U}} \rangle$, by Theorem 6.5(2), with $\boldsymbol{\Sigma} \succeq \boldsymbol{\Sigma}^\circ$

$\Leftarrow$ $exit\ A\ \langle \frac{\boldsymbol{\Sigma}^\circ}{1/A,B \bullet \boldsymbol{U}} \rangle$, by ($S_1$:conj:2)

$\Longleftarrow$ $call\ A\ \langle \frac{\boldsymbol{\Sigma}^{\circ\circ}}{1/A,B \bullet \boldsymbol{U}} \rangle$, by Theorem 6.5(2), with $\boldsymbol{\Sigma}^\circ \succeq \boldsymbol{\Sigma}^{\circ\circ}$

$\Leftarrow$ $call\ A, B\ \langle \frac{\boldsymbol{\Sigma}^{\circ\circ}}{\boldsymbol{U}} \rangle$, by ($S_1$:conj:1)

The converse (starting from $exit\ A$ and $exit\ B$) yields itself as well. Similarly for disjunction.

Summary of S_1:PP operational semantics

- focus on general goals[¶] instead of selected atoms
- forward and backward derivation steps treated equally
- modularity of derivation: parts of execution can be abstracted
- lean data structure for "state of computation":
  - one ancestor stack, A-stack
  - one (generalized) environment, B-stack

[¶] there are precursors in advocating the idea of general goals: [CvEHL92], [TB93]

Back to motivation: Capturing non-interference

Push events (call, redo) are more amenable to forward steps, and pop events (exit, fail) are more amenable to backward steps.

Example: Assume run-time test Cc succeeds exactly once.

$$
\begin{array}{l}
call\ Cc, G\ \langle \frac{=}{U} \rangle \\
\quad \rightarrowtriangle \quad call\ Cc\ \langle \frac{=}{Cc, G \bullet U} \rangle \\
\quad \longrightarrow \quad exit\ Cc\ \langle \frac{=}{Cc, G \bullet U} \rangle \\
\quad \rightarrowtriangle \quad call\ G\ \langle \frac{=}{Cc, G \bullet U} \rangle
\end{array}
\qquad
\begin{array}{l}
fail\ Cc, G\ \langle \frac{=}{U} \rangle \\
\quad \lefttriangle \quad fail\ Cc\ \langle \frac{=}{Cc, G \bullet U} \rangle \\
\quad \longleftarrow \quad redo\ Cc\ \langle \frac{=}{Cc, G \bullet U} \rangle \\
\quad \lefttriangle \quad fail\ G\ \langle \frac{=}{Cc, G \bullet U} \rangle
\end{array}
$$

$$
\begin{array}{l}
exit\ Cc, G\ \langle \frac{=}{U} \rangle \\
\quad \lefttriangle \quad exit\ G\ \langle \frac{=}{Cc, G \bullet U} \rangle
\end{array}
\qquad
\begin{array}{l}
redo\ Cc, G\ \langle \frac{=}{U} \rangle \\
\quad \rightarrowtriangle \quad redo\ G\ \langle \frac{=}{Cc, G \bullet U} \rangle
\end{array}
$$

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