A concise proof of Gaussian smoothing

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Abstract

The nonlinear Rauch-Tung-Striebel smoother is derived by a gaussian assumption. Using the general properties of conditional expectations and variances, and the theorem on normal correlation, recursive formulas for the conditional mean and covariance matrix are obtained. Furthermore, in the nongaussian case, coupled smoother equations are derived.

Keywords

Nonlinear smoothing
Conditional expectations
Gaussian assumption
Theorem on normal correlation

1 Introduction

The purpose of this note is to present a method of obtaining filtering and smoothing results based on general properties of conditional expectations and variances. Then, concrete results are obtained by inserting gaussian approximations, in which the conditional expectation is a linear function of the conditioning variables and the conditional variance does not explicitly depend on this data (linear regression, theorem on normal correlation; cf. Liptser and Shiryayev (2001, ch. 13)). From this general setup, the well known nonlinear Rauch-Tung-Striebel smoother follows immediately (Rauch et al.; 1965; Särkkä and Hartikainen; 2010). In the nongaussian case, the regression is nonlinear and coupled smoother equations are obtained.

We consider the nonlinear state space model with states $y_k \in \mathbb{R}^p$ and measurements $z_k \in \mathbb{R}^d, k = 0, ..., T$,

$$y_{k+1} = f(y_k, \zeta_k) \quad (1)$$

$$z_k = h(y_k, \epsilon_k). \quad (2)$$

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The error sequences $\zeta_k \sim N(0,I)$, $\epsilon_k \sim N(0,R)$ are assumed to be mutually and temporally independent. We want to derive the fixed interval smoother equations with measurements $z^k = (z_k, ..., z_0)$, $z \equiv z^T$, $k = T-1, ..., 0$,

$$\begin{aligned} E[y_k|z] &= E[y_k|z^k] + G_k\{E[y_{k+1}|z] - E[y_{k+1}|z^k]\} \\ \mathrm{Var}(y_k|z) &= \mathrm{Var}(y_k|z^k) + G_k\{\mathrm{Var}(y_{k+1}|z) - \mathrm{Var}(y_{k+1}|z^k)\}G_k' \end{aligned}$$

with a minimum of assumptions.

2 Gaussian smoothing

2.1 Conditional expectations

First we state general formulas regarding conditional expectations (cf. Rao; 1973, 2b.3)

$$\begin{aligned} E[y] &= E[E[y|x]] \\ \mathrm{Var}(y) &= E[\mathrm{Var}(y|x)] + \mathrm{Var}(E[y|x]), \end{aligned}$$

where y and x are arbitrary random vectors. The second equation is the well known variance decomposition used, e.g., in regression analysis, where the first term on the right side is the 'residual' variance and the second is the 'explained' variance (by the conditioning variable x). More generally, preconditioning on z, one can write

$$E[y|z] = E[E[y|x,z]|z] \tag{3}$$
$$\mathrm{Var}(y|z) = E[\mathrm{Var}(y|x,z)|z] + \mathrm{Var}(E[y|x,z]|z). \tag{4}$$

In the smoother equations, the variables will be chosen as $y = y_k, x = y_{k+1}, z = z^T = (z_T, ..., z_0)$. Thus we have

$$\begin{aligned} E[y_k|z] &= E[E[y_k|y_{k+1}, z]|z] \\ \mathrm{Var}(y_k|z) &= E[\mathrm{Var}(y_k|y_{k+1}, z)|z] + \mathrm{Var}(E[y_k|y_{k+1}, z]|z) \end{aligned}$$

2.2 Markov property

Since y_k is a Markov process, we can write

$$\begin{aligned} E[y_k|y_{k+1}, z^T] &= E[y_k|y_{k+1}, z^k] \\ \mathrm{Var}(y_k|y_{k+1}, z^T) &= \mathrm{Var}(y_k|y_{k+1}, z^k), \end{aligned}$$

$z^k := (z_k, ...z_0)$, dropping measurements from the future (see appendix). Thus we obtain

$$E[y_k|z] = E[E[y_k|y_{k+1}, z^k]|z] \tag{5}$$
$$\mathrm{Var}(y_k|z) = E[\mathrm{Var}(y_k|y_{k+1}, z^k)|z] + \mathrm{Var}(E[y_k|y_{k+1}, z^k]|z). \tag{6}$$

This is the starting point for backward recursions involving $E[y_{k+1}|z]$ and $\mathrm{Var}(y_{k+1}|z)$.

2.3 Gaussian assumption

For gaussian variables, one can express the conditional expectations using the 'theorem on normal correlation', i.e.

$$\begin{aligned} E[y|x] &= E[y] + \text{Cov}(y,x)\text{Cov}(x,x)^{-1}[x - E(x)] \\ \text{Var}(y|x) &= \text{Var}(y) - \text{Cov}(y,x)\text{Cov}(x,x)^{-1}\text{Cov}(y,x)' \end{aligned}$$

(Liptser and Shiryayev; 2001, ch. 13). This is also the best *linear* estimate of y, given x. Note that the conditional variance does not depend explicitly on x (it is not random). Preconditioning on z, one obtains

$$\begin{aligned} E[y|x,z] &= E[y|z] + \text{Cov}(y,x|z)\text{Cov}(x,x|z)^{-1}[x - E(x|z)] \\ \text{Var}(y|x,z) &= \text{Var}(y|z) - \text{Cov}(y,x|z)\text{Cov}(x,x|z)^{-1}\text{Cov}(y,x|z)'. \end{aligned}$$

This is the form of the measurement update in the gaussian filter.

2.4 Smoother recursions

Inserting the variables $y = y_k, x = y_{k+1}, z = z^k = (z_k, ..., z_0)$ in the normal correlation equations, one obtains

$$E[y_k|y_{k+1}, z^k] = E[y_k|z^k] + G_k\{y_{k+1} - E[y_{k+1}|z^k]\} \tag{7}$$

$$\text{Var}(y_k|y_{k+1}, z^k) = \text{Var}(y_k|z^k) - G_k\text{Var}(y_{k+1}|z^k)G_k', \tag{8}$$

where

$$G_k := \text{Cov}(y_k, y_{k+1}|z^k)\text{Var}(y_{k+1}|z^k)^{-1} \tag{9}$$

is the smoother gain. Thus, using the gaussian assumption, the variable y_{k+1} appears linearly on the right hand side, which yields immediately a recursion for $E[y_k|z]$ and $\text{Var}(y_k|z)$. Together with the update equations (5–6), one obtains the nonlinear Rauch-Tung-Striebel smoother

$$\begin{aligned} E[y_k|z] &= E[y_k|z^k] + G_k\{E[y_{k+1}|z] - E[y_{k+1}|z^k]\} \\ \text{Var}(y_k|z) &= \text{Var}(y_k|z^k) - G_k\text{Var}(y_{k+1}|z^k)G_k' + G_k\text{Var}(y_{k+1}|z)G_k'. \end{aligned}$$

With the usual notation $\mu_{k|k} = E[y_k|z^k]$, $\mu_{k+1|k} = E[y_{k+1}|z^k]$, $\mu_{k|T} = E[y_k|z^T]$ etc. this reads

$$\mu_{k|T} = \mu_{k|k} + G_k\{\mu_{k+1|T} - \mu_{k+1|k}\} \tag{10}$$

$$\Sigma_{k|T} = \Sigma_{k|k} - G_k\Sigma_{k+1|k}G_k' + G_k\Sigma_{k+1|T}G_k'. \tag{11}$$

Again, as in eqn. (4), we have the form 'residual variance plus explained variance', where y_{k+1} is the predictor variable. In contrast to Särkkä and Hartikainen (2010), the variance equation (11) is explicitly derived.

2.5 Filter recursions

The terms $\mu_{k|k}, \mu_{k+1|k}, \Sigma_{k|k}, \Sigma_{k+1|k}$ are known from the filter recursions (time update)

$$\begin{aligned}\mu_{k+1|k} &= E[f(y_k, \zeta_k)|z^k]\\ \Sigma_{k+1|k} &= \mathrm{Var}(f(y_k, \zeta_k)|z^k)\end{aligned}$$

and (measurement update, normal correlation)

$$\begin{aligned}\mu_{k+1|k+1} &= \mu_{k+1|k} + K_k(z_{k+1} - E[z_{k+1}|z^k])\\ \Sigma_{k+1|k+1} &= \Sigma_{k+1|k} - K_k \mathrm{Var}(z_{k+1}|z^k)K_k',\end{aligned}$$

with the Kalman gain

$$\begin{aligned}K_k &= \mathrm{Cov}(y_{k+1}, z_{k+1}|z^k)\mathrm{Var}(z_{k+1}|z^k)^{-1}\\ &= \mathrm{Cov}(y_{k+1}, h(y_{k+1}, \epsilon_{k+1})|z^k)\mathrm{Var}(h(y_{k+1}, \epsilon_{k+1})|z^k)^{-1}.\end{aligned}$$

Here, the state space equations (1–2) were inserted. The smoother gain is given explicitly as

$$G_k = \mathrm{Cov}(y_k, f(y_k, \zeta_k)|z^k)\Sigma_{k+1|k}^{-1}.$$

The conditional expectations in the filter and smoother can be evaluated using Taylor expansion (extended and second order Kalman filter) or numerical integration (unscented, Gauss-Hermite or cubature Kalman filter).

3 Nongaussian case

As noted, the general equations (5–6) are basic for the smoothing algorithm. The only assumption involved is the Markov property of the state y_k. In the nongaussian case, the conditional expectation $E[y_k|y_{k+1}, z^k]$ will be a nonlinear function of (y_{k+1}, z^k) and $\mathrm{Var}(y_k|y_{k+1}, z^k)$ explicitly depends on the condition (cf. equations 7–8). For example, the quadratic regression function

$$E[y_k|y_{k+1}, z^k] \approx a + b\, y_{k+1} + c\,(y_{k+1} - \mu_{k+1|T})^2$$

(scalar notation, dropping the z dependence) gives the expression

$$\begin{aligned}\mu_{k|T} = E[y_k|z] &= E[a + b\, y_{k+1} + c\,(y_{k+1} - \mu_{k+1|T})^2 \mid z]\\ &= a + b\,\mu_{k+1|T} + c\,\Sigma_{k+1|T},\end{aligned}$$

which yields a coupling of the backward recursions. Similarly, the variance equation (6) will involve 4th moments which can be factorized approximately.

4 Conclusion

The main advantage of the approach described in this note is the transparent statement of assumptions (Markov property, normal correlation), which together with the general properties of conditional expectations yields a short and complete derivation of the backward smoother equations. In the case of a nonlinear regression, coupled smoother equations are obtained.

Appendix

i) The variance decomposition (4) is given as

$$\begin{aligned}
\mathrm{Var}(y|z) &= E[y^{\otimes 2}|z] - (E[y|z])^{\otimes 2} \\
&= E[E[y^{\otimes 2}|x,z]|z] - (E[E[y|x,z]|z])^{\otimes 2} \\
&= E[\mathrm{Var}[y|x,z]|z] + E[(E[y|x,z])^{\otimes 2}|z] - (E[E[y|x,z]|z])^{\otimes 2} \\
&= E[\mathrm{Var}(y|x,z)|z] + \mathrm{Var}(E[y|x,z]|z),
\end{aligned}$$

where $y^{\otimes 2} = y \otimes y \in \mathbb{R}^{p\times p}$.
ii) The Markov property for

$$E[y_k|y_{k+1}, z^T] = E[y_k|y_{k+1}, z^k],$$

$z^T = (z_T, ..., z_0)$, is proved by considering the state space model (1–2). The future measurements $(z_{k+1}, ..., z_T)$ can be expressed by $(y_{k+1}, ..., y_T)$ and the measurements errors $(\epsilon_{k+1}, ..., \epsilon_T)$. Moreover, $(y_{k+2}, ..., y_T)$ is a function of y_{k+1} and the process errors $\zeta_{k+1}, ..., \zeta_{T-1}$. Thus, the conditional expectation $E[y_k|y_{k+1}, z^k, \epsilon_{k+1}, ..., \epsilon_T, \zeta_{k+1}, ..., \zeta_{T-1}] = E[y_k|y_{k+1}, z^k]$, since y_k does not depend on future error variables. Also we have

$$\begin{aligned}
\mathrm{Var}(y_k|y_{k+1}, z^T) &= E[(y_k - E[y_k|y_{k+1}, z^T])^{\otimes 2}|y_{k+1}, z^T] \\
&= E[(y_k - E[y_k|y_{k+1}, z^k])^{\otimes 2}|y_{k+1}, z^k] \\
&= \mathrm{Var}(y_k|y_{k+1}, z^k).
\end{aligned}$$

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