# Endogenous Uncertainty and Optimal Monetary Policy

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**Abstract**

In the so-called new neoclassical synthesis, expected future output and inflation enter the model equations for demand and supply, since households and firms take into account forecasts of endogenous variables in their consumption and pricing decisions. However, the linear model equations of the new neoclassical synthesis reflect the behavior of risk-neutral economic subjects and firms, since market participants are indifferent to deviations of actual values of endogenous variables from previously predicted levels, which is in contrast to the risk-aversion reflected in the utility functions used in the underlying microfoundation. In this paper, we show that risk-averse economic subjects and firms form rational expectations not only regarding the expected values but also regarding uncertainty in future variables, impacting the model equations for demand and supply and the conduct of monetary policy (JEL D81, E10, E52).

## I Introduction

The so-called new neoclassical synthesis has become a major issue of research in the area of monetary policy since the late 1990s (for an overview cf Walsh, 2003; Woodford, 2003) and has established itself as a standard model in macroeconomics literature. The model equations of the new neoclassical synthesis are based on a microfoundation, i.e. they are derived from models describing the behavior of economic subjects and firms. The common approach for deriving the so-called forward-looking IS curve that forms part of the new model is to consider the utility function of a representative economic subject, that depends on various endogenous variables, i.e. consumption, real money balances, etc. By maximizing the present value of the household's future utility under the household's inter-temporal budget constraints, a path is obtained for the corresponding endogenous variables chosen by a rational economic subject. The concept of the utility function of an economic subject is not only the basis for the derivation of a forward-looking IS curve, but can also be used to obtain a microfounded economic loss function that is used to assess the appropriateness of different monetary policy regimes from a welfare point of view (cf Woodford, 2003).

Analogously, the pricing behavior of firms is modeled assuming forward-looking and profit maximizing firms under partly inflexible prices. The so-called Calvo model yields a forward-looking Phillips curve (also referred to as inflation adjustment curve, the IA curve) containing expected future inflation.

The model equations for demand (IS curve) and supply (IA curve) are typically presented in the following form using the output gap $x_t = y_t - \bar{y}_t$ (denoting the difference between actual output $y_t$ and natural output $\bar{y}_t$), inflation rate $\pi_t$ and the real interest rate $r_t$ as endogenous variables:

$$\text{IS curve: } x_t = E_t x_{t+1} - a_1(r_t - \bar{r}_t) \quad (1)$$

$$\text{IA curve } \pi_t = \beta E_t \pi_{t+1} + \varphi x_t + u_t \quad (2)$$

with the natural interest rate $\bar{r}_t$ and positive coefficients $a_1$ and $\varphi$, the discount factor $\beta \leq 1$ and inflation shocks $u_t$. To conclude, the model equations of the new neoclassical synthesis for demand and supply contain expectations of future endogenous variables.

In recent years, the inclusion of uncertainty into monetary policy research has become a focus of interest in the area of the new neoclassical theory. Various authors analyze a monetary authority with model uncertainty in their framework, i.e. they assume that the "real" model for the economy can differ from the model perceived and used by the central bank. Further, the central bank's model error is assumed to be exogenously stochastic. The intention of various researchers (cf Hansen and Sargent, 2001; Dennis, 2007 and Svensson and Williams, 2007) is to develop a monetary policy that is effective under this type of model uncertainty. The focus of our paper is to review a completely different aspect of uncertainty, i.e. the way economic subjects and firms, being well aware of the risk of shocks in the economy, will take uncertainty into account in their consumption and production planning, and the resulting impact on welfare and the conduct of monetary policy.

To start with, the fact that higher statistical moments of the probability distribution of future values of endogenous variables do not enter the model equations (1) and (2) results in a contradiction to their microfoundation: The underlying utility functions of firms and economic subjects are non-linear and typically risk-averse (for details s. below), whereas the model equations (1) and (2) only contain expected values of endogenous variables and thus describe risk-neutral market participants. This wedge is justifiable in a system of linear (or linearized) model equations with exogenous shocks (i.e. the probability distribution representing uncertainty is not determined by the system or the monetary policy function), where statistical moments representing uncertainty are constant and hence can be dropped in the model equations. This simplification represents a common approach to model the economy as an aggregation of the behavior of a large number of individual economic subjects and in many situations is a justified way to derive model equations that allow a meaningful description of macroeconomic dependencies, as can bee seen from the success of the new neoclassical synthesis in recent years. For instance, the role of expectations of forward-looking economic subjects and firms is reflected in the linearized model equations in a meaningful way.

However, the aim of our paper is to investigate in how far economic results have to be modified when the afore-mentioned simplifications are replaced by a more thorough model where uncertainty is determined by the system itself and hence cannot be described by constant terms in the equations for demand and supply. Consequently, higher statistical moments have to enter the resulting model equations to take into account firms and economic subjects, which will no longer be risk-neutral. In this case, economic subjects and firms will form rational expectations not only regarding the expected values of endogenous variables, but also regarding higher statistical moments, taking uncertainty regarding endogenous variables into account in their resource planning and pricing. Further, we will show that risk-averse market participants enable the monetary authority to influence the economy by manipulating the trade-off in the level of uncertainty in future endogenous variables, i.e. the probability distribution experienced by economic subjects becomes a function of the parameters of monetary policy.

To develop model equations taking into account risk-aversion, we extend the existing microfoundation of the new neoclassical synthesis to obtain model equations reflecting the impact of uncertainty on rational economic subjects. We will follow closely the well-known presentation of the new neoclassical synthesis according to Walsh (2003) and show that the understanding of economic subjects and firms of possible future deviations between predicted and actual endogenous variables influences present demand for goods and the present price-setting behavior of firms, in addition to the influence of expected future inflation and output already contained in the linear model equations of the new neoclassical model framework: For example, higher expected future output (and thus income) already increases present demand according to the IS curve (1). Analogously, an increase in inflation expectations increases present inflation according to the IA curve (2).

To be more precise, we will derive non-risk-neutral model equations for demand and supply by extending the model of Walsh (2003) by the following two fundamental assumptions:

1. Costs of re-allocation: Consumers as well as firms form expectations regarding the future value of endogenous variables to make decisions for the allocation of future resources in the present period. If in a future period the actual values of the endogenous variables differ from previous expecta-

tions, economic subjects or firms will (or might be even forced to) change ex-post the allocation of resources performed in the past, which typically incurs frictions costs of re-allocating resources, e.g. a household has to cancel mortgage or leasing contracts, sell investment products, look for a new job in a different location etc. Since economic subjects and firms are aware of possible future costs of changing their allocation plans, they will be risk-averse, i.e. they will prefer a policy regime that protects them from large fluctuations of endogenous variables around their mean values to minimize costs of re-allocation in future periods.

2. Non-linearity of preferences and profits: The utility functions used to describe the behavior of economic subjects are typically concave, which implies a risk-averse behavior when variables entering the utility function are stochastic – the reason being the well-known fact that the expected utility is less than the utility evaluated at the expected value of the stochastic input variables. Thus, economic subjects modeled by a concave utility function will prefer a policy regime that reduces the volatility of endogenous variables around mean values. Similarly, the behavior of firms is described by a maximization of future profits, resulting (as we discuss below) in model equations that are non-linear in the endogenous variables, which again means firms will not be indifferent to uncertainty in endogenous variables.

The inclusion of costs of re-allocation is also considered by other authors. In particular, Smets and Wouters (2003) and (2007) and Christiano, Eichenbaum and Evans (2005) develop a model where adjustments to the utilization of the capital stock of households incurs costs of re-allocation and where the empirically observed persistence of consumption is introduced into the model framework by external habit formation. However, the authors follow the common practice of using a log-linearized model framework where uncertainty does not appear in the form of volatilities of stoachastic variables. Our description of costs of re-allocation is different in two aspects: First of all, the persistence of consumption is not introduced by external habit formation but is explained by the costs economic subjects face when they have to alter long-term consumption plans, such as leasing or mortgage contracts and hence provides an alternative explanation for households' consumption persistence. Secondly, we include costs of re-allocation into the model for both private households on the demand side and for firms on the supply side.

To conclude, our goal is to derive model equations for demand and supply as well as an economic loss function in a similar way as in the setup of the new neo-classical synthesis, but which reflect the aversion of economic subjects and firms regarding uncertainty in future values of endogenous variables. Consequently, the model equations will contain a measure of uncertainty for endogenous variables that (as we will discuss below) can be influenced by monetary policy. Thus, monetary authorities will not only be concerned about stabilizing economic subjects' expectations regarding future inflation and output, but also about the uncertainty in these variables inherent in the underlying system and perceived by economic subjects.

As an introductory example, we briefly re-visit the concept of a risk-averse utility function to illustrate the impact uncertainty can have on the planning of consumption within a very simple two-period model. The purpose of the example is to illustrate the impact of the costs of re-allocation after shocks and non-linearities in the decision rules of economic subjects on their optimal resource allocation. Afterwards, we discuss a more general multi-period model setup that is consistent with the microfoundation used within the new neoclassical synthesis to derive model equations for demand and supply in section two. On the demand side, we will show that increasing uncertainty regarding future income or inflation reduces current demand for goods, reflecting the risk-aversion of economic subjects in their consumption planning. On the supply side we will demonstrate that increasing uncertainty regarding future demand reduces the average productivity of firms and thus reduces the natural output level, as uncertainty results in mis-investments of firms and thus an inefficient resource allocation. Further, we show that uncertainty regarding future inflation creates incentives for firms to "over-price" their products and thus creates inflationary pressures, due to the non-linearity of firms' profit functions. Section three derives the corresponding economic loss function. We will show that the loss function is based on the economic loss commonly used within the setup of the new neoclassical synthesis, but contains additional terms representing losses due to the adverse impact of uncertainty on the productivity of firms. The conduct of monetary policy under uncertainty is developed in section four – we will see that the monetary authority faces a trade-off between minimizing the afore-mentioned welfare losses due to uncertainty of future output and uncertainty regarding future inflation. The conclusion in section five contains a summary of key findings and an outlook on future research.

## A *Introductory Example (maybe dropped if paper too long)*

In our introductory example, we analyze an economic subject in a very simple two-period model setup. We let $u(C_t)$ denote the utility of an economic subject at time $t = 1, 2$, depending (to start with) only on the household's consumption $C_t$. To include uncertainty into the discussion, we assume that income in period one (i.e. the present period) is known (fixed), whereas in the second period an unpredictable income shock $\epsilon$ to the previously expected income $Y_2$ can occur, not allowing the economic subject to allocate efficiently the consequences of the shock between period one and two, as the shock in period two was not foreseen in period one. In the two-period model, the shock has to be absorbed in the second period, whereas in the multi-period model derived below shocks can be re-allocated optimally among all future periods.

Regarding income shocks, we want to take into account possible costs that occur if an economic subject has to re-allocate its consumption or investment plans if a significant unexpected income decrease occurs. For instance, an economic subject has to cancel mortgage or leasing contracts, sell investment products etc. to compensate the adverse income shock, thus incurring costs. To be precise, if an unexpected favorable income shock $\epsilon > 0$ occurs in the second period, no costs of re-allocation are assumed necessary and thus the budget of the economic subject is increased by the same amount $\epsilon$. However, in case an adverse income shock occurs $\epsilon < 0$, the economic subject will face additional costs for re-allocating its budget, resulting in an effective budget decrease of $g(\epsilon) \leq \epsilon$, i.e. the effective budget impact $g(\epsilon)$ can be summarized as follows:

$$g(\epsilon) = \begin{cases} \epsilon & \text{if } \epsilon \geq 0; \\ \leq \epsilon & \text{if } \epsilon < 0. \end{cases} \quad (3)$$

Thus, the optimization problem of the economic subject reads:

$$\begin{aligned} \text{Max } U &= u(C_1) + \beta E_\epsilon u(C_2 + g(\epsilon)) \quad (4) \\ \text{Budget constraint } (Y_1 - C_1)(1 + r) &= C_2 - Y_2 \end{aligned}$$

Here future utility is discounted with $\beta < 1$. The budget constraint shows that the amount of income $Y_1 - C_1$ not spent in the first period can then be used for consumption in the second period including interest earned $r$, where the second period is described by an expected income $Y_2$, planned (expected) consumption $C_2$ and actual consumption $C_2 + g(\epsilon)$.

Further, the income shock does not appear in the inter-temporal budget constraint but directly in the utility function of the second period to deprive the household of the possibility to distribute the income shock optimally between the two periods[1]. The optimization (4) is used by the economic subject in period one (the present period) to plan its path of consumption for both periods. To perform the optimization (4), we expand the utility of the second period in terms of $\epsilon$:

$$\begin{aligned} U &= u(C_1) + \beta E_\epsilon \left( u(C_2) + u'(C_2)g'(0)\epsilon + \frac{1}{2}\left(u''(C_2)g'(0)^2 + u'(C_2)g''(0)\right)\epsilon^2 \right) \\ &= u(C_1) + \beta u(C_2) + \underbrace{\frac{1}{2}\left(u''(C_2)g'(0)^2 + u'(C_2)g''(0)\right)}_{=:Z(C_2)} \sigma_\epsilon^2 \end{aligned} \tag{5}$$

with $\sigma_\epsilon$ denoting the standard-deviation of $\epsilon$ and $E_\epsilon \epsilon = 0$. With the usual concavity assumptions $u_C > 0$, $u_{CC} < 0$ representing the risk-aversion of the economic subject and the properties $g' > 0$ and $g'' \leq 0$ following from equation (3), we conclude that

$$Z(C_2) < 0, \tag{6}$$

i.e. the expected utility is reduced by shocks (although we assumed $E_\epsilon \epsilon = 0$), which is due to the concavity of the utility function. Consequently, the expected utility of the second period can be approximated as:

$$u_2(C_2) = u(C_2) + Z(C_2)\sigma_\epsilon^2 \tag{7}$$

Solving the optimization problem (4) yields the standard condition of equal marginal utility of both periods:

$$\frac{u'(C_1)}{u_2'(C_2)} = \frac{u'(C_1)}{u'(C_2) + Z'(C_2)} = \beta(1+r) \tag{8}$$

[1]In the multi-period model introduced below, the period-t income shock will appear in the budget-constraint between period $t$ and $t+1$, i.e. the household can shift the impact of the shock into the future but not into the past.

Using a period utility of a form commonly used within the new neoclassical synthesis (cf McCallum and Nelson, 1999; Walsh, 2003), i.e.

$$u(C) = \frac{\sigma}{\sigma - 1} C^{\frac{\sigma-1}{\sigma}} \Longrightarrow u''' = \frac{\sigma + 1}{\sigma^2} C^{-\frac{2\sigma+1}{\sigma}} > 0 \tag{9}$$

implies $Z_C > 0$, i.e. the marginal utility of the second period increases due to uncertainty. To be precise, the marginal utility of the second period (7) reads:

$$u_2'(C_2) = u'(C_2) + \frac{1}{2}\left(u_2'''(C_2)g'(0)^2 + u''(C_2)g''(0)\right)\sigma_\epsilon^2 \tag{10}$$

$$= C_2^{\frac{-1}{\sigma}}\left(1 + \left(\underbrace{\frac{\sigma+1}{2\sigma^2}g'(0)^2\frac{1}{C_2^2} - \frac{1}{2\sigma}g''(0)\frac{1}{C_2}}_{=:K(C_2)>0}\right)\sigma_\epsilon^2\right) \tag{11}$$

Thus, we can expect a shift of consumption from period one to period two compared with the allocation of consumption without uncertainty, as additional consumption in period two has become relatively more attractive. Taking the log of equation (8) and using $c_i = \log C_i$ we receive using the approximation $\log(1 + x) \sim x$:

$$\log u'(C_1) = \log u_2'(C_2) + \log\beta + r \tag{12}$$

$$\Longrightarrow \frac{-1}{\sigma}c_1 = \frac{-1}{\sigma}c_2 + \log(1 + K(C_2)\sigma_\epsilon^2) + \log\beta + r \tag{13}$$

$$\Longrightarrow c_1 = c_2 - \sigma k(c_2)\sigma_\epsilon^2 - \sigma\log\beta - \sigma r \tag{14}$$

with the following definition:

$$k(c_2) := K(C_2 = e^{c_2}) = \frac{\sigma+1}{2\sigma^2}e^{-2c_2}g'(0)^2 - \frac{1}{2\sigma}g''(0)e^{-c_2} > 0 \tag{15}$$

Performing the standard steps to extend (14) to a demand function for output $y$, we obtain an IS curve for an arbitrary period $t$ of the form:

$$y_t = E_t y_{t+1} - \tilde{k}\sigma_\epsilon^2 - \sigma r_t \tag{16}$$

Here we used a constant coefficient $\tilde{k}$ for the uncertainty term $\sigma_\epsilon^2$. Equation (16) is a simple forward-looking IS curve with uncertain future income, where the concavity of the underlying period utility implies a behavior of economic subjects that is known as risk-aversion. To be precise, as in the new neoclassical synthesis present demand increases with future income expectations, but equation (16) additionally decreases with the level of future income uncertainty $\sigma_\epsilon^2$. Hence, consumption is shifted from the first to the second period when uncertainty is taken into account, since the marginal utility of the second period is increased – the reason being that additional consumption in the second period is an "insurance" against the downside risk of shocks. This effect is even enhanced when possible costs for re-allocating resources after shocks are taken into account, represented by the term $g''(0)$ in equation (10).

As mentioned before, in case of purely exogenous shocks the parameter $\sigma_\epsilon$ in equation (16) is constant and thus can be dropped in the model equations, which is no longer the case for endogenous uncertainty parameters (e.g. when the monetary authority can influence the level of uncertainty), which then have to enter the model equations representing risk-averse economic subjects and firms as we will show below. When the probability distribution of income and inflation shocks can be influenced by monetary policy, rational economic subjects will anticipate this effect and take it into account in their planning.

# II The Model

In the following, we derive the impact of uncertainty on demand and supply and the economic loss function in a more general setup than used in the introductory example.

## A *Model Setup for Demand*

At first, we will generalize and extend the analysis of the introductory example in the following ways to derive a forward-looking risk-averse IS curve:

- Economic subjects are described by a utility function, depending on consumption and real money balances and can hold interest rate bearing bonds.

- We will consider a multi-period model setup, i.e. economic subjects have the possibility to re-optimize their future consumption plan after shocks.

- Since the new neoclassical synthesis contains expected future income and inflation as model parameters, we assume that two types of shocks can occur in a period $t$ that are unforeseen in previous periods: An income shock $\epsilon_t^Y$ and an inflation shock $\epsilon_t^\pi$, i.e. the actual income in period $t$ is $Y_t + \epsilon_t^Y$ and the actual inflation $\pi_t + \epsilon_t^\pi$, with $\pi_t$ and $Y_t$ denoting the previously expected values.

- The existence and consequences of possible shocks is taken into account by economic subjects and firms in their consumption and pricing decisions.

In the following, we consider an economic subject with a period-utility $u_t(C_t, M_t)$ with $t = 0, \ldots, \infty$ depending on consumption $C_t$ and real money balances $M_t$ and the possibility of holding interest-bearing bonds $B_t$ with real interest rate $r_t$. Consumption $C_t$ can either be a single good or a Dixit-Stiglitz aggregate. Thus, the inter-temporal budget-constraint of period $t$ planned in period $t-1$ reads:

$$Y_t - C_t = (1 + \pi_t)M_{t+1} - M_t + \frac{B_{t+1}}{1 + r_t} - B_t \tag{17}$$

Equation (17) basically states that the expected income surplus $Y_t - C_t$ not used for consumption in period $t$ can be used to either increase expected real money balances or bond holdings, where $(1 + \pi_t)$ denotes the amount of money an economic subject has to save at the beginning of period $t$ to own one real money unit at the beginning of period $t+1$, whereas $\frac{1}{1+r_t}$ denotes the price of a bond at time $t$ that pays off one real money unit at time $t+1$. Equation (17) represents the budget-constraint of period $t$ as planned in the previous period $t-1$ based on the economic subject's expectations.

In the following, the planned (expected) values for consumption, income, real money balances and bond holdings are indicated by $C_t$, $Y_t$, $M_t$ and $B_t$, whereas deviations from expected values are explicitly shown as a function of stochastic variables. When an income or inflation shock occurs in period $t$, the economic subject can re-calculate the optimal path of future consumption, real money balances and bond holdings given the values of present income and inflation in period $t$. Extending equation (17), the budget constraint taking into account an inflation shock $\epsilon_t^\pi$ and an income shock $\epsilon_t^Y$ at period $t$ reads:

$$\begin{aligned} Y_t + \epsilon_t^Y - C_t &= (1+\pi_t+\epsilon_t^\pi)M_{t+1} - M_t + \frac{B_{t+1}}{1+r_t} - B_t \qquad (18) \\ \Longleftrightarrow Y_t - C_t &= (1+\pi_t)M_{t+1} - M_t + \frac{B_{t+1}}{1+r_t} - B_t + \epsilon_t \qquad (19) \end{aligned}$$

with the aggregated budget shock

$$\epsilon_t := \epsilon_t^\pi M_{t+1} - \epsilon_t^Y \qquad (20)$$

Equation (17) denotes the budget planned at time $t-1$, whereas equation (18) denotes the actual budget that the economic subject faces at time $t$ including shocks that were not known in the previous period. The inter-temporal character of equation (18) enables the economic subject to distribute the two shocks into the future. As can be seen from equation (19), both the income and the inflation shock impact the disposable income in the budget constraint and can therefore be aggregated into one budget shock $\epsilon_t := \epsilon_t^\pi M_{t+1} - \epsilon_t^Y$, which we will do in the following to simplify notation. Further, as in the introductory example, we will assume that the economic subject will face possible re-allocation costs in case the budget shock is unfavorable as indicated in equation (3). Thus, we will modify the budget constraint (19), replacing the aggregated income shock $\epsilon_t$ by the effective budget impact $g(\epsilon_t)$ to obtain:

$$Y_t - C_t = (1+\pi_t)M_{t+1} - M_t + \frac{B_{t+1}}{1+r_t} - B_t + g(\epsilon_t) \qquad (21)$$

In the following, we assume that a budget shock $\epsilon_0$ occurs at time $t = 0$ and we calculate the way the economic subject re-optimizes its consumption plan for all periods $t \geq 0$, resulting in adjusted values $C_t(\epsilon_0)$, $M_t(\epsilon_0)$ and $B_t(\epsilon_0)$ for planned consumption, real money balances and bond holdings. In appendix A we show that after re-allocating the shock optimally the utility can be approximated in the following way:

$$E_\epsilon u_t(C_t(\epsilon_0), M_t(\epsilon_0)) \approx u_t(C_t(0), M_t(0)) + Z_t^0 \sigma_\epsilon^2 \tag{22}$$

for all periods $t \geq 0$. The result is analogous to our introductory example (5) – the main difference is the fact that the economic household can distribute the shock over consumption, money and bond holdings between the present and all future periods. The coefficient $Z_t^0 < 0$ denotes the impact of shocks at period zero on the period $t \geq 0$. The budget uncertainty is represented by the term

$$\sigma_\epsilon^2 = E(\epsilon_0^Y + M_1 \epsilon_0^\pi)^2 = \left(\sigma^Y\right)^2 + M_1^2 \left(\sigma^\pi\right)^2 \tag{23}$$

which we derived using equation (20) with $(\sigma^Y)^2$ and $(\sigma^\pi)^2$ denoting the expected future uncertainty of output and inflation. Further, we assumed stochastic independence between inflation and income shocks and volatilities $\sigma^Y$ and $\sigma^\pi$ constant in time to simplify notation. Summing up all future periods we obtain:

$$E_\epsilon U(\epsilon_0) = E_\epsilon \sum_{t=0}^{\infty} \beta^t u_t(C_t(\epsilon_0), M_t(\epsilon_0)) \approx U(0) + Z^0(\sigma_Y^2 + M_1^2 \sigma_\pi^2) \tag{24}$$

$$\text{with } Z^0 := \sum_{t=0}^{\infty} \beta^t Z_t^0 \quad \text{and} \quad U(0) = \sum_{t=0}^{\infty} \beta^t u_t(C_t(0), M_t(0)) \tag{25}$$

Equation (24) can be interpreted as a Taylor-curve, showing the trade-off regarding output- and inflation-uncertainty (volatility) in the utility function of economic subjects. The expected impact of possible period-0 shocks indicated by equation (24) is anticipated by rational economic subjects in period $t = -1$ and taken into account in their consumption plan, thus entering the IS curve. Consequently, analogous to the IS curve (16) based on the simple example (5), we obtain an IS curve that contains the uncertainty regarding inflation and output as position parameters with a more complicated coefficient due to the possibility of re-distributing shocks into the future. Put simply, we can expect a forward-looking IS curve, which we write in the standard form for an arbitrary period $t$ and explicitly denoting expected future variables with the expectation operator $E_t$:

$$y_t = E_t y_{t+1} - \tilde{k}((\sigma^Y)^2 + E_t M_{t+1}^2 (\sigma^\pi)^2) - \sigma r_t \tag{26}$$

Hence, present demand is reduced when economic subjects are uncertain about future income or future inflation. As we discuss below, uncertainty in inflation and output is determined by the economic system and the conduct of monetary policy.

So far we have considered shocks occurring at time $t = 0$ that were not foreseen at time $t = -1$. In principle, at time $t = -1$ an economic subject can expect shocks to occur in all future periods. Thus, for each shock $\epsilon_s = \epsilon_s^\pi M_{s+1} - \epsilon_s^Y$ occurring in a period $s$ impacting the budget-constraint of period $s$, the economic subject can re-calculate the optimal future path of endogenous variables, impacting the expected utility analogous to equation (22) with a coefficient $Z_t^s$ denoting the impact of shocks at time $s$ on a future period $t \geq s$, where we assume constant shock volatilities as mentioned above. Thus, the total impact of potential future shocks that are unforeseen at period $t = -1$ reads:

$$EU = U(0) + \sum_{s=0}^{\infty} \beta^s \sum_{t=s}^{\infty} \beta^{t-s} Z_t^s \left( (\sigma^Y)^2 + M_{s+1}^2 (\sigma^\pi)^2 \right) \tag{27}$$

Equation (27) is more complex than (24), where only shocks in the present period are taken into account, but results in an IS curve analogous to (26).

## B *Model Setup for Supply*

The IA curve within the new neoclassical synthesis is based on the Calvo model of inflexible prices with firms acting under monopolistic competition. We follow closely the derivation of the IA curve according to Walsh (2003) with the intention of elaborating the effects of costs of re-allocation after the occurrence of inflation and output shocks and non-linearities in the decision rules of firms, analogous to the demand side.

In the model economy, firm $j$ produces good $c_{jt}$ in period $t$ using the production technology $c_{jt} = a_t N_{jt}$ with labor force $N_{jt}$ and productivity $a_t$. Further, marginal costs of production are assumed to equal the real wage $W_t/P_t$. In the model setup presented in Walsh (2003), the demand for good $c_{jt}$ is assumed to be known in advance to ensure that the necessary amount of labor $N_{jt} = c_{jt}/a_t$ is employed in time, which is in fact an unrealistic simplification, since companies typically operate under uncertainty regarding future demand for their goods but have to make investment and employment decisions beforehand. This implies the risk of over- or underestimating future demand and thus being over- or underinvested. Hence, as for the demand side described above, we would like to incorporate uncertainty regarding both future demand and inflation into the behavior of rational forward-looking firms.

In order to take into account uncertainty regarding future demand and the rigidity of firms' resources, we extend the model in the following way: In period $t$ each firm $j$ estimates the demand $c^e_{jt+1}$ for its good in the following period $t+1$. Based on this estimate the firm employs the required amount of labor in period $t$ for the following period $N_{jt+1}$. In period $t+1$ the firm faces an effective demand $c_{jt+1}$ that differs from the previously expected demand by a random variable $\epsilon_{jt+1}$:

$$c_{jt+1} = c^e_{jt+1} + \epsilon_{jt+1} \tag{28}$$

In the new neoclassical model (cf Walsh, 2003) the demand for labor $N_{jt+1}$ and the marginal production costs were given by

$$N_{jt+1} = \frac{c^e_{jt+1}}{a_{t+1}} \tag{29}$$

$$\phi^0_{t+1} = \frac{W_{t+1}}{P_{t+1}a_{t+1}} \tag{30}$$

and were perfectly predictable. However, taking into account the uncertainty condition (28) this remains true only if actual demand equals predicted demand. If actual demand turns out to be below expectations, firms employed more labor than necessary according to equation (29), resulting in higher marginal costs (marginal costs and unit costs coincide in our model setup). In the worst case (if corrective measures cannot be taken in period $t+1$), the marginal costs are increased by a factor $\frac{c^e_{jt+1}}{c_{jt+1}} > 1$. On the other hand, if demand turns out to be higher than expected, the employed resources are insufficient to produce the amount of demanded goods. We assume that firms can hire additional resource in period $t+1$ in the very short term at higher costs. To conclude, in both cases the actual marginal costs will be a function of the expectation error with the minimum at $\epsilon_{jt+1} = 0$ given by equation (30). In the following we assume that the marginal costs are a smooth function of $\epsilon_{jt+1}$ and thus can be expanded around $\epsilon_{jt+1} = 0$ up to second order terms:

$$\phi_{t+1}(\epsilon) = \phi^0_{t+1}(1 + \phi^1_{t+1}\epsilon^2_{jt+1}) \tag{31}$$

Here $\phi^0_{t+1}$ is given by (30) and $\phi^1_{t+1} > 0$ denotes the sensitivity of wage costs with respect to unpredicted changes in demand. Consequently, the expected marginal costs taken into account by firms in their pricing decisions (calculated as average over all firms in the economy in a given period, assuming that the expectation error $\epsilon_{jt+1}$ of each firm is idiosyncratic with the same volatility $\sigma^2_\epsilon$) read

$$\phi_{t+1} = \phi^0_{t+1}(1 + \phi^1_{t+1}\sigma^2_\epsilon) \tag{32}$$

where we assumed a constant variance $\sigma^2_\epsilon$ of the shock $\epsilon_{jt+1}$ and $E\epsilon_{jt+1} = 0$. Having derived a model for the costs of re-allocating resources after the occurrence of unforeseen shifts in demand, the next step is to quantify the impact on the supply side of the economy. Therefore, we recall and extend the relationship between real marginal cost and productivity:

$$\phi_{t+1} = \phi^0_{t+1}(1 + \phi^1_{t+1}\sigma^2_\epsilon) = \frac{W_{t+1}}{P_{t+1}a_{t+1}}(1 + \phi^1_{t+1}\sigma^2_\epsilon) = \frac{W_{t+1}}{P_{t+1}\frac{a_{t+1}}{1+\phi^1_{t+1}\sigma^2_\epsilon}} \tag{33}$$

Equation (33) implies that uncertainty regarding future demand results in an increase in real marginal costs and thus a decrease of productivity. This result is economically appealing, because uncertainty implies (on average) misinvestments (as described above, firms can be over- or underinvested in labor) and thus lowers labor productivity. The adverse influence of uncertainty on productivity is also reflected in the natural level of output and the demand for labor, as we briefly outline in the following: Per definition the natural output denotes the output with fully flexible prices. In this case firms set profit-maximizing markups, which amount to (neglecting uncertainty)

$$\mu = \frac{1}{\phi_{t+1}} \tag{34}$$

(for details cf Walsh, 2003). According to equation (33) we have:

$$\frac{1}{\mu} = \frac{W_{t+1}}{P_{t+1}\frac{a_{t+1}}{1+\phi^1_{t+1}\sigma^2_\epsilon}} \Longrightarrow \frac{W_{t+1}}{P_{t+1}} = \frac{a_{t+1}}{\mu}\frac{1}{1 + \phi^1_{t+1}\sigma^2_\epsilon} \tag{35}$$

Further, to model the impact of uncertainty on the real wage and on productivity, we have to extend the utility function of an economic subject, taking into account the labor supply $N_t$ of the household, i.e. we have a utility of the form $u(C_t, M_t, N_t)$ with $U_N < 0$, since the household prefers leisure to work. Further, if we assume that the household's real income $Y_t$ equals the real wage earned

$W_t N_t / P_t$, then for the household's optimal allocation of resources the real wage equals the rate of substitution between consumption and leisure $-U_N/U_C$, which yields in combination with equation (35)

$$\frac{-U_N}{U_C} = \frac{W_{t+1}}{P_{t+1}} = \frac{a_{t+1}}{\mu} \frac{1}{1 + \phi^1_{t+1} \sigma^2_\epsilon} \tag{36}$$

Since the model assumes a flexible labor market with flexible wages, equation (36) can be interpreted as the determination of the real wage and consequently of labor demand and output. We conclude that the real wage $\frac{W_{t+1}}{P_{t+1}}$ has to compensate the adverse effect of a demand uncertainty on productivity (i.e. the real wage decreases in $\sigma^2_\epsilon$). Because of the optimal allocation of time to work and leisure, working becomes relatively less attractive and hence labor supply as well as output is reduced.

Further, for the model setup of the new neoclassical synthesis, there is a linear relationship between percentage changes of output and percentage fluctuations in productivity (for a detailed derivation cf Walsh, 2003). Since the reduction of productivity due to shocks is proportional to $\sigma^2_\epsilon$, i.e.

$$\frac{a}{1 + \phi^1 \sigma^2_\epsilon} \approx a(1 - \phi^1 \sigma^2_\epsilon)$$

we conclude that in our model framework the natural output level is an analogous function of uncertainty:

$$\bar{y} = \bar{y}^0 (1 - \bar{y}^1 \sigma^2_\epsilon) \tag{37}$$

with $\bar{y}^0$ denoting natural output in absence of uncertainty and $\bar{y}^1 > 0$ denoting the sensitivity of natural output with respect to uncertainty.

Having derived a model for firms' marginal costs taking into account costs of re-allocation after unpredicted shifts in demand, we would like to derive the impact on the IA curve of the new neoclassical synthesis, where we will take into account non-linearities in the decision rule, analogous to the demand side.

Therefore, we follow closely the derivation of the IA curve based on the Calvo model of inflexible prices presented in Walsh (2003). As for the demand side, taking into account second order terms in the derivation results in terms representing the uncertainty in future values of endogenous variables. As we show in appendix B, the IA curve based on firms that maximize their future profits with uncertain knowledge of future price levels reads:

$$\pi_t = \beta E_t \pi_{t+1} + \varphi x_t + \zeta \sigma_\pi^2 + u_t \tag{38}$$

The IA curve (38) represents the forward-looking pricing behavior of firms, taking into account expected future inflation $E_t \pi_{t+1}$ and the current output-gap $x_t$ into pricing decisions (and thus driving inflation). Compared with the standard IA curve of the new neoclassical synthesis (2), we have obtained an extra term $\zeta \sigma_\pi^2$ with a constant $\zeta > 0$, which is due to the fact that firms' future profits are a non-linear function of the future price level. Analogous to the demand side, uncertain future price levels appearing in the profit equation result in a second order statistical moment in the model equation, the economic reason being that firms determining their price level under uncertain information regarding future inflation tend to "over-price" their products, as explained in appendix B.

The cost-push inflation shock $u_t$ in the IA curve (38) with standard deviation $\sigma_u$ can impact both inflation and output, depending on the conduct of monetary policy, and results in volatile inflation rates with standard deviation $\sigma_\pi$ and volatile output with standard deviation $\sigma_x$. To be precise, the monetary authority faces the problem to distribute shocks $u_t$ between output $x_t$ and inflation $\pi_t$ and hence distributing the uncertainty $\sigma_u$ between $\sigma_\pi$ and $\sigma_x$, as shown below.

It is interesting to analyze the steady-state behavior of the IA curve, i.e. by setting $\pi^e = \pi_t = E_t\pi_{t+1}$ in equation (38) we obtain

$$\pi^e = \frac{\varphi x}{1-\beta} + \frac{\zeta}{1-\beta}\sigma_\pi^2 \tag{39}$$

This means that the long-term Phillips curve (i.e. the long term inflation-output trade-off) is shifted by a term proportional to $\sigma_\pi^2$, resulting in a positive steady-state inflation rate even if output is at the natural level $x = 0$. Thus, the IA curve with uncertainty rids a weakness of the standard IA curve (2) used within the new neoclassical synthesis, which predicts a steady-state inflation rate of zero in absence of shocks, contradicting empirical findings that most modern economies tend to have a positive inflation rate even with output at its natural level (cf Walsh, 2003).

# III Policy Function

A key component for the analysis of monetary policy is the derivation of a global loss function, which is used to assess and compare the appropriateness of different monetary policy regimes from a welfare point of view. Therefore, Woodford (2003) uses a utility function of a representative household with a Dixit-Stiglitz aggregate of consumption, money holding and labor input as a starting point and performs a Taylor-expansion of second order around the equilibrium state to obtain the well-known loss function:

$$L_t := \pi_t^2 + \lambda(x_t - x^*)^2 \tag{40}$$

Here $x_t = y_t - \bar{y}_t$ represents the difference between actual and natural output level and $x^* = y^e - \bar{y}_t$ denotes the difference between efficient output $y^e$ and natural output. As we showed in equation (37), natural output is no longer constant, but a function of the volatility in output, because unexpected shifts in output reduce the average productivity of firms. The output volatility is not exogenous but determined by the monetary policy parameters and the underlying model equations, as we discuss below. Since natural output depends on output volatility, whereas the efficient output is constant (the efficient output is defined as the output with an optimal allocation of resources, i.e. when firms are not over- or underinvested due to uncertainty), the same dependency holds for the gap $x^*$, i.e. we have:

$$x^* = y^e - \bar{y}_t = x_0^* + x_1^* \sigma_x^2 \tag{41}$$

where we used the volatility of output as a proxy for the uncertainty regarding future demand and with $x_1^* > 0$, as higher output uncertainty widens the gap between efficient and natural output level. Thus, we can conclude that we obtain an extended loss function of the form

$$L_t := \pi_t^2 + \lambda(x_t - x_0^* - x_1^* \sigma_x^2)^2 \tag{42}$$

Hence, compared with the standard loss function with constant $x^*$, equation (42) additionally takes into account losses due to a decrease in average productivity when future demand for goods is uncertain, because firms have invested inefficiently – the more the higher the value of $\sigma_x$.

# IV Monetary policy under uncertain future inflation and income

After we have derived the impact of uncertain future inflation and income on demand and supply as well as the loss function of the economy (42), we would like to analyze the influence on the conduct of monetary policy. We start with a discretionary regime, before considering simple rule-based policies. In the new neoclassical synthesis, the behavior of the monetary authority is typically analyzed minimizing a loss function (we will use the function (42) in the following) with the IA curve (we use the form (38)) as a period constraint (cf Clarida, Gali and Gertler, 1999; Walsh, 2003; Woodford, 2003).

## A *Discretionary Policy*

In the discretionary regime, the monetary authority minimizes the loss (42) for given values of expected inflation $E_t\pi_{t+1}$. The mathematical problem that occurs when we try to minimize equation (42) is the occurrence of terms representing output and inflation volatility $\sigma_x$ and $\sigma_\pi$, since these volatilities are only known when the discretionary reaction to shocks is known, which itself is a result of the minimization.

To promote an analytic solution, we assume that the monetary authority will conduct its policy in such a way that output reacts linearly to an inflation shock $u_t$ in the IA curve (38), which is in fact the case for the discretionary policy without uncertainty (cf Clarida, Gali and Gertler, 1999), i.e. we assume that:

$$x_t = w_0 + w_1 u_t \tag{43}$$

with constants $w_i$ to be derived. Plugging (43) into (38), the output shock volatility needed in (42) reads

$$\sigma_x^2 = w_1^2 \sigma_u^2 \tag{44}$$

and the resulting inflation shock volatility becomes:

$$\sigma_\pi^2 = (1 + \varphi w_1)^2 \sigma_u^2 \tag{45}$$

which yields the loss function

$$L_t = E\left(\pi_t^2 + \lambda(x_t - x^*)^2\right)$$

$$
\begin{aligned}
&= E\Big((\varphi x_t + \beta E_t\pi_{t+1} + \zeta\sigma_\pi^2 + u_t)^2 + \lambda(x_t - x_0^* - x_1^*\sigma_x^2)^2\Big) \\
&= E\Big((\varphi(w_0 + w_1 u_t) + \beta E_t\pi_{t+1} + \zeta(1+\varphi w_1)^2\sigma_u^2 + u_t)^2 \qquad (46) \\
&+ \lambda(w_0 + w_1 u_t - x_0^* - x_1^* w_1^2\sigma_u^2)^2\Big)
\end{aligned}
$$

Deriving (46) with respect to the unknown parameters $w_0$ and $w_1$ yields two non-linear equations for the determination of these parameters:

$$
\frac{\partial L_t}{\partial w_0} = \frac{\partial L_t}{\partial w_1} = 0
$$

The solution for the two parameters is quite complex and non-linear, but can be expanded in a Taylor-series around the state without uncertainty (i.e. $\zeta = x_1^* = 0$):

$$
\begin{aligned}
w_0 &\approx \frac{\lambda x^* - \varphi\beta E_t\pi_{t+1}}{\varphi^2+\lambda} - \sigma_u^2 \frac{\varphi + \frac{\varphi^5}{(\varphi^2+\lambda)^2} - \frac{-2\varphi^3}{\varphi^2+\lambda}}{\varphi^2+\lambda}\zeta + \sigma_u^2 \frac{\lambda\varphi^2}{(\varphi^2+\lambda)^3} x_1^* \\
w_1 &\approx \frac{-\varphi}{\varphi^2+\lambda} + \frac{2\varphi\lambda(\varphi x_0^* + \beta E_t\pi_{t+1})}{(\varphi^2+\lambda)^3}(\varphi x_1^* - \lambda\zeta)
\end{aligned}
$$

with the first term on the right hand side representing the solution without uncertainty (cf Walsh, 2003). We conclude that the reaction to shocks defined by the coefficient $w_1$ is modified compared with the policy without uncertainty, where the direction of change determined by the term $\varphi x_1^* - \lambda\zeta$ depends on the relative weighting of $x_1^*$ and $\zeta$, i.e. the sensitivity coefficients of uncertainty regarding output volatility and inflation volatility in the model economy. Hence, the optimal discretionary regime is influenced by losses due to inflation uncertainty and output uncertainty in opposite ways: Losses due to output uncertainty would require a stronger stabilization of output at the expense of inflation volatility, whereas the reduction of losses due to inflation uncertainty would require a stronger inflation stabilization – the term $\varphi x_1^* - \lambda\zeta$ shows the trade-off between the two losses. Further, we obtain the long-term inflation expectations $\pi^e$ defined by the assumption $\pi^e = \pi_t = E\pi_{t+1}$ in equation (38):

$$\pi^e \approx \frac{\varphi\lambda x_0^*}{\lambda(1-\beta)+\varphi^2} + \frac{\sigma_u^2\lambda(x_1^*\varphi^3+\zeta\lambda^2)}{(\lambda(1-\beta)+\varphi^2)(\varphi^2+\lambda)^2} \tag{47}$$

With the first term on the right hand side representing the solution in the absence of uncertainty (cf Walsh, 2003). Compared with the discretionary solutions without uncertainty we obtain an additional inflation bias proportional to $\sigma_u^2$, that exists even in case $x^* = 0$, where the inflation bias in the standard model vanishes. The inflation bias in the standard model setup of the new neoclassical synthesis is due to the well-known time-inconsistency of discretionary monetary policy and can be circumvented by rule-based policies. However, the reason for the additional inflation bias appearing in equation (47) is the non-linear profit function of firms, which results in an "over-pricing" and hence an inflation bias when the stochastic character of future inflation is taken into account by firms in their pricing decisions. This inflation bias cannot be avoided by a rule-based policy as we show below.

## B *Rule-based Policy*

As we can see from the IA curve (38), a key question of monetary policy as regards shocks is the trade-off between stabilizing output and inflation, i.e. the monetary authority can decide in how far inflation shocks $u_t$ are absorbed by the inflation rate $\pi_t$ or by output $x_t$. Consequently, a straightforward way of prescribing a rule-based policy is to explicitly define the share $\eta$ of the shock $u_t$ that has to be absorbed by output, as proposed in Clarida, Gali and Gertler (1999):

$$x_t = -\eta u_t \quad \Longrightarrow \quad \sigma_x^2 = \eta^2 \sigma_u^2 \tag{48}$$

Here output is simply counteracting inflation shocks with the factor $\eta > 0$. Plugging (48) into the IA curve (38) yields

$$\pi_t = \varphi x_t + \beta E_t \pi_{t+1} + \zeta \sigma_\pi^2 + u_t = \beta_t \pi_{t+1} + \zeta \sigma_\pi^2 + (1 - \varphi\eta) u_t \tag{49}$$

$$\Longrightarrow \quad \sigma_\pi^2 = (1 - \varphi\eta)^2 \sigma_u^2 \tag{50}$$

The factor $\eta$ determines the distribution of the volatility $\sigma_u$ between $\sigma_x$ and $\sigma_\pi$ and the resulting economic losses due to output uncertainty and inflation uncertainty. The major difference of a rule-based policy to the abovementioned discretionary policy is the fact that the monetary authority minimizes the present value of all future losses and takes into account its influence on expectations. To be more precise, inflation expectations can be found by iterating equation (49) forward in time (where we assume the shocks $u_t$ not to be auto-correlated and thus $Eu_{t+i} = 0$ to simplify notation):

$$E\pi_t = \sum_{i=0}^{\infty} \beta^i E[\varphi x_{t+i} + \zeta \sigma_\pi^2 + u_{t+i}] \tag{51}$$

$$= \sum_{i=0}^{\infty} \beta^i [(1 - \varphi\eta) E u_{t+i} + \zeta \sigma_\pi^2] \tag{52}$$

$$= \frac{\zeta \sigma_\pi^2}{1 - \beta} \tag{53}$$

$$= \quad \frac{\zeta(1-\varphi\eta)^2\sigma_u^2}{1-\beta} \tag{54}$$

Thus, the present value of future economic losses reads using the relationship (41):

$$\begin{aligned}
L &= E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda(x_{t+i} - x^*)^2] \\
&= E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2 - 2\lambda x^* x_{t+i} + \lambda(x^*)^2] \\
&= E_t \sum_{i=0}^{\infty} \beta^i \lambda[(x^*)^2 + 2\eta x^* E_t u_{t+i}] + E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2] \\
&= \frac{\lambda(x^*)^2}{1-\beta} + \sum_{i=0}^{\infty} \beta^i E_t \left( \left( \beta E_t \pi_{t+i+1} + \zeta\sigma_\pi^2 + (1-\varphi\eta)u_{t+i} \right)^2 + \lambda\eta^2 u_{t+i}^2 \right) \\
&= \frac{\lambda(x_0^* - x_1^*\sigma_x^2)^2}{1-\beta} + \sum_{i=0}^{\infty} \beta^i E_t \left( \left( \frac{\zeta(1-\varphi\eta)^2\sigma_u^2}{1-\beta} + (1-\varphi\eta)u_{t+i} \right)^2 + \lambda\eta^2 u_{t+i}^2 \right) \\
&= \frac{\lambda(x_0^* - x_1^*\eta^2\sigma_u^2)^2}{1-\beta} + \frac{\zeta^2(1-\varphi\eta)^4\sigma_u^4}{(1-\beta)^3} + \frac{(1-\varphi\eta)^2\sigma_u^2}{1-\beta} + \frac{\lambda\eta^2\sigma_u^2}{1-\beta}
\end{aligned} \tag{55}$$

Hence, the loss function with uncertainty shows additional terms proportional to powers of $\sigma_u$. The optimal rule defined by the loss-minimizing value of $\eta$ can be obtained by deriving equation (55) with respect to $\eta$:

$$\eta_{opt} = \frac{\varphi}{\varphi^2+\lambda} + \frac{2\varphi\lambda x_0^* x_1^*}{(\varphi^2+\lambda)^2} - \frac{2(\varphi^2(2\varphi^2+\lambda) + x_0^* x_1^* \lambda(10\varphi^2 - 2\lambda) - \lambda^2)\varphi\sigma_u^2}{(\varphi^2+\lambda)^3(1-\beta)^2}\zeta^2 \tag{56}$$

with the first term on the righthand side representing the solution in absence of uncertainty as found in Clarida, Gali and Gertler (1999). As for the discretionary regime, the conduct of monetary policy is influenced by inflation uncertainty and output uncertainty in opposite ways: The optimal value of $\eta$ increases in $x_1^*$, i.e. the sensitivity of the economy regarding output uncertainty and hence making output stabilization more attractive. At the same time, $\eta$ decreases with $\zeta$, i.e.

the sensitivity regarding inflation uncertainty, making inflation stabilization increasingly attractive. Since the effect of inflation uncertainty is quadratic in the inflation sensitivity coefficient $\zeta$ but linear in the output sensitivity coefficient $x_1^*$, at least for low levels of sensitivity we can expect output stabilization to become more attractive than without taking uncertainty into account.

## C *Credibility Shocks*

The model we derived for demand and supply contains the volatilities of inflation and output expected and used by economic subjects and firms in their decision rules. Further, in our analysis of discretionary or rule-based monetary policy, we assumed that economic subjects and firms are fully rational regarding their expectations of future output and inflation uncertainty, i.e. they build their uncertainty expectations using the underlying economic system, resulting in expectations regarding output and inflation uncertainty (44) and (45) in the discretionary regime and (48) and (50) in the rule-based regime. Henceforth, expected volatilities coincide with the volatilities in the underlying model equations.

So far, we have only assumed shocks to the expected values of output and inflation, i.e. in a given period the actual values of output and inflation can deviate from previous expectations, whereas the expected output and inflation volatility were constant and thus always correct (i.e. coinciding with the volatilities in the underlying model equations). However, to be consistent regarding the role of expectations and shocks, we have to allow shocks to both expected mean values and expected volatilities around mean values. For instance, a shock to output uncertainty would mean that economic subjects would suddenly feel less secure about the path of future income and use a higher volatility $\sigma_x^2$ than prescribed by the economic model equations in their decision making process, resulting in a reduction of present demand according to the IS curve (1). A similar shock can occur to the expected inflation uncertainty in the IA curve (2), pushing present inflation upwards. These shocks could be characterized as credibility shocks, since in such a scenario the economic subjects' trust in the ability of the monetary authority to stabilize the fluctuations of endogenous variables at the level predicted by the model equations of the economic system deteriorates, which would result in a decrease of present demand in the IS curve and an increase of present inflation in the IA curve.

As regards the conduct of monetary policy, it is important for the monetary authority to be able to distinguish between shocks to endogenous variables and credibility shocks, since they require different types of monetary reactions. For instance, according to the IA curve (38) a sudden increase in inflation could either be due to a cost-push shock $u_t$ or to a credibility shock expressed as a sudden increase of $\sigma_\pi$. The cost-push shock would typically cause a monetary reaction (i.e. an interest rate increase), distributing the impact of the shock between variations in output and inflation, as discussed before. However, if inflation is caused by an increase of inflation uncertainty $\sigma_\pi$ used in the pricing decision of firms, the monetary authority does not face a trade-off between inflation and output stabilization, but can decrease inflationary pressures by taking measures to increase its credibility. For instance, the monetary authority can make its forecasts about the future path of endogenous variables and its decisions and reactions to the predicted states of the economy more transparent to reduce risks for firms adjusting their prices and thus reducing firms' incentives for "over-pricing" their products.

An important field for future research is the question how monetary authorities can distinguish between shocks to endogenous variables and credibility shocks and the design of a monetary policy regime that responds optimally to both types of shocks.

# V Conclusion

The model equations of the new neoclassical synthesis are derived from a log-linearization and hence reflect a risk-neutral behavior of demand and supply. Although this is a justifiable simplification for discussing a large variety of economic problems, it is worth analyzing the model implications when the afore-mentioned simplification is replaced by a model including the risk-averse behavior of economic subjects and firms. Our analysis is based on the basic assumption that uncertainty expressed as deviations between actual and previously predicted variables causes economic losses for both demand and supply and has to be taken into account by monetary authorities. Our model is based on three fundamental assumptions:

- Firms as well as economic subjects face costs of re-allocating resources when unpredicted changes in endogenous variables occur. Such frictions are completely neglected in the standard model setup of the new neoclassical synthesis. Further, the costs of re-allocation of economic subjects we introduced provide an alternative explanation for the empirically observed consumption persistence of households to the model of external habit formation.

- The underlying decision rules of firms and economic subjects used in the microfoundation are typically non-linear and thus optimal decisions cannot be described by expected future values of endogenous variables only, but have to take into account uncertainty in future variables. The standard model of the new neoclassical synthesis is based on a log-linearization yielding risk-neutral model equations for demand and supply, because they contain expected future variables only. This is a justified simplification to describe a huge variety of ecnomic dependecies, as can be seen from the progress in research in the area of new neoclassical synthesis in recent years. However, risk-neutrality contradicts the underlying utility and pricing models used within the microfoundation and is in contrast to the empirical observation of typically risk-averse market participants.

- The probability distributions of future values of endogenous variables are not exogenous (as it is typically assumed when discussing model uncertainty within the standard model of the new neoclassical synthesis), but determined by the economic system itself, especially the conduct of monetary policy.

The model we presented taking into account uncertainty in the microfoundation of economic subjects and firms shows several results not contained in the linear model equations of the new neoclassical synthesis: First of all, the level of uncertainty in endogenous variables is taken into account by economic subjects when planning future paths for consumption, money and bond holdings and by firms when making pricing decisions. Hence, economic subjects are forward-looking not only regarding expected values of future inflation and output but also regarding uncertainty in inflation and income. As a consequence, present demand (IS curve) is shifted into future periods when economic subjects face uncertainty regarding their future income, the reason being the concavity of the utility function resulting in a risk-averse behavior of economic subjects. As regards inflation (IA curve), uncertainty can explain a positive inflation bias even where $x^* = 0$ (efficient natural output), which prevails under both discretionary and rule-based policies. This inflation bias exists in addition to the inflation bias occurring in the standard model of the new neoclassical synthesis due to the well-known time-inconsistency problem, which can be avoided by a rule-based policy. The reason for this additional inflation bias is the fact that profit-maximizing firms, being aware of uncertain future prices, tend to "over-price" their products compared to the standard Calvo model. Consequently, the model we presented can explain positive inflation rates even in situations where output is at its natural level and monetary policy is rule-based, which is in accordance with empirical observations, since most modern economies show a positive inflation rate throughout the business cycle (cf Walsh, 2003).

Moreover, deviations between actual and predicted values cause economic losses due to inefficient consumption planning and resource allocation appearing in the loss function of the economy, which have to be taken into account by policymakers when searching for the optimal monetary policy from a welfare point of view. Both consumers and firms prefer a regime of low levels of uncertainty to minimize frictions due to the mis-allocation of resources.

As regards the conduct of monetary policy, policy makers can influence the behavior of economic subjects and firms not only by manipulating their expectations about future income and inflation, but also by determining the level of uncertainty in output and inflation that is perceived or anticipated by economic subjects. In our analysis it turns out that the monetary authority faces a trade-off between minimizing losses due to uncertainty in output and uncertainty regarding inflation.

Moreover, in the model we developed, shocks not only appear in the form of deviations between predicted and actual values of endogenous variables, but also regarding the level of uncertainty expected by economic subjects, which we referred to as credibility shocks. In such a scenario, the economic subjects' trust in the ability of the monetary authority to stabilize the fluctuations of endogenous variables at the level predicted by the model equations deteriorates, which results in a decrease of present demand (due to the risk-averse behavior of consumers in the IS curve (26)) and an increase of present inflation (because of the incentives of firms to "over-price" their products as shown in the IA curve (38)). As a consequence, it is essential for the conduct of monetary policy to be able to distinguish between shocks to endogenous variables and credibility shocks, since both require different reactions by the monetary authority and face different trade-offs.

An interesting area of future research would be to discuss monetary rules that respond optimally to both shocks to endogenous variables and credibility shocks and the way monetary authorities are able to distinguish these two types of shocks in their empirical research. The latter is especially important since misinterpretations of the causes for shocks would provoke inefficient reactions of the monetary authority.

# Appendix A Multi-period model with uncertainty

To calculate the impact of shocks on the planning of an economic subject in a certain period, we will make the following assumptions to simplify notation without loss of generality: Expectations are made at time $t = -1$. At time $t = 0$ an inflation shock $\epsilon_0^\pi$ and an income shock $\epsilon_0^Y$ occur, which are then distributed optimally among the present and all future periods $t \geq 0$. Thus, the optimization problem expressed as Lagrange-function in period $t = 0$ when the shocks occur reads:

$$L := \sum_{t=0}^{\infty} \beta^t \left( u(C_t, M_t) + \lambda_t [Y_t + \delta_{t0} g(\epsilon_0) - (C_t + (1+\pi_t)M_{t+1} - M_t + \frac{B_{t+1}}{1+r_t} - B_t)] \right) \tag{57}$$

with

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{58}$$

We would like to understand the influence of the two shocks summarized in $\epsilon_0 = \epsilon_0^\pi M_1 - \epsilon_0^Y$ at time $t = 0$ on the optimal future path of consumption, real money and bond holding, i.e. $(C_t(\epsilon_0), M_t(\epsilon_0), B_t(\epsilon_0))$. The optimality conditions derived from (57) are:

$$\begin{aligned} u_M(C_{t+1}(\epsilon_0), M_{t+1}(\epsilon_0)) &= ((1+\pi_t)(1+r_t) - 1) u_C(C_{t+1}(\epsilon_0), M_{t+1}(\epsilon_0)) \\ u_C(C_t(\epsilon_0), M_t(\epsilon_0)) &= \beta(1+r_t) u_C(C_{t+1}(\epsilon_0), M_{t+1}(\epsilon_0)) \\ Y_t + \delta_{t0} g(\epsilon_0) &= C_t(\epsilon_0) + (1+\pi_t) M_{t+1}(\epsilon_0) - M_t(\epsilon_0) + \frac{B_{t+1}(\epsilon_0)}{1+r_t} - B_t(\epsilon_0) \end{aligned}$$

Further, we assume a separable utility function $u_{cM} \equiv 0$. Deriving the optimality conditions with respect to $\epsilon_0$ yields[2]:

$$\begin{aligned}
u_{MM}^{t+1} M_{t+1}' &= [(1+\pi_t)(1+r_t)-1] u_{cc}^{t+1} C_{t+1}' \\
&\implies M_{t+1}' = \frac{(1+\pi_t)(1+r_t)-1}{u_{MM}^{t+1}} u_{cc}^{t+1} C_{t+1}' \qquad (59)\\
u_{cc}^t C_t' &= \beta(1+r_t) u_{cc}^{t+1} C_{t+1}' \implies C_{t+1}' = \frac{u_{cc}^t}{\beta(1+r_t)u_{cc}^{t+1}} C_t' \qquad (60)\\
0 &= \delta_{t0} g'(0) - C_t' - (1+\pi_t) M_{t+1}' + M_t' - \frac{B_{t+1}'}{1+r_t} + B_t' \qquad (61)
\end{aligned}$$

Equations (59) and (60) can be combined to:

$$M_{t+1}' = \frac{[(1+\pi_t)(1+r_t)-1] u_{cc}^t}{\beta(1+r_t) u_{MM}^{t+1}} C_t' \qquad (62)$$

Plugging (59) and (60) into (61) yields:

$$\begin{aligned}
&\delta_{t0} g'(0) + C_t' \underbrace{\left( \frac{(1+\pi_{t-1})(1+r_{t-1})-1}{u_{MM}^t} u_{cc}^t - (1+\pi_t) \frac{(1+\pi_t)(1+r_t)-1}{\beta(1+r_t) u_{MM}^{t+1}} u_{cc}^t - 1 \right)}_{:k_t} \\
&- \frac{B_{t+1}'}{1+r_t} + B_t' = 0 \qquad (63)
\end{aligned}$$

Relationship (63) can be used to calculate the impact of the (aggregated) shock $\epsilon_0$ on present and future periods. For $t = 0$ the amount of bonds is given by the investment decision in period $t = -1$, thus we have $B_0' = 0$. Consequently equation (63) yields:

$$B_1' = (1+r_0)(g'(0) + k_0 C_0') \qquad (64)$$

For period $t = 1$ we conclude:

$$k_1 C_1' - \frac{B_2'}{1+r_2} + B_1' = 0$$

[2]In the following, we use the abbreviations: $\frac{dC_t(\epsilon_0)}{d\epsilon_0} =: C_t'$ and $u_{cc}(C_t(\epsilon_0), M_t(\epsilon_0)) =: u_{cc}^t$ etc.

$$\begin{aligned}
\Longleftrightarrow k_1 \frac{u_{cc}^0}{\beta(1+r_1)u_{cc}^1} C_0' + (1+r_0)(g'(0)+k_0 C_0') - \frac{B_2'}{1+r_2} &= 0 \\
\Longleftrightarrow (1+r_2)\left(k_1 \frac{u_{cc}^0}{\beta(1+r_1)u_{cc}^1} C_0' + (1+r_0)(g'(0)+k_0 C_0')\right) &= B_2'
\end{aligned}$$

Equation (63) can be iterated into the future to obtain $B_t'$ for all future periods $t = 1..\infty$. To sum up, we get the following result:

- At the time of the shock $t = 0$ the household changes its consumption in the same period expressed by the derivative $C_0'$.

- The utility-optimal adjustment of real money balances in period $t$ is given by equation (59).

- The part of the shock $\epsilon_0$ that is not absorbed by the adjustment of consumption and real money holding in period $t = 0$ is shifted into future periods by an adjustment of the amount of bonds held by the economic subject, expressed by equation (64).

- In the optimal regime, the impact of the shock at time $t = 0$ on the consumption of all future periods is given recursively by condition (60), where all $C_t'$ have the same sign as $C_0'$. The adjustment of real money holding for all future periods is given analogously by equation (59) – once again the sign of the change is the same for all periods.

- Hence, analogous to the simple introductory example approximated in equation (5), the corresponding utility $u_t(C_t, M_t)$ for the present and all future periods $t \geq 0$ can be expressed by a Taylor-expansion for $\epsilon_0$, where the coefficients of the Taylor polynomial have the same sign in all periods. Thus, performing the same steps as for the simple two period model (5) we get an expression of the form

$$E_\epsilon u_t(C_t(\epsilon_0), M_t(\epsilon_0)) \approx u_t(C_t(0), M_t(0)) + Z_t^0 \sigma_\epsilon^2 \tag{65}$$

for each period $t \geq 0$.

# Appendix B Derivation of the IA curve

In the Calvo model we refer to in the following, in each period a fixed percentage $1 - \omega$ of randomly chosen firms can adjust their prices, the remaining firms are bound to the prices of the previous period. When firms are able to adjust prices, they maximize the present value of future profits, where a future period $i$ is discounted with $\Delta_{i,t+i}$ and weighted with the probability $\omega^i$ of not being able to adjust the price set today within the next $i$ periods. Following closely the derivation of the new IA curve in Walsh (2003) the expected future profit reads:

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \phi_{t+i} c_{jt+i} \right] \\ = \quad & E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\Theta} - \phi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\Theta} \right] C_{t+i} \end{aligned}$$

Here we used the demand for the composite good $c_{jt+i}$ of the Dixit-Stiglitz aggregate $C_{t+i}$ according to Walsh (2003):

$$c_{jt+i} = \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\Theta} C_{t+i}$$

Deriving the present value of future profits with regards to the price $p_{jt}$ to determine the optimal price $p_t^*$ chosen by all firms adjusting prices in the present period yields:

$$
\begin{aligned}
& E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1-\Theta) \left( \frac{p_t^*}{P_{t+i}} \right)^{-\Theta} \frac{1}{P_{t+i}} + \phi_{t+i} \Theta \left( \frac{p_t^*}{P_{t+i}} \right)^{-\Theta-1} \frac{1}{P_{t+i}} \right] C_{t+i} = 0 \\
\Longrightarrow \quad & E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\Theta} \left[ \left( \frac{p_t^*}{P_{t+i}} \right) (1-\Theta) + \phi_{t+i} \Theta \left( \frac{p_t^*}{P_{t+i}} \right)^{-\Theta} \frac{1}{p_t^*} \right] C_{t+i} = 0 \\
\Longrightarrow \quad & p_t^* E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\Theta} (1-\Theta) \left( \frac{1}{P_{t+i}} \right)^{1-\Theta} C_{t+i} \\
= \quad & -E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\Theta} C_{t+i} \phi_{t+i} \left( \frac{1}{P_{t+i}} \right)^{-\Theta} \Theta \\
\Longrightarrow \quad & Q_t := \frac{p_t^*}{P_t} = \frac{\Theta}{\Theta - 1} \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\Theta} \phi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\Theta} C_{t+i}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\Theta} \left( \frac{P_{t+i}}{P_t} \right)^{\Theta-1} C_{t+i}}
\end{aligned}
\tag{66}
$$

Here we used the definition of the discount factor $\Delta_{i,t+i} = \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\Theta}$ according to Walsh (2003). In the following, we will approximate all future endogenous variables around the steady-state, treating them as random variables under the expectation operator $E_t$. For the price level we make use of the fact that at time $t$ the average price-level is the weighted average of the price $p_t^*$ chosen by $1 - \omega$ percent of the firms that can adjust their price in period $t$ and the price level of the previous period, maintained by the remaining $\omega$ percent of firms that cannot adjust their prices in period $t$. To be precise, we have (for details cf Walsh, 2003)

$$
\begin{aligned}
P_t^{1-\Theta} &= (1-\omega)(p_t^*)^{1-\Theta} + \omega P_{t-1}^{1-\Theta} \\
\Longrightarrow 1 &= (1-\omega) Q_t^{1-\Theta} + \omega \left( \frac{P_{t-1}}{P_t} \right)^{1-\Theta}
\end{aligned}
\tag{67}
$$

(68)

Expanding (67) around the steady-state $Q_t = p_t^*/P_t = 1$ up to second order terms yields (with $\hat{q}$ denoting percentage changes of $Q$ and the inflation rate $\pi_t = P_t/P_{t-1}$):

$$\hat{q}_t = \frac{\omega}{1-\omega}\pi_t + \frac{\omega(\Theta - 2(1-\omega))}{2(1-\omega)^2}\pi_t^2 \tag{69}$$

The main difference between our model (69) and the original model of Walsh (2003) is the fact it contains second order terms in the approximation to take into account the non-linearity of firms' pricing decision. Moreover, analogous to the derivation of the IA curve provided by Walsh (2003) we expand consumption $C_t$, the price level $P_t$ and the marginal costs $\phi_{t+i}$ around the steady-state characterized by $Q_t = \mu\phi = 1$, where we include second order terms for price level and inflation neglected in the original derivation cited above. Consequently, equation (66) can be approximated as:

$$\begin{aligned}
& \left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)(1+\hat{q}_t) + C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i[(1-\sigma)E_t\hat{c}_{t+i} + (\Theta-1)(E_t\hat{p}_{t+i} - \hat{p}_t) \\
+ \quad & \frac{1}{2}\Theta(\Theta-1)(E_t\hat{p}_{t+i} - \hat{p}_t)^2] \\
= \quad & \mu\frac{C^{1-\sigma}}{1-\omega\beta}\phi + \mu\phi C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i[E_t\hat{\phi}_{t+i} + (1-\sigma)E_t\hat{c}_{t+i} + \Theta(E_t\hat{p}_{t+i} - \hat{p}_t) \\
+ \quad & \frac{1}{2}(\Theta-1)(\Theta-2)(E_t\hat{p}_{t+i} - \hat{p}_t)^2] \\
\Longrightarrow \quad & \frac{1}{1-\omega\beta}\hat{q}_t = \sum_{i=0}^{\infty}\omega^i\beta^i[E_t\hat{\phi}_{t+i} + E_t\hat{p}_{t+i} - \hat{p}_t + (1-\Theta)(E_t\hat{p}_{t+i} - \hat{p}_t)^2] \\
\Longrightarrow \quad & \hat{q}_t + \hat{p}_t = (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i[E_t\hat{\phi}_{t+i} + E_t\hat{p}_{t+i} + (1-\Theta)(E_t\hat{p}_{t+i} - \hat{p}_t)^2] \\
\Longrightarrow \quad & \hat{q}_t + \hat{p}_t = (1-\omega\beta)(\hat{\phi}_t + \hat{p}_t) + \omega\beta(E_t\hat{q}_{t+1} + E_t\hat{p}_{t+1}) \\
\Longrightarrow \quad & \hat{q}_t = (1-\omega\beta)\hat{\phi}_t + \omega\beta(E_t\hat{q}_{t+1} + E_t\hat{p}_{t+1} - \hat{p}_t)
\end{aligned}$$

$$= \quad (1-\omega\beta)\hat{\phi}_t + \omega\beta(E_t\hat{q}_{t+1} + E_t\pi_{t+1})$$

Using (69) to substitute $\hat{q}$ yields:

$$\begin{aligned}
\frac{\omega}{1-\omega}\pi_t &= (1-\omega\beta)\hat{\phi}_t + \omega\beta\left[\left(1+\frac{\omega}{1-\omega}\right)E_t\pi_{t+1}\right] + \frac{\omega(\Theta-2(1-\omega))}{2(1-\omega)^2}(\omega\beta E_t\pi_{t+1}^2 - \pi_t^2) \\
\Longrightarrow \pi_t &= \underbrace{\frac{(1-\omega)(1-\omega\beta)}{\omega}}_{=:\kappa}\hat{\phi}_t + \beta E_t\pi_{t+1} + \frac{\Theta-2(1-\omega)}{2(1-\omega)}(\omega\beta E_t\pi_{t+1}^2 - \pi_t^2) \\
&= \beta E_t\pi_{t+1} + \kappa\hat{\phi}_t + \frac{\Theta-2(1-\omega)}{2(1-\omega)}(\omega\beta(\sigma_\pi^2 - (E_t\pi_{t+1})^2) - \pi_t^2) \qquad (70)
\end{aligned}$$

where we used $E_t\pi_{t+1}^2 = \sigma_\pi^2 + (E_t\pi_{t+1})^2$ and assumed a constant volatility $\sigma_\pi^2$. Equation (70) represents a quadratic equation in $\pi_t$ and $\pi_{t+1}$. Since we are only interested in the link between present inflation $\pi_t$ and expectations regarding future inflation $E_t\pi_{t+1}$ and its uncertainty $\sigma_\pi^2$, we drop the quadratic inflation terms – the qualitative behavior of the system is unchanged by this simplification. Further, if the labor market is flexible, we can express the term $\kappa\hat{\phi}_t$ by a term proportional to the output gap, i.e. $\varphi x_t$ as shown in Walsh (2003), which is a substitution commonly used within the new neoclassical synthesis. Thus, we obtain the IA curve (38) including uncertainty regarding inflation:

$$\pi_t = \beta E_t\pi_{t+1} + \varphi x_t + \zeta\sigma_\pi^2 + u_t \qquad (71)$$

with

$$\zeta = \frac{\Theta-2(1-\omega)}{2(1-\omega)}\omega\beta \qquad (72)$$

As regards the order of magnitude of $\zeta$, matching the IA curve to empirical data (for details cf Woodford, 2003) yields $\varphi \approx 0.024$, which means that $\omega$ is close to one and hence the coefficient $\zeta$ of uncertainty regarding future inflation can be deemed positive. Consequently, present inflation $\pi_t$ is increased by both higher expected future inflation $E_t\pi_{t+1}$ and uncertainty about future inflation, i.e. firms setting prices today tend to increase prices more if they are not certain about their inflation forecast. The reason can be understood by equation (69) – the optimal price chosen is a convex function of inflation. Consequently, when inflation is a stochastic variable, the expected optimal price will be greater than the price formula (69) evaluated at the expected inflation rate:

$$E\hat{q}(\pi_t) > q(\hat{E}\pi_t)$$

Hence, we conclude that profit-maximizing firms tend to "over-price" their products when the future price level is uncertain.

# References

Christiano, Lawrence; Eichenbaum, Martin and Evans, Charlie. *"Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." Journal of Political Economy, February 2005, vol. 113 (1), pp. 1-45.*

Clarida, Richard; Gali, Jordi and Gertler, Mark. *"The Science of Monetary Policy: A New Keynesian Perspective." Journal of Economic Perspectives, December 1999, 37(4), pp. 1661-1707.*

Clarida, Richard; Gali, Jordi and Gertler, Mark. *"Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." Quarterly Journal of Economics, February 2000, 115(1), pp. 147-180.*

Dennis, Richard. *"Model Uncertainty and Monetary Policy." Federal Reserve Bank of San Francisco, May 2007, Working Paper 2007-09.*

Fendel, Ralf. *"New directions in stabilization policies." BNL Quarterly Review no. 231, December 2004, pp. 365-394.*

Gali, Jordi. *"New perspectives on monetary policy, inflation and business cycle." M. Dewatripont, L. Hansen and S. Turnovsky eds, Advances in Economic Theory, vol. III, 2000, Cambridge University Press, Cambridge, pp. 51-197.*

Gali, Jordi; Gertler, Mark and Lopez-Salido, David. *"Markups, Gaps, and the Welfare Costs of Business." the Review of Economics and Statistics, November 2007, 89(1), pp. 44-59.*

Goodfriend, Marvin. *"Monetary Policy in the New Neoclassical Synthesis: A Primer." Federal Reserve Bank of Richmond Quarterly, September 2002, pp. 21-45.*

Hansen, Lars Peter and Sargent, Thomas. *"Robust Control and Model Uncertainty." American Economic Review, May 2001, pp. 60-66.*

King, Robert. *"The New IS-LM Model: Language, Logic and Limits." Federal Reserve Bank of Richmond Quarterly, June 2000, pp. 45-103.*

McCallum, Bennett and Nelson, Edward. *"An optimizing IS-LM specification for monetary policy and business cycle analysis." Journal of Money, Credit, and*

*Banking, August 1999, pp. 296-316.*

Smets, Frank and Wouters, Raf. *"An estimated dynamic stochastic general equilibrium model of the Euro area" Journal of the European Economic Association, May 2003, 1(5), pp. 1123-1175.*

Smets, Frank and Wouters, Raf. *"Shocks and frictions in US business cycles: A* Bayesian DSGE approach." *American Economic Review, June 2007, 97(3), pp. 586-606.*

*Svensson, Lars and Williams, Noah.* "Bayesian and Adaptive Optimal Policy under Model Uncertainty." *CFS Working Paper No. 2007/11*, February 2007.

*Walsh, Carl. "Monetary Theory and Policy."* 2nd Edition, *MIT Press Cambridge , 2003.*

*Woodford, Michael. "The Taylor Rule and Optimal Monetary Policy." American Economic Review,* January 2001, *91(2), pp. 232-237.*

*Woodford, Michael.* "Interest and Prices." *Princeton University Press*, *2003.*